Food Webs Subject To Stoichiometric Constraints

As human activities continue to alter environmental balances and nutrient cycles, it is becoming vital to understand how these changes can impact the environment. Ecological Stoichiometry (ES) offers a conceptual framework to investigate the impact of elemental imbalances on populations while simultaneously considering how population dynamics regulate nutrient cycling. The REU students will develop novel models of food webs that explicitly track essential elements, such as Carbon (C) and Phosphorus (P) across ecological trophic levels.

The theory of ES [11] has deepened understandings of ecological dynamics. This theory considers the balance of chemical elements and how the relative abundance of essential elements in organisms affects ecological dynamics. There are large amounts of data that support ES [1, 3], and a wide variety of stoichiometric food web models [6, 2, 10]. These stoichiometric models incorporate the effects of both food **quantity** and food **quality** into a single framework. Several stoichiometric models incorporate some aspects of community food web structure, including structures of two trophic levels [10], three trophic levels [9], competing primary producers [5, 8] and competing grazers [7]. These existing community structure models provide evidence that stoichiometry can drastically change population dynamics. It may play an important role in explaining biodiversity and can provide mechanisms for deterministic extinction. However, current stoichiometric models lack complexities of community networks and population structures.

The REU team will fill these gaps by formulating new stoichiometric models to investigate how these important aspects of community structure influence population dynamics and nutrient cycling when subject to stoichiometric constraints. The team will build systems of differential equations, incorporating ecological structures subject to stoichiometric constraints, that present worthwhile mathematical challenges, see System 1 for example modeling framework. The team will use analytical analyses to explore the existence and stability of equilibria and periodic solutions, computational tools to conduct parameter sensitivity analyses, and bifurcation theory.

$$\frac{dx}{dt} = bx\left(1 - \frac{x}{\min\{K, (P - \theta_y y - \theta_z z)/q\}}\right) - f(x)y \tag{1a}$$

$$\frac{dy}{dt} = \min\left\{\hat{e}, \frac{Q}{\theta_y}\right\} f(x)y - g(y)z - dy \tag{1b}$$

$$\frac{dz}{dt} = \min\left\{\hat{e}_z, \frac{\theta_y}{\theta_z}\right\} g(y)z - d_z z \tag{1c}$$

Given the incredible degree of biodiversity and complex trophic interactions observed in nature we propose to use ecological networks to incorporate more complex structure into models. Ecological network research can help us understand how complexities observed in nature can persist and influence ecosystem function [4]. We will consider a wide variety of species whose interactions range from generalists to specialists. This will result in a sizable number of models of ecological networks of varying number of species, interactions, and degrees of nestedness. Figure 2 depicts two example ditrophic ecological networks. In this new modeling framework stoichiometric imbalances must be considered between every interaction. We will also consider tritrophic models and use ecological trophic transfer efficiencies as important gauges of ecosystem function [9]. This framework will allow us to investigate how the limitations of essential elements influence and propagate throughout larger community structures, and how stoichiometric constraints

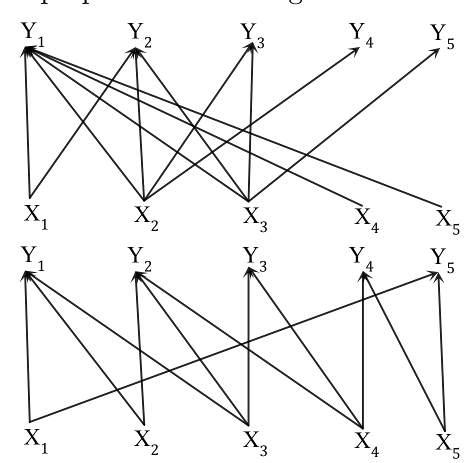

Fig. 1: Nested (top) and unnested (bottom) networks with the same number of species and links.

can disassemble networks and lead to extinction. We will consider the influence of stoichiometric constraints on biodiversity, regions of coexistence, food chain efficiency, and nutrient cycling.

Timeline: Week 1: Learn about ecological communities and review existing models developed in the ES framework, computational training in Matlab. Weeks 2-3: Formulate new models of food webs that include trophic interactions across community networks. Weeks 4-5: Parameterize and validate the model with existing empirical data. Weeks 6-7: Analyze the model analytically with tools from dynamical systems theory, as well as numerically, with MatLab simulations. Week 8: Summarize and interpret results and write paper.

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