Operator Learning for Inverse PDE Problems

PDE inverse problems are generally formulated as a constrained optimization problems. Given a computational domain Ω and a parametric PDE

$$\mathscr{A}_a u = f$$

where $\mathscr{A}_a$ is a differential operator parametrized by a function $a(x)$, $u(x)$ is the solution to the PDE, f denotes the source and boundary conditions. The parameter function $a(x)$ encodes desired media knowledge of the computational domain Ω. In practice, a number of simulations are performed experimentally with different source and boundary setups f_i. In each simulation, measurements of the solution $u_i(x; a, f_i)$ are taken on the boundary, and we denote them as $\mathscr{M} u_i$. For convenience, we define the so-called forward operator $\mathscr{F}_a$ that maps sources f to measurements $\mathscr{M} u$. The collections of data form a data set for the inverse problem,

$$\text{Data} = \{(x_i, y_i) \mid x_i = f_i, y_i = \mathscr{F}_a x_i + \varepsilon\}$$

where ε is the additive noise. The main goal of the inverse problem is to reconstruct the unknown parameter function $a(x)$ from a finite data set.

The inverse reconstruction is usually done via a regularized optimization problem,

$$\min_a \frac{1}{n} \sum_{i=1}^{n} \|\mathscr{F}_a x_i - y_i\|^2 + \mathscr{R}(a)$$

where $\mathscr{R}$ is a regularizer function that encodes prior knowledge of the parameter function $a(x)$.

We will test the operator learning towards two inverse problems: the electrical impedance tomography (EIT) and Optical Tomography (OT).