

**MATH 1351 TI-85 EXERCISE VII**  
**Derivatives as the limit of the difference quotient**

In Section 1 of Chapter 3 we learned that the derivative of a function  $f$  at the input value  $x$  is the limiting value of the slopes of the chords drawn from  $(x, f(x))$  to nearby points on the graph. This slope from  $(x, f(x))$  to the nearby point  $(x+h, f(x+h))$  is the *difference quotient*

$$\text{DQ:} \quad (f(x+h) - f(x)) / h$$

for small positive and negative values of  $h$ . And its' limiting value we defined to be the slope of the *tangent line* to the graph at  $(x, f(x))$ .

Also in Section 1, we saw how to find a formula for this derivative in terms of the value  $x$  for some very simple functions. Here we will try to conjecture what such a derivative formula might be for a much more sophisticated function; namely, we'll consider the function  $f(x) = \ln(x)$ . We'll investigate the limit of the difference quotient at the three values  $x = 1, 2, \& 3$ .

1. Compute each of the difference quotients for  $f(x) = \ln(x)$  directly:

for $x = 1$ & $h = 10^{-10}$	DQ = _____
for $x = 1$ & $h = -10^{-10}$	DQ = _____
for $x = 2$ & $h = 10^{-10}$	DQ = _____
for $x = 2$ & $h = -10^{-10}$	DQ = _____
for $x = 3$ & $h = 10^{-10}$	DQ = _____
for $x = 3$ & $h = -10^{-10}$	DQ = _____

2. Sketch the graph of each DQ for each value of  $x = 1, 2, \& 3$  and use **ZOOM** and **TRACE** to estimate the value near 0. (The variable in each DQ is  $h$ , but the TI-85 recognizes only  $x$  as an independent variable. So we have to replace  $h$  in the above DQ with  $x$ . This shouldn't be too confusing since the  $x$  in the above DQ is to be replaced with the numbers 1, 2, & 3 anyway.)

So the graph of DQ corresponding to the point  $(1, \ln(1))$  is the graph of

$$y = (\ln(1+x) - \ln(1)) / x \quad \text{and} \quad \text{limit as } x \rightarrow 0 \simeq \underline{\hspace{2cm}}$$

At the point  $(2, \ln(2))$  the graph is that of

$$y = (\ln(2+x) - \ln(2)) / x \quad \text{limit as } x \rightarrow 0 \simeq \underline{\hspace{2cm}}$$

At the point  $(3, \ln(3))$  the graph is that of

$$y = (\ln(3+x) - \ln(3)) / x \quad \text{limit as } x \rightarrow 0 \simeq \underline{\hspace{2cm}}$$

3. Finally, simply sketch the graph of  $y = \ln(x)$  in the **ZDECIM** window. Choose **MATH** from the screen menu, then choose **dy/dx** from the MATH screen menu. **TRACE** is automatically activated. Move the cursor to  $x=1$  and press **ENTER**. Do the same for each of  $x=2$  and  $x=3$ . We obtain  $dy/dx = \underline{\hspace{1cm}}$  at  $x=1$ ,  $dy/dx = \underline{\hspace{1cm}}$  at  $x=2$ , and  $dy/dx = \underline{\hspace{1cm}}$  at  $x=3$ . What appears to be the relationship between the value of  $x$  and that of  $dy/dx$  for  $y = \ln(x)$ ? \_\_\_\_\_

---