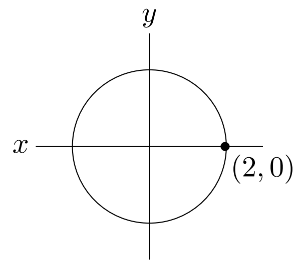
Math 1452: Polar Coordinates and Graphing

What are polar coordinates? Polar coordinates replace the typical (x, y) points with (r, θ) points. Instead of starting from the origin and moving left/right and up/down like the traditional x and y coordinates, polar coordinates pick a certain angle θ on the unit circle and then move r units out in the direction of that angle.

Which functions are more easily represented with polar coordinates? Functions that are curved and symmetric work well in polar coordinates. A circle with radius 2 can be represented by the equation $x^2 + y^2 = 4$ and the graph

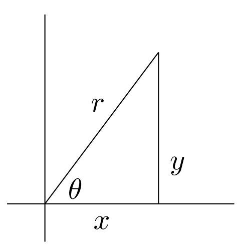


This graph has a radius of 2 at every angle θ, so it is represented by the polar equation $r = 2$.

How can polar coordinates make integral applications easier? At this point in the course, much of our time has been spent using integrals to find area. If we were asked to set up an integral to find the area of the region bounded by $x^2 + y^2 = 4$, we would run into a problem since this area is *neither* vertically simple nor horizontally simple. Instead, we can set up an integral using polar coordinates as

$$\int_0^{2\pi} 2d\theta = 2\theta \Big|_0^{2\pi} = 2 \cdot 2\pi - 2 \cdot 0 = 4\pi$$

How can I convert from rectangular to polar coordinates? We can perform this conversion using a right triangle and SOHCAHTOA. Consider a triangle in the (x, y)-plane below:



$\sin(\theta) = \frac{y}{r}$ $\qquad$ $\cos(\theta) = \frac{x}{r}$ $\qquad$ $\tan(\theta) = \frac{y}{x}$

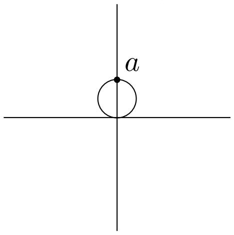
Solving for x and y, we obtain the coordinate conversions $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Using these formulas, we can rewrite the equation $x^2 + y^2 = 4$ as

$$\begin{aligned} x^2 + y^2 &= 4 \\ (r\cos(\theta))^2 + (r\sin(\theta))^2 &= 4 \\ r^2\cos^2(\theta) + r^2\sin^2(\theta) &= 4 \\ r^2\left(\cos^2(\theta) + \sin^2(\theta)\right) &= 4 \\ r^2 &= 4 \\ r &= 2 \end{aligned}$$

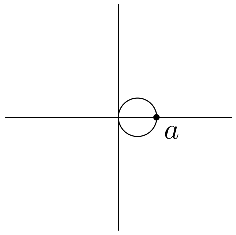
What are some common polar functions? On the next page, we list the different types of polar graphs you may be asked to draw and use. Notice that the equations using $\sin(\theta)$ are centered on the y-axis and the equations using $\cos(\theta)$ are centered on the x-axis.

