Artificial intelligence (AI) based on deep learning (DL) has revolutionized the development of many fields of science and engineering tackling fundamental challenges surprisingly.

Project 1: **Friedrichs learning for Solving High-Dimensional PDEs.**
Friedrichs learning is a novel learning methodology that can learn the weak solutions of challenging PDEs. Unlike strong solutions that are differentiable and satisfy the PDE in common sense, weak solutions are functions for which the derivatives may not all exist but which are nonetheless deemed to satisfy the PDE in some precisely defined sense. These solutions are important because many PDEs in modeling real-world phenomena do not have sufficiently smooth solutions. Inspired by conventional PDE solvers in the weak form and Friedrichs' theory, the project will propose to reformulate the minimization problem in DL for PDEs to a minimax problem to identify weak solutions. Preliminary results for linear elliptic PDEs and div-curl systems have justified the efficiency of the proposed minimax model. The project will investigate the extension to hyperbolic systems and fully nonlinear PDEs.

Project 2: **Friedrichs learning for Data-Driven Recovery and Prediction of PDEs.**
Data-driven recovery and prediction of ordinary differential equations (ODEs)/PDEs via recurrent neural networks (RNNs) have been an active field in recent years with empirical success. The main difference of this project from previous works lies in the fact that the project will learn the governing PDEs from raw data in the weak sense, where the data of the PDE solutions can be heavily noisy and the PDE solutions can be discontinuous. Existing machine learning algorithms in the strong form are not well-developed in these two cases while our novel Friedrichs learning can provide meaningful modeling solutions. Friedrichs learning for PDE recovery and prediction has a wide range of applications, e.g., mathematical biology, plasma physics, and turbulent flow.

Project 3: **Theoretical principles of deep learning in Friedrichs learning.**
The success of DL has not been fully understood, not to mention the application of DL in Friedrichs learning. Given a computational budget, the fundamental questions of DL are: what is the best learning error and how to numerically achieve it? The answers to these questions form the theoretical foundation of DL-based scientific computing including Friedrichs learning. The project will develop approximation, optimization, and generalization theories in the weak formulation to answer these questions. The theories in the weak form require theoretical innovation in the strong form. Therefore, the project will also facilitate the theoretical understanding of existing DL algorithms for mathematical problems in strong form.