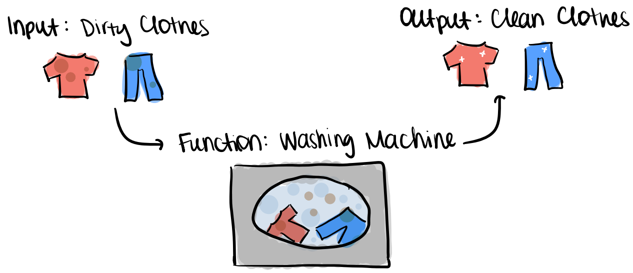
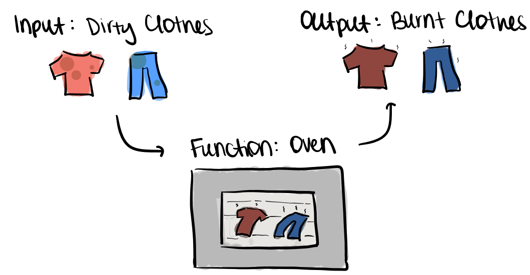
Math 1451: Function Review

What is a function? We can think of a function like a *washing machine*. Using mathematical language, the dirty clothes represent the *input* of the function and the clean clothes represent the *output* of the function. Depending on the function of each machine, the input is transformed in different ways to different outputs. For example, if we input our dirty clothes into an oven, the output would be burnt clothes instead of clean clothes.





Why are functions important? One of the main things we talk about in calculus is the derivative, which allows us to model how things change over time. We compute derivatives by looking at a specific function evaluation called the difference quotient, which we will discuss later in this worksheet. In the meantime, let's focus on function evaluation.

How do we evaluate functions? Let's look at some mathematical examples, starting by defining the function $f(x) = 3x$. This function is defined as taking our input and transforming it by multiplying it by 3. Let's see what happens when we put different inputs into this function.

$$\begin{aligned} f(x) &= 3x \\ f(5) &= 3 \cdot 5 = 15 \\ f(\pi) &= 3 \cdot \pi = 3\pi \\ f\left(\frac{\pi}{2}\right) &= 3 \cdot \frac{\pi}{2} = \frac{3\pi}{2} \end{aligned}$$

So far, this is pretty standard function evaluation. Where things may get tricky is when we replace x with another variable or combination of variables instead of replacing x with a numerical value. Remember, no matter the input, the function f is defined to multiply that input by 3.

$$\begin{aligned} f(y) &= 3y \\ f(\Delta x) &= 3\Delta x \\ f(x+y) &= 3(x+y) = 3x + 3y \\ f\left(\frac{x}{y}\right) &= 3 \cdot \left(\frac{x}{y}\right) = \frac{3x}{y} \end{aligned}$$

As you can see above, no matter the input, the function f always multiplies the input by 3. This concept works similarly even for other functions.

Now, let's look at the function $f(x) = x^2$, the function that takes our input and squares it.

$$\begin{aligned} f(x) &= x^2 \\ f(5) &= 5^2 = 25 \\ f(\pi) &= \pi^2 \\ f\left(\frac{\pi}{2}\right) &= \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4} \end{aligned}$$

Again, let's replace x with another variable or combination of variables.

$$\begin{aligned} f(y) &= y^2 \\ f(\Delta x) &= (\Delta x)^2 \\ f(x+y) &= (x+y)^2 = x^2 + 2xy + y^2 \\ f\left(\frac{x}{y}\right) &= \left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2} \end{aligned}$$

Why is function evaluation important? One of the main things we talk about in calculus is the derivative. The derivative is defined by looking at some important function evaluations called the difference quotient, described below.

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{and} \quad \frac{f(x) - f(c)}{x - c}$$

To calculate the difference quotient, it is important that we are able to evaluate our function as described above. For the function $f(x) = 3x$, the difference quotients look like

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{3(x+\Delta x) - 3x}{\Delta x} = \frac{\cancel{3x} + 3\Delta x - \cancel{3x}}{\Delta x} = \frac{3\cancel{\Delta x}}{\cancel{\Delta x}} = 3$$

$$\frac{f(x) - f(c)}{x - c} = \frac{3x - 3c}{x - c} = \frac{3\cancel{(x-c)}}{\cancel{x-c}} = 3$$

For the function $f(x) = x^2$, the difference quotients look like

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{x^2}}{\Delta x} = \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = 2x + \Delta x$$

$$\frac{f(x) - f(c)}{x - c} = \frac{x^2 - c^2}{x - c} = \frac{(x+c)\cancel{(x-c)}}{\cancel{x-c}} = x + c$$

Although these expressions can be intimidating at first, evaluating them starts with remembering that a function is simply a transformation of your input, regardless of how complicated that input may look. After that first step, we just need to make sure that we remember our algebra skills and write down all of our steps to make sure we do not make an algebra error.