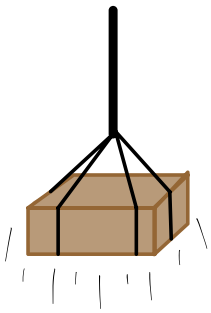
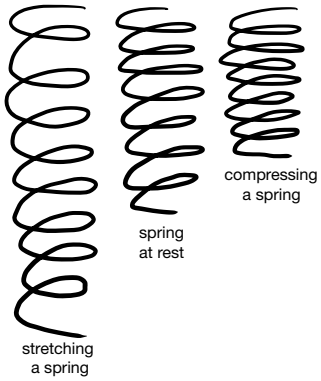
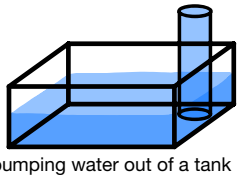
Math 1452: Work Word Problems

What is work? In general, work is a measure of how much effort it takes to apply some force over a certain distance. Below are some illustrations of work:



lifting a box





pumping water out of a tank

How do we calculate work? You may be familiar with the following formula for simple work:

$$\text{Work} = \text{Force} \times \text{Distance}$$

This work is *simple* because the force is constant over the entire distance. If the force changes depending on the distance, we can no longer use multiplication to calculate work. Instead we treat distance as a variable, write the force as a function depending on distance, and add this up using an integral. Remember from Calculus I that an integral is just a summation where the summand changes. Below are formulas for the types of work illustrated above, all of which depend on distance:

Type of Work	Formula for Force
Lifting Objects	$F =$
Using Springs	$F =$
Pumping Liquid	$F =$

Notice that two of the above equations depend on weight density. In some questions, the weight density will be given to you and in others, it is a constant for you to remember.
The weight density of water is 62.4 pounds per cubic foot.

How do I set up my integral? The integrand will be one of the formulas for force listed above using either x or y variables.

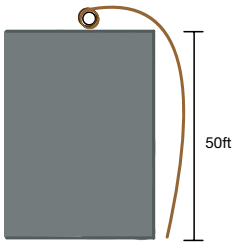
For most of these questions, the lower bound of the integral will be 0 and the upper bound depends on the type of work:

Type of Work	Upper Bound
Lifting Objects	
Using Springs	
Pumping Liquid	

What unit do I use for work? Work is measured in joules, so the answer for each work question will be the evaluation of our integral followed by "joules".

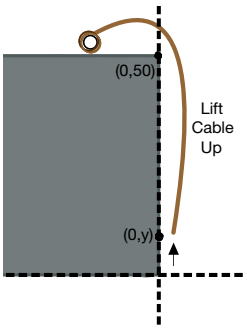
Example 1. A cable is 50 meters long and has a density of 3 kg/m. If the cable is hanging over the side of the building, then what is the work done in pulling the cable up?

Let's start by pulling out all the numbers from the question and drawing a sketch of the scenario:



Quantity	Description
50 meters	
3 kg/m	

1. **Identifying the integrand:** We need to know the **weight density** and the **distance**.
   (a) **Weight Density:**
   (b) **Distance:**



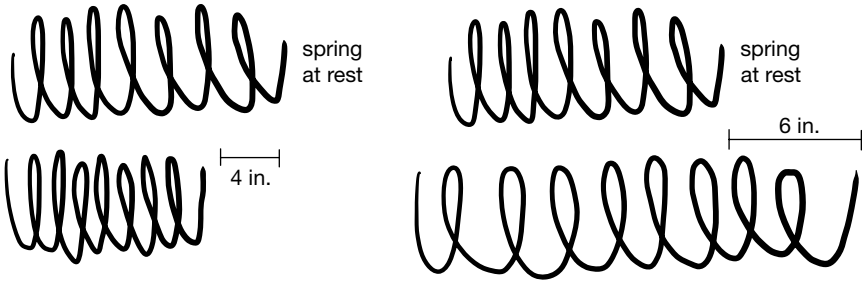
2. **Identifying the bounds:** Since the cable will lift 50 feet, our lower bound is 0 and our upper bound is 50.

Now that we have identified our integrand and our bounds, we set up the integral as

Example 2. A force of 3 lbs is required to hold a spring that has been compressed 4 inches from its natural length. Find the work done in stretching the spring 6 inches from its natural length.

Let's start by pulling out all the numbers from the question and drawing a sketch of the scenario:

Quantity	Description
3 lbs	
4 inches	
6 inches	



1. **Identifying the integrand:** We need to know the **spring constant** and the **distance**.
   (a) **Spring Constant:** For all questions with springs, there will be a sentence in the question that gives the force required to stretch or compress the spring a certain length.

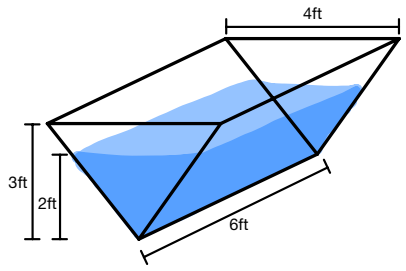
   (b) **Distance:** Since the spring is being stretched and compressed *horizontally*, we will use x as our variable to represent the distance that the spring has been stretched.
2. **Identifying the bounds:** Since the spring will compress 6 inches, our lower bound is 0 and our upper bound is 6.

Now that we have identified our integrand and our bounds, we set up the integral as

Example 3. A tank is shaped like a trough with a triangular base. It is 3 feet high, 6 feet long, and 4 feet wide. It contains water to a depth of 2 feet. Find the work required to pump all the water to the top of the tank.

Let's start by pulling out all the numbers from the question and drawing a sketch of the scenario:

Quantity	Description
3 feet	
4 feet	
6 feet	
2 feet	

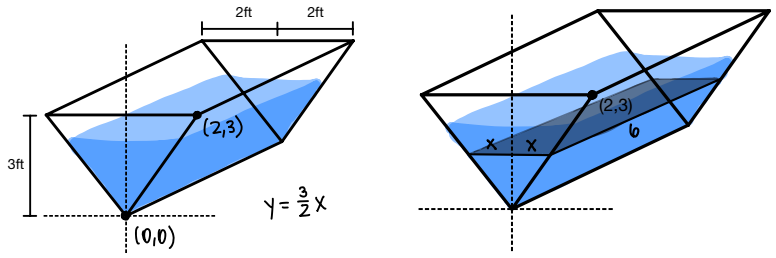


1. **Identifying the integrand:** We need to know the **weight density**, **distance**, and the **area of a slice**.

   (a) **Weight Density:**

   (b) **Distance:**

   (c) **Area of a Slice:**



2. **Identifying the bounds:** Since the water level is 2 feet, our lower bound is 0 and our upper bound is 2.

Now that we have identified our integrand and our bounds, we set up the integral as