

Math 1451: Definite Integration by Substitution

In these examples, we will explore two *different* ways to evaluate definite integrals using substitution. Recall that indefinite integration by substitution is defined as follows:

If we are given a composite function $f(x) = g(u(x))$ and if G is an antiderivative of g , then

$$\int f(x)dx = \int g(u) \frac{du}{dx} dx = \int g(u)du = G(u) + C$$

Let's look at the example $\int (4x - 5)^3 dx$.

1. Identify $f(x)$, $g(x)$, and $u(x)$
2. Differentiate $u(x)$
3. Rewrite and integrate

Now, the process changes slightly for definite integrals: Let's look at the example $\int_1^2 (4x - 5)^3 dx$. The first two steps are the same as above, but now we must make a choice in how to rewrite the integral. We can either change the bounds using $u(x)$ or use substitution on an indefinite integral and then evaluate using the original bounds on x .

Integrate $g(u)$ with u bounds

Integrate $f(x)$ with x bounds

Integrate $g(u)$ with x bounds

