

MAPLE Worksheet Number 9  
Integrals in Calculus via Antiderivatives

Recall that  $F(x) = \int f(x) dx$  means  $\frac{\partial}{\partial x} F(x) = f(x)$ . We call  $F(x)$  the "indefinite integral" of  $f(x)$  with respect to  $x$ . Sometimes we also call  $F(x)$  the "antiderivative" of  $f(x)$  with respect to  $x$ . Notice that if  $C$  is constant with respect to  $x$  then

$$\frac{\partial}{\partial x} (F(x) + C) = \frac{\partial}{\partial x} F(x) = f(x).$$

hence the use of the word "indefinite" to describe  $F(x)$ . So the more common notation for the indefinite integral is

$$\int f(x) dx = F(x) + C.$$

Later it will be important for us to remember to include the integration constant  $C$ . The MAPLE command for computing the indefinite integral is analogous to that for computing the derivative of  $f$  with respect to  $x$ :

`int(expression,variable of integration);`

Of course the derivative and indefinite integral are "inverse operations" of each other, that is,

$$\frac{\partial}{\partial x} \int f(x) dx = f(x) \quad \mathbf{and} \quad \int \frac{\partial}{\partial x} F(x) dx = F(x)$$

and MAPLE knows this. As we did in exercise 7 perform the following commands, remember we haven't defined  $f$  or  $F$  to be any particular functions.

```
[ > Diff(int(f(x),x),x)=diff(int(f(x),x),x);
[ > Int(diff(F(x),x),x)=int(diff(F(x),x),x);
```

Use MAPLE to compute each of the following indefinite integrals, ckecking each result with differentiation.

```
[ > \int x^3 dx
[ > \int sin(ax) dx
[ > \int ln(x) dx
[ > \int e^(3x) dx
```

Notice that MAPLE left off the integration constant each time, or more accurately, MAPLE defaults to integration constant 0. Sometimes this causes MAPLE some grief. Consider the following integration, first do it "by hand" and include the answer, then use MAPLE to compute what is supposed to be the same answer.

```
[ > Int(2*x*(x^2+1)^20,x)=int(2*x*(x^2+1)^20,x);
```

Woah! Try factoring this.

```
[ > ans:=factor(rhs(%));
```

Well, this isn't much help. Let's make sure it is right by differentiating it to see if we get what we started with.

$$\left[ \begin{array}{l} > \frac{\partial}{\partial x} ans \\ > \text{factor}\left(\frac{\partial}{\partial x} ans\right) \end{array} \right]$$

OK that seems to work, so what's going on? To answer this note what integration constant you got when you did the calculation "by hand." Notice the constant term in the above integral is 0. Try adding your constant to it and factoring again.

$$\left[ > \text{factor}(ans + \text{your constant}); \right]$$

Both answers are correct, they represent the same indefinite integral, I personally prefer the one we get by hand.

Now we'll do some more complicated integration problems, you know, like those "neat" ones from Cal II. Try these.

$$\left[ > \int \sin(x)^2 \cos(x) dx \right]$$

What integration method was used here? (Above.)

$$\left[ > \int \sin(x)^2 \cos(x)^2 dx \right]$$

$$\left[ > \int x e^x dx \right]$$

$$\left[ > \int x^3 e^x dx \right]$$

What integration method was used in these last two examples? (Above.)

$$\left[ > \int \frac{1}{\sqrt{1-x^2}} dx \right]$$

$$\left[ > \int \frac{x}{\sqrt{1-x^2}} dx \right]$$

$$\left[ > \int \frac{x^3}{\sqrt{1-x^2}} dx \right]$$

What integration method(s) was(were) used in these last two examples? (Above.)

We can compute double integrals just as easily as we computed mixed derivatives. Compute the following multiple integration:

$$\left[ > \iint xy + x^3 - y^2 + 2 dx dy \right]$$

Compute the same integral with the order of integration reversed. What do you observe?

Does the order of integration ever make any difference? Actually in definite integration sometimes it make a lot of difference when trying to compute "by hand.". Try to compute the following double

integration problem by hand:  $\int_0^1 \int_y^1 e^{(x^2)} dx dy$  . What is the problem you encounter immediately? Now

reverse the order of integration and compute the same integral. (Remember you must draw the region of integration and choose the appropriate limits of integration in the new order.)

Now compute the integral in both orders with MAPLE. Include both results. What do you observe?

There are lots of continuous functions which simply don't have a "closed form antiderivative." By this we mean there are functions  $f(x)$  for which there is no formula expression in  $x$ ,  $F(x)$ , for which

$\frac{\partial}{\partial x} F(x) = f(x)$  . We have already seen one such function, namely,  $e^{(x^2)}$  ,another is  $\sin(x^2)$  . Ask

MAPLE to compute the indefinite integral of both and display the result. Of course the Fundamental

Theorem tells us there is a function whose derivative is  $\sin(x^2)$  ,namely  $\int_0^x \sin(t^2) dt$  is such a

function, but we would have to use some numerical integration technique to compute its value for particular values of  $x$ . For example use the trapezoidal rule to compute the value for  $x=5$ .

There are other integrals that MAPLE needs some help in solving. For example ask MAPLE

to compute  $\int h dx$  for the function  $h$  defined as

$$\left[ \begin{array}{l} > h := \frac{1}{\sqrt{1+\sqrt{x}}} \end{array} \right.$$

Load the student package and try the following command sequence:

```
[ > changevar(1+sqrt(x)=u^2,Int(h,x),u);
[ > value(%);
[ > ans:=subs(u=sqrt(1+sqrt(x)),%);
```

We can check the answer by differentiating.

```
[ > diff(ans,x);
```

This doesn't look right, try simplifying it.

```
[ > simplify(%);
```

Now ask MAPLE to compute  $\int g dx$  for the function  $g$  defined by

```
[ > g := ln(x+sqrt(1+x^2))
[ > int(g,x);
```

Again, MAPLE needs help. This time we'll try integration by parts. Recall the formula

$$\int u dv = uv - \int v du .$$

Our only option seems to be to let  $u = \ln(x + \sqrt{1 + x^2})$  and  $dv = dx$  . Try the following

```
[ > intparts(int(g,x),ln(x+sqrt(1+x^2)));
```

Try simplifying this and computing the value of the result to see if MAPLE can handle it now. Of course

you are to check your answer by differentiation (and simplification if necessary).

Use the same method to compute  $\int \arcsin(x)^2 dx$  . Again check you answer via differentiation.