Math 1452: Integrating Powers of Trig Functions

A common integral type has the integrand as a power of complementary trig functions, such as the examples below:

$$\int \sin^2(x)\cos^2(x)dx, \quad \int \sec^4(x)\tan^3(x)dx, \quad \int \csc^3(x)\cot(x)dx$$

Each of these integrals can be solved using the integral techniques from Calculus I, using u-substitution and trig identities. Our goal is to factor out one of the following expressions:

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with the remaining trig functions in our integrand having an even power. We will look at each pair of complementary trig functions separately.

Powers of sin(x) and cos(x). These simplifications will depend on if the powers of $\sin(x)$ and $\cos(x)$ are even or odd and will rely on the trig identity $\sin^2(x) + \cos^2(x) = 1$.

i. If $\cos(x)$ has odd power:

ii. If $\sin(x)$ has odd power:

iii. If both of the powers are even:

Powers of sec(x) and tan(x). These simplifications will depend on if the powers of $\sec(x)$ and $\tan(x)$ are even or odd and will rely on the trig identity $\tan^2(x) + 1 = \sec^2(x)$.

a. If $\sec(x)$ has even power:

b. If $\tan(x)$ has odd power:

c. If $\tan(x)$ has even power:

Powers of cot(x) and csc(x). These simplifications work in the exact same manner as the box above after replacing $\sec(x)$ with $\csc(x)$ and replacing $\tan(x)$ with $\cot(x)$, making sure to now use the trig identity $1 + \cot^2(x) = \csc^2(x)$.

Example 1. Evaluate the integral $\int \sin^3(x)\cos^4(x)dx$.

Example 2. Evaluate the integral $\int \sec^4(x)\tan^3(x)dx$.

Example 3. Evaluate the integral $\int \csc^3(x)\cot(x)dx$

There are just some examples of how to apply these integration techniques. The best way to improve your skills at integration is to continue practicing. More examples using these integration techniques can be found here: https://tutorial.math.lamar.edu/Classes/CalcII/IntegralsWithTrig.aspx