Math 1452: Integration by Parts

What is integration by parts? Let's recall the definition of integration by parts:

If $u(x)$ and $v(x)$ are functions with continuous derivatives, then

$$\int udv = uv - \int vdu$$

How do I know which part of my integral is the u and which is the dv? Once you identify that an integral will use integration by parts, there is a heirarchy we can use to identify which part of our integral is the u:

L
I
P
E
T

To identify u, we go down the list and check if our integrand has that type of function. If so, we set that function to be u and the rest of the integrand becomes dv. In the following examples, we identify u and dv for the integrals listed:

$\int xe^x dx$	$\int x^2 \ln(x) dx$	$\int \arcsin(x) dx$	$\int x\sin(x) dx$

In each of the examples above, we can integrate the identified dv using an antidifferentiation rule. This is called *basic* integration by parts and is one of three types of integration by parts, listed below:

1. Basic:
2. Repeated:
3. Circular:

We work an example of each type of integration by parts and then go on to discuss how to apply integration by parts to a definite integral.

Basic Integration by Parts:

Example 1. Evaluate the integral $\int x\sin(x)dx$ using integration by parts.

$u =$ $\quad dv =$

$du =$ $\quad \int dv =$

$v =$

$$\int udv = uv - \int vdu$$

Repeated Integration by Parts:

Example 2. Evaluate the integral $\int x^2 \sin(x)dx$ using integration by parts.

$u =$ $dv =$ $\int udv = uv - \int vdu$

$du =$ $\int dv =$

$v =$

Now we use integration by parts again.

$u =$ $dv =$ $\int udv = uv - \int vdu$

$du =$ $\int dv =$

$v =$

Therefore we have

Circular Integration by Parts:

Example 3. Evaluate the integral $\int e^x \sin(x)dx$ using integration by parts.

$u =$ $dv =$ $\int udv = uv - \int vdu$

$du =$ $\int dv =$

$v =$

Now we use integration by parts again.

$u =$ $dv =$ $\int udv = uv - \int vdu$

$du =$ $\int dv =$

$v =$

We have arrived back at our original integral, so the rest of this question is solved using algebra.