

MATH 1351 TI-85 EXERCISE XIII
Area estimations with the TI-85

Name: _____ SID: _____

The idea is to use sums of *left boxes* and sums of *right boxes* to estimate the area under the graph of a positive function $y = f(x)$, between the values $x = a$ and $x = b$. We recall the syntax:

$$\text{left box approximation} = \text{sum seq}(\text{delta-}x \, f(a + I \, \text{delta-}x), I, 0, N-1, 1)$$

$$\text{right box approximation} = \text{sum seq}(\text{delta-}x \, f(a + I \, \text{delta-}x), I, 1, N, 1).$$

Exercise 1. Attach a picture that illustrates the validity of the following statements.

A. If the function $f(x)$ is increasing on the interval $[a, b]$, then

$$lt \, box \, appx < actual \, area < rt \, box \, appx.$$

B. If the function $f(x)$ is decreasing on the interval $[a, b]$, then

$$rt \, box \, appx < actual \, area < lt \, box \, appx.$$

Thus the possible error in using either approximation is less than $|rt \, box \, appx - lt \, box \, appx|$.

Exercise 2. Use these ideas to estimate the area under the graph of $y = 1/x$, between $x = 1$ and $x = 3$. (Compute the *lt* and *rt box appx*'s until their difference is less than .005.) Record your data in the table provided.

# of boxes	<i>lt box appx</i>	<i>rt box appx</i>	possible error

Final estimate of the area (rounded off to 2 decimal places) is _____

Exercise 3. Estimate the area under the graph of $y = e^{(x^2)}$, between $x = 0$ and $x = 2$. Again record your data in the table provided.

# of boxes	<i>lt box appx</i>	<i>rt box appx</i>	possible error

Final estimate of the area (rounded off to 2 decimal places) is _____

Exercise 3. Suppose you know that some number is between 1 and 5, but that's all you know about it. What would be your "best" guess as to the number? (That is, knowing only that the number is between 1 and 5, what guess would be guaranteed to minimize the possible error?) _____

We know that the actual area (for an increasing or decreasing, *monotone*, function) is between *lt box appx* and *rt box appx*. So the "obvious" best guess is half way between them. This will be called the *trapezoidal approximation*:

$$\text{trap appx} = (\text{lt box appx} + \text{rt box appx}) / 2 .$$

Exercise 2'. From the above data, use the *trapezoidal rule* to approximate the area under the graph of $y = 1/x$, between $x = 1$ and $x = 3$. What is the fewest number of "boxes" required to make the *trap appx* within 2 decimal places of the answer you obtained previously? _____

Exercise 3'. From the above data, use the *trapezoidal rule* to approximate the area under the graph of $y = e^{(x^2)}$, between $x = 0$ and $x = 2$. What is the fewest number of "boxes" required to make the *trap appx* within 2 decimal places of the answer you obtained previously? _____

Exercise 4. Compare each of your above approximations with the answer obtained by using the TI-85 numerical integrator directly. It is accessed by choosing **2nd CALC** from the keyboard then **fnInt** from the screen menu. The syntax is

$$\text{fnInt}(f(x) , x , a , b) ,$$

read "the definite integral of $f(x)$, with respect to x from $x = a$ to $x = b$."

Exercise 5. Why do you think the above rule is called the *trapezoidal rule*?