Math 1452: Area and Volume Formulas

How do I find the area between curves?

1. Sketch the region and identify the intersection points of the curves.
2. Determine if the region is vertically or horizontally simple.
3. Write the integrand by identifying...
   (a) the top and bottom functions of the region if it is vertically simple.
   (b) the left and right functions of the region if it is horizontally simple.
4. Write the bounds using...
   (a) the x-values of the intersection points if the region is vertically simple.
   (b) the y-values of the intersection points if the region is horizontally simple.
5. Evaluate the integral to solve.

How do I find the volume of a region using cross sections? Volume by cross sections follows the same steps as finding area, but the integrand is an area formula instead of a length.

How do I find the volume of a region rotated around an axis? There are three options: disks, washers, and shells. If the question leaves the method up to you, you must determine if the region is vertically or horizontally simple and compare the region to the axis of revolution.

	Vertically Simple	Horizontally Simple
Vertical Axis of Revolution	Shells	Disks/Washers
Horizontal Axis of Revolution	Disks/Washers	Shells

Below, we list the formulas on area and volume that you may be asked to know and use:

	Vertically Simple	Horizontally Simple
Area of a Region	$\int_{\text{left}}^{\text{right}} (\text{top} - \text{bottom})dx$	$\int_{\text{bottom}}^{\text{top}} (\text{right} - \text{left})dy$
Volume by Cross Sections	$\int_{\text{left}}^{\text{right}} A(x)dx$	$\int_{\text{bottom}}^{\text{top}} A(y)dy$
square:	$A(x) = (\text{top} - \text{bottom})^2$	$A(y) = (\text{right} - \text{left})^2$
semicircle:	$A(x) = \frac{\pi}{8}(\text{top} - \text{bottom})^2$	$A(y) = \frac{\pi}{8}(\text{right} - \text{left})^2$
equilateral triangle:	$A(x) = \frac{\sqrt{3}}{4}(\text{top} - \text{bottom})^2$	$A(y) = \frac{\sqrt{3}}{4}(\text{right} - \text{left})^2$
circle:	$A(x) = \frac{\pi}{4}(\text{top} - \text{bottom})^2$	$A(y) = \frac{\pi}{4}(\text{right} - \text{left})^2$
Volume by Revolution	$\int_{\text{left}}^{\text{right}} A(x)dx$	$\int_{\text{bottom}}^{\text{top}} A(y)dy$
disks:	$A(x) = \pi(f(x))^2$	$A(y) = \pi(f(y))^2$
shells:	$A(x) = 2\pi x(\text{top} - \text{bottom})$	$A(y) = 2\pi y(\text{right} - \text{left})$
washers:	$A(x) = \pi(R^2 - r^2)$	$A(y) = \pi(R^2 - r^2)$