Project 3: Einstein paradigm of the non-linear Brownian motion, Theory and application

The project will explore new method of modeling of biological systems based on Einstein paradigm of Brownian motion. In his celebrated thesis Albert Einstein introduced the paradigm of a biological system as a collection of particles. These particles are moving around equilibrium point featured by random variables, such as a length of ”free jumps”. Random process is characterized by two parameters - time interval ”before collision” of particles, and frequency of the free jumps. Based on Einstein transparent ideas we will build models of various biological systems and interpret results of virtual experiments. We will derive system of partial differential equations to model chemotactic processes, evolution of the living organisms, ecological processes, cancer cells proliferation, atherogenesis of the cardiovascular system, virus’s proliferation in urban areas. We will build new models in the form of the coupled systems of non-linear partial differential equations (PDE). Based on the PDE driven models, we will assimilate observed statistical data to saturate parameters of the system in order to interpret important features of the processes which include different traveling bands, index of the cells proliferation, index of the aggregation of the biological particles in create new body, disease proliferation index. We will develop how these characteristics of the processes can be introduced based on PDE driven models. We will use PDE technique to investigate stability of the mathematical models with respect to parameters of the equation and initial data.

Timeline: Week 1: Basic principles of random processes such as expected value, deviations, moments of higher order and their relation to biological processes. Weeks 2-3: Einstein conservation of mass equation for dynamical system. Weeks 4-6: Fundamentals of Taylor series, Caratheodory and Lagrange mean value theorems in order to derive system of partial differential equations. Analytic method to construct solutions which exhibits biological features of interest. Weeks 7-8: Methods for comparison using observed data and indirect measurements of the parameters of biological processes through solution of partial differential equations. Qualitative basics of theory of PDE and energy method. Basics of the Kolmogorov deep learning algorithm.

References

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[3] A. Einstein, *Investigations on the Theory of Brownian Movement*, 1926.

[4] R. Feynman, *The Brownian Movement*, 1964.

[5] V. Brattka, *From Hilbert’s 13th Problem to the theory of neural networks: constructive aspects of Kolmogorov’s Superposition Theorem,* Kolmogorov’s Heritage in Mathematics, Springer, Berlin, Heidelberg, 2007.