Binomial Theorem and Pascal's Triangle

(number3.mws)

1. Use the MAPLE command **expand( );** to calculate the expansion of $(a+b)^n$ for values n=1..10 . In each case use the MAPLE command **coeffs( );** to pick off the coefficients. For example try

```
> expand((a+b)^5);
  coeffs(%);
```

The BINOMIAL THEOREM gives a formula for finding the coefficients of the expansion of $(a+b)^n$ for any n. It is relatively obvious that the coefficient of $a^k\, b^{(n-k)}$ for any value of k from 0 to n is given by the number of ways we can choose k of the a's from the n a's in the expansion $(a+b)\,(a+b)$ ....($a+b$), n times .
The number of combinations of n objects choosing k at a time, *nCk*, is given by the formula

$$nCk = \frac{n!}{k!\,(n-k)!}\ .$$

2. Verify that the MAPLE syntax for computing *nCk* is **binomial(n,k);** (In other words, show that binomial(n,k) = $\frac{n!}{k!\,(n-k)!}$ for all integers n$\geq$k.)

3. State the BINOMIAL THEOREM and use MAPLE to demonstrate it.

4. Compute $\sum_{k=0}^{n} \mathrm{binomial}(n, k)$ for n=1 to 10.

5. What general formula does the above calculation suggest.

6. Use MAPLE to verify the formula for all n.

7. Use the BINOMIAL THEOREM to "prove" the general formula.

Recall the definition of Pascal's Triangle.

8. Use MAPLE to construct the first 10 rows of Pascal's Triangle.

9. Look for patterns in Pascal's triangle and demonstrate them using MAPLE.