

Math 1320: Evaluating Algebraic Expressions

What is an algebraic expression? An algebraic expression is an expression made up of variables (x), numbers, and operations ($+$, $-$, \cdot , \div , and exponents). We know how to add, subtract, multiply, and divide numbers, but what if we have a combination of these operations? We can think of order of operations as the rules that tell us the order in which we should evaluate an algebraic expression with multiple operations. The order of operations is as follows:

- 1) **Parentheses:** evaluate operations within the innermost parentheses then work outward
- 2) **Exponents:** evaluate all exponents
- 3) **Multiplication and Division:** evaluate in the order in which they occur, from left to right
- 4) **Addition and Subtraction:** evaluate in the order in which they occur, from left to right

Why is evaluating algebraic expressions important? In the course we will eventually learn to model real events with algebraic expressions. We need to have a universal order of operations to help us make sense of those models. Consider the following:

Correct	Incorrect
$8 \div 2 - 1 + 3$	$8 \div 2 - 1 + 3$
$= 4 - 1 + 3$	$= 4 - 1 + 3$
$= 3 + 3$	$= 4 - 4$
$= 6$	$= 0$

As you can see from the example above, we get different values when we do not follow order of operations. On the right, the addition was done before subtraction. Since subtraction comes first (from left to right) we need to evaluate this first, then perform the addition (left).

In this course, we will find the value of expressions for a specific value of a variable. So, first we will need to replace the variable with the number it represents, then use order of operations. Let's look at some algebraic expressions to help us practice.

Example 1. Evaluate $4 + 2(x - 4)^2 - 3$ for $x = 7$.

$$\begin{aligned}
 &4 + 2(x - 4)^2 - 3 \\
 &= 4 + 2(7 - 4)^2 - 3 && \text{Replace } x \text{ with } 7 \\
 &= 4 + 2(3)^2 - 3 && \text{Evaluate inside the parentheses: } 7 - 4 = 3 \\
 &= 4 + 2(9) - 3 && \text{Evaluate the exponent: } 3^2 = 9 \\
 &= 4 + 18 - 3 && \text{Multiply} \\
 &= 22 - 3 && \text{Add} \\
 &= 19 && \text{Subtract}
 \end{aligned}$$

Example 2. Evaluate $[4 - x(2 - 1)] + 4^2 \div 8 \cdot 3$ for $x = 3$

$$\begin{aligned}
 &[4 - x(2 - 1)] + 4^2 \div 8 \cdot 3 \\
 &= [4 - 3(2 - 1)] + 4^2 \div 8 \cdot 3 && \text{Replace } x \text{ with } 3 \\
 &= [4 - 3(1)] + 4^2 \div 8 \cdot 3 && \text{Work in the innermost parentheses: } 2 - 1 = 1 \\
 &= [4 - 3] + 4^2 \div 8 \cdot 3 && \text{Evaluate in the brackets: Multiply } 3(1) = 3 \\
 &= 1 + 4^2 \div 8 \cdot 3 && \text{Evaluate inside the brackets: Subtract } 4 - 3 = 1 \\
 &= 1 + 16 \div 8 \cdot 3 && \text{Evaluate the exponent: } 4^2 = 16 \\
 &= 1 + 2 \cdot 3 && \text{Divide} \\
 &= 1 + 6 && \text{Multiply} \\
 &= 7 && \text{Add}
 \end{aligned}$$