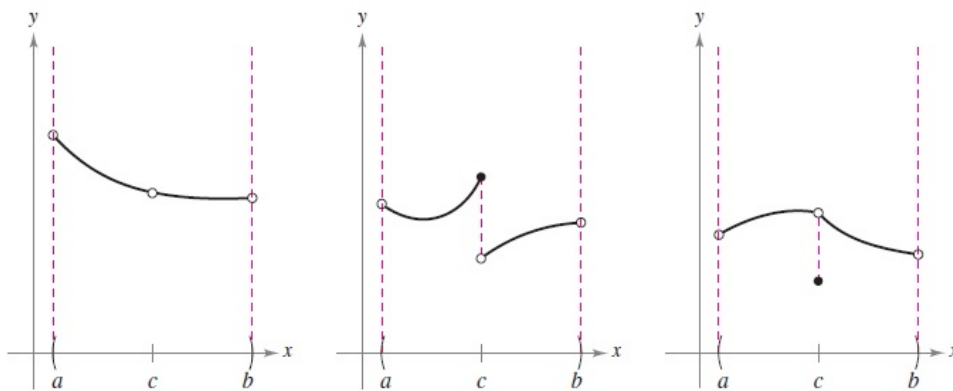


Math 1451: Continuity with Piecewise Functions

What is continuity? A function is continuous at the point $x = c$ if there is no interruption in the graph at that point, i.e., the graph is not broken and there are no holes, jumps, or gaps. The images below represent three cases in which a function is not continuous at $x = c$.



If we do not know the graph of the function, we can use analytical methods to determine continuity. An equivalent definition of continuity is that a function is continuous at $x = c$ if ...

- 1.
- 2.
- 3.

When any of these conditions fail, the function is no longer continuous.

Which functions are continuous? Luckily for us, many of the functions we look at in calculus are continuous, such as polynomial functions, radical functions, and the trigonometric functions $\sin(x)$ and $\cos(x)$. **Which functions are *not* continuous?** Some functions that tend to not be continuous are rational functions, the trigonometric functions $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$, and piecewise functions. In this worksheet, we will look specifically at piecewise functions.

What questions may I be asked about continuity of piecewise functions? There are two main question types you will be asked about continuity of piecewise functions:

- 1.
- 2.

For these questions, it is important to remember that if the function has different behavior to the left and right of the point $x = c$, the limit does not exist. We will make sure the limit exists by computing the lefthand limit and the righthand limit and making sure they are the same.

Example 1. Determine if the function $f(x)$ is continuous. If it is not, state the values of x at which the function is not continuous.

$$f(x) = \begin{cases} x + 1, & x \leq 1 \\ x^2 - 1, & x > 1 \end{cases}$$

We only need to be concerned about continuity at $x = 1$, where the function changes between the two polynomials. We work through the three steps to check continuity:

1. Verify that $f(1)$ is defined.

2. Verify that $\lim_{x \rightarrow 1} f(x)$ exists.

To do this, we take the lefthand limit of the function and righthand limit of the function to verify that they are the same.

3.

Example 2. Determine the value(s) of a that makes the function $f(x)$ continuous.

$$f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$$

We only need to be concerned about continuity at $x = 2$. We solve for the variable a that ensures that $f(x)$ is defined at $x = 2$ and $\lim_{x \rightarrow 2} f(x)$ exists:

1. Verify that $f(2)$ is defined.

2. Verify that $\lim_{x \rightarrow 2} f(x)$ exists.

To do this, we take the lefthand limit of the function and righthand limit of the function to verify that they are the same.

3.