Circle

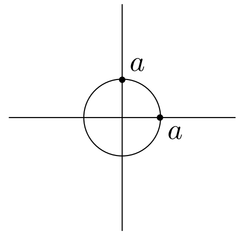
$r = a\sin(\theta)$



$r = a\cos(\theta)$

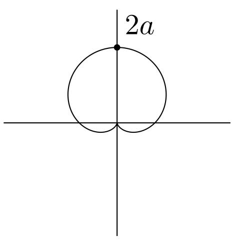


$r = a$

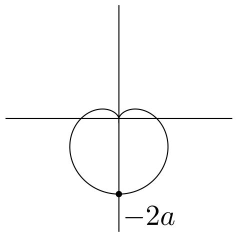


Cardiod

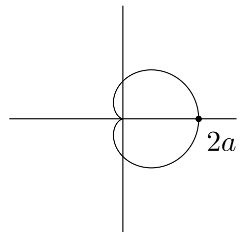
$r = a(1 + \sin(\theta))$



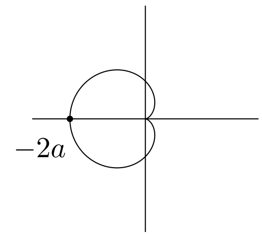
$r = a(1 - \sin(\theta))$



$r = a(1 + \cos(\theta))$



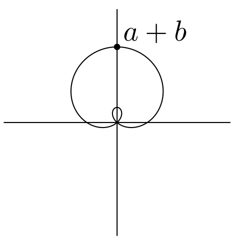
$r = a(1 - \cos(\theta))$



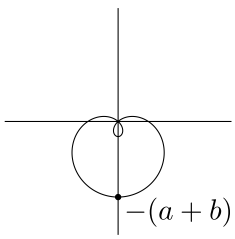
Limacon

$\frac{b}{a} < 1$

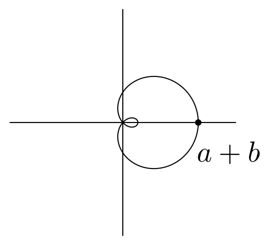
$r = b + a\sin(\theta)$



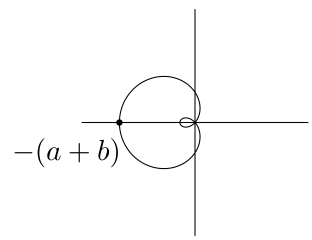
$r = b - a\sin(\theta)$



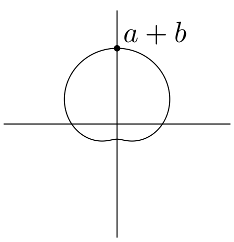
$r = b + a\cos(\theta)$

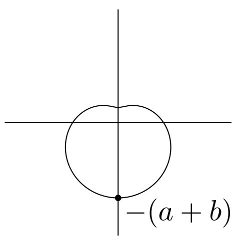


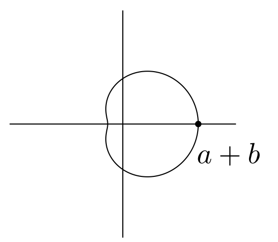
$r = b - a\cos(\theta)$

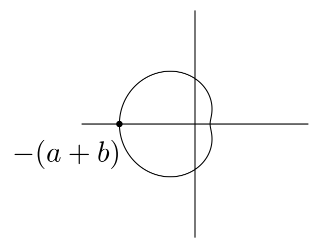


$\frac{b}{a} > 1$



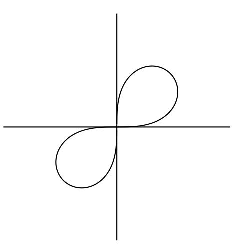




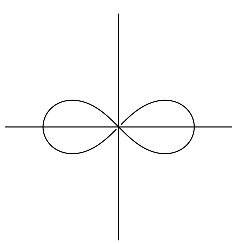


Lemniscate

$r^2 = a^2\sin(2\theta)$



$r^2 = a^2\cos(2\theta)$

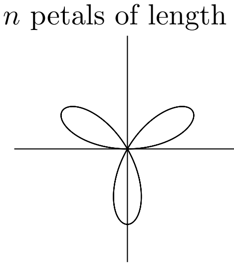


Rose

n is odd

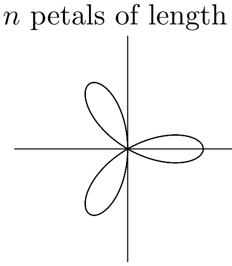
$r = a\sin(n\theta)$

n petals of length a



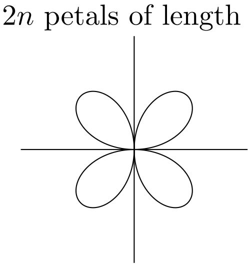
$r = a\cos(n\theta)$

n petals of length a



n is even

$2n$ petals of length a



$2n$ petals of length a