Math 1451: Methods to Evaluate Integrals

What strategies do we have to evaluate integrals? When an integral is too complicated to immediately solve by antidifferentiation, we often have two choices: ***simplify*** the integrand before applying an antidifferentiation rule, or perform u-***substitution***. Sometimes, more than one method may work for us to evaluate the integral. Let's look at an example:

Ex. Evaluate the integral $\int (x+1)^2 dx$.
Simplifying:

$$\begin{aligned}\int (x+1)^2 dx &= \int x^2 + 2x + 1 dx \\ &= \int x^2 dx + 2\int x dx + \int 1 dx \\ &= \frac{1}{3}x^3 + x^2 + x + C\end{aligned}$$

Ex. Evaluate the integral $\int (x+1)^2 dx$.
Substituting:

$$\begin{aligned} u &= x+1 \\ \frac{du}{dx} &= 1 \\ du &= dx \end{aligned}$$

$$\begin{aligned} &\int (x+1)^2 dx \\ =&\int u^2 du \\ =&\frac{1}{3}u^3 + C \\ =&\frac{1}{3}(x+1)^3 + C \end{aligned}$$

Although it may not seem like it at first glance, in this example, both methods give us the same answer. Mutliplying out the answer on the right gives us

$$\frac{1}{3}(x+1)^3 + C = \frac{1}{3}(x^3 + 3x^2 + 3x + 1) + C = \frac{1}{3}x^3 + x^2 + x + C,$$

which is the answer on the left. Let's look at another example:

Ex. Evaluate the integral $\int (x+2)(x^2+4x)dx$.
Simplifying:

$$\begin{aligned}\int (x+2)(x^2+4x)dx &= \int x^3 + 6x^2 + 8x dx \\ &= \frac{1}{4}x^4 + 2x^3 + 4x^2 + C \\ &= \frac{1}{4}(x^4 + 8x^3 + 16x^2) + C\end{aligned}$$

Ex. Evaluate the integral $\int (x+2)(x^2+4x)dx$.
Substituting:

$$\begin{aligned} u &= x^2 + 4x \\ \frac{du}{dx} &= 2x + 4 \\ du &= (2x+4)dx \\ dx &= \frac{du}{2x+4} \end{aligned}$$

$$\begin{aligned} &\int (x+2)(x^2+4x)dx \\ =&\int (x+2)\cdot u \cdot \frac{du}{2x+4} \\ =&\int (x+2)\cdot u \cdot \frac{du}{2(x+2)} \\ =&\int \frac{1}{2}udu \\ =&\frac{1}{4}u^2 + C \\ =&\frac{1}{4}(x^2+4x)^2 + C \\ =&\frac{1}{4}(x^4 + 8x^3 + 16x^2) + C \end{aligned}$$

Again, we can see that both methods give the same answer, once we rewrite each answer slightly. Based on this observation, keep in mind that your answer may look slightly different than an answer someone else gets, but this may just be because you chose to work the question in a different method.