

## Math 1452: Summary of Integration Techniques

**Which integral rules should I have memorized?** To succeed in a typical Calculus II course, you should have the following integral rules memorized:

1.  $\int 0 du = C$
2.  $\int u^n du = \frac{u^{n+1}}{n+1} + C$  for all  $n \neq -1$
3.  $\int u^{-1} du = \int \frac{1}{u} du = \ln |u| + C$
4.  $\int e^u du = e^u + C$
5.  $\int \sin(u) du = -\cos(u) + C$
6.  $\int \cos(u) du = \sin(u) + C$
7.  $\int \tan(u) du = -\ln |\cos(u)| + C = \ln |\sec(u)| + C$
8.  $\int \cot(u) du = -\ln |\csc(u)| + C = \ln |\sin(u)| + C$
9.  $\int \sec(u) du = \ln |\sec(u) + \tan(u)| + C$
10.  $\int \sec^2(u) du = \tan(u) + C$
11.  $\int \sec(u) \tan(u) du = \sec(u) + C$
12.  $\int \csc(u) du = -\ln |\csc(u) + \cot(u)| + C$
13.  $\int \csc^2(u) du = \cot(u) + C$
14.  $\int \csc(u) \cot(u) du = -\csc(u) + C$
15.  $\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin\left(\frac{u}{a}\right) + C = \sin^{-1}\left(\frac{u}{a}\right) + C$
16.  $\int \frac{-1}{\sqrt{a^2 - u^2}} du = \arccos\left(\frac{u}{a}\right) + C = \cos^{-1}\left(\frac{u}{a}\right) + C$
17.  $\int \frac{1}{a^2 + u^2} du = \arctan\left(\frac{u}{a}\right) + C = \tan^{-1}\left(\frac{u}{a}\right) + C$
18.  $\int \frac{-1}{a^2 + u^2} du = \operatorname{arccot}\left(\frac{u}{a}\right) + C = \cot^{-1}\left(\frac{u}{a}\right) + C$
19.  $\int \frac{1}{|u|\sqrt{u^2 - a^2}} du = \operatorname{arcsec}\left(\frac{u}{a}\right) + C = \sec^{-1}\left(\frac{u}{a}\right) + C$
20.  $\int \frac{-1}{|u|\sqrt{u^2 - a^2}} du = \operatorname{arccsc}\left(\frac{u}{a}\right) + C = \csc^{-1}\left(\frac{u}{a}\right) + C$

We can also factor out scalars and integrate term-by-term, which is detailed in the three rules below:

1.  $\int cf(u) du = c \int f(u) du$
2.  $\int f(u) + g(u) du = \int f(u) du + \int g(u) du$
3.  $\int f(u) - g(u) du = \int f(u) du - \int g(u) du$

**What strategies do we have to evaluate integrals?** When an integral is too complicated to immediately solve by an antiderivative rule, we have various choices:

1. Simplify the integral in any way possible.
  - Long division
  - Complete the square
  - Rationalize the numerator
  - Rationalize the denominator
  - Trig identity
2. Perform  $u$ -substitution.
3. Perform integration by parts.
  - $\int u dv = uv - \int v du$
4. Identify the trigonometric form.
  - $\int \sin^m(x) \cos^n(x) dx$
  - $\int \sec^m(x) \tan^n(x) dx$
  - $\int \csc^m(x) \cot^n(x) dx$
5. Identify the radical form and use trig substitution.
  - $\sqrt{a^2 - x^2}$  with substitution  $x = a \sin(\theta)$
  - $\sqrt{a^2 + x^2}$  with substitution  $x = a \tan(\theta)$
  - $\sqrt{x^2 - a^2}$  with substitution  $x = a \sec(\theta)$
6. Perform partial fraction decomposition according to the factorization of the denominator.
  - Distinct linear terms
  - Repeated linear terms
  - Distinct quadratic terms
  - Repeated quadratic terms

Strategies 1 and 2 were covered in Calculus I. Worksheets and videos for how to use strategies 4-6 can be found at the same location you found this worksheet.