Math 1452: Infinite Series Tests

In this worksheet, we will look at different methods to determine converge for infinite series. First, we will look at the four known types of series. Next, we will describe six tests to analyze positive term series. Finally, we will describe four tests to analyze series with positive *and* negative terms.

p-Series	q-Log Series
The general form of this series is $\sum_{k=1}^{\infty} \frac{1}{k^p}$	The general form of this series is $\sum_{k=1}^{\infty} \frac{\ln(k)}{k^q}$
for some power p .	for some power q .
• If $p > 1$, then the series will converge.	• If $q > 1$, then the series will converge.
• If $p \leq 1$, then the series will diverge.	• If $q \leq 1$, then the series will diverge.
Geometric Series	Telescoping Series
The general form of this series is $\sum_{k=1}^{\infty} a_k$	The general form of this series is $\sum_{k=1}^{\infty} a_k$
where $a_k = ar^k$ for some real numbers a and r .	where a_k is a difference of fractions. We expand
• If $r < 1$, then the series will converge to $\frac{a}{1-r}$.	the summation and cancel common terms to find
• If $r \ge 1$, then the series will diverge.	a general formula for the n^{th} parital sum S_n .
	• If $\lim_{n\to\infty} S_n < \infty$, then the series converges.
	• If $\lim_{n \to \infty} S_n = \infty$, then the series diverges.

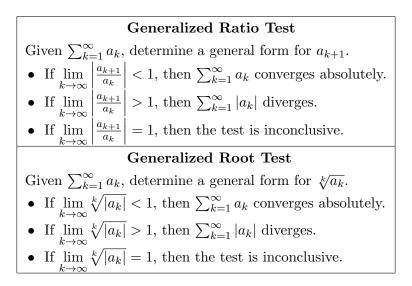
Known Types of Series:

Tests for Positive	Term Series:
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Divergence Test	Integral Test
Consider a series $\sum_{k=1}^{\infty} a_k$.	Consider a series $\sum_{k=1}^{\infty} a_k$.
• If $\lim_{n \to \infty} a_k = 0$, then the test is inconclusive.	• If $\int_1^\infty a(x) dx$ converges, the series converges.
• If $\lim_{n \to \infty} a_k \neq 0$, then the series diverges.	• If $\int_1^\infty a(x) dx$ diverges, the series diverges.
Ratio Test	Root Test
Given $\sum_{k=1}^{\infty} a_k$, find a general form for a_{k+1} .	Given $\sum_{k=1}^{\infty} a_k$, find a general form for $\sqrt[k]{a_k}$.
• If $\lim_{k \to \infty} \frac{a_{k+1}}{a_k} < 1$, then $\sum_{k=1}^{\infty} a_k$ converges.	• If $\lim_{k \to \infty} \sqrt[k]{a_k} < 1$, then $\sum_{k=1}^{\infty} a_k$ converges.
• If $\lim_{k \to \infty} \frac{a_{k+1}}{a_k} > 1$, then $\sum_{k=1}^{\infty} a_k$ diverges.	• If $\lim_{k \to \infty} \sqrt[k]{a_k} > 1$, then $\sum_{k=1}^{\infty} a_k$ diverges.
• If $\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = 1$, then the test is inconclusive.	If $\lim_{k \to \infty} \sqrt[k]{a_k} = 1$, then the test is inconclusive.
Direct Comparison Test	Limit Comparison Test
Given $\sum_{k=1}^{\infty} a_k$, choose a similar series $\sum_{k=1}^{\infty} b_k$.	Given $\sum_{k=1}^{\infty} a_k$, choose a similar series $\sum_{k=1}^{\infty} b_k$.
• If $a_k < b_k$ and $\sum_{k=1}^{\infty} b_k$ converges,	• If $\lim_{k \to \infty} \frac{a_k}{b_k} = 0$ and $\sum_{k=1}^{\infty} b_k$ converges,
then $\sum_{k=1}^{\infty} a_k$ converges.	then $\sum_{k=1}^{\infty} a_k$ converges.
• If $a_k > b_k$ and $\sum_{k=1}^{\infty} b_k$ diverges,	• If $\lim_{k \to \infty} \frac{a_k}{b_k} = \infty$ and $\sum_{k=1}^{\infty} b_k$ diverges,
then $\sum_{k=1}^{\infty} a_k$ diverges.	then $\sum_{k=1}^{\infty} a_k$ diverges.
	• If $\lim_{k \to \infty} \frac{a_k}{b_k} = L$ and $0 < L < \infty$,
	then $\sum_{k=1}^{\infty} a_k$ copies $\sum_{k=1}^{\infty} b_k$.

Tests for a Series with Positive and Negative Terms:

For a series $\sum_{k=1}^{\infty} a_k$ with positive and negative terms, we will check for two types of convergence: absolute and conditional. To check for absolute convergence, we look at the series $\sum_{k=1}^{\infty} |a_k|$ and use any of the tests above or one of the following:



What if $\sum_{k=1}^{\infty} |a_k|$ diverges? In this case, we switch to checking for conditional convergence, which uses one final test described below:

Alternating Series Test Given $\sum_{k=1}^{\infty} (-1)^k a_k$, find a general form for a_{k+1} . If $\lim_{k \to \infty} a_k = 0$ and $a_{k+1} < a_k$, then $\sum_{k=1}^{\infty} a_k$ converges conditionally.