

factor(%);

We know $\int 2x(x^2+1)^{10} dx = \frac{(x^2+1)^{11}}{11}$. Right? So why doesn't MAPLE factor it this way? The key lies in the constant of integration.

What is the constant term in the MAPLE answer above?

What is the constant term in our answer above?

Complete the following sequence putting the result in the appropriate box.

g:=int(2*x*(x^2+1)^10,x);

factor(g+(1/11));

diff(%,x);

factor(%);

So adding our constant to the MAPLE answer yields our answer. But recall the indefinite integral is only defined “up to a constant”. This is because the derivative of any constant is 0 and the indefinite integral is looking for a function whose derivative is the expression inside the integral sign. MAPLE makes the classic mistake of assuming the integration constant to be 0.

Continue the above exercise first anticipating the answer, using paper and pencil if necessary, then executing the MAPLE command.

diff(sin(x^2),x);

diff((x^3)(2x+1)^10,x);

diff((2x+1)/(3x+1),x);

simplify(%);

diff(x/exp(x),x);

simplify(%);

Remember that integration and differentiation are the inverses of each other.

diff(int(x^2+1,x),x);

int(diff(x^2+1,x),x);

Why didn't we get back x^2+1 this last time?

int((x+1)/(x+2),x);

diff(%,x);

simplify(%);

int(x^n,x);

diff(x^n,x);

simplify(%)

int(sin(a*x),x);

diff(tan(x),x);

simplify(%);
 diff(exp(ax^2+bx+c),x);

What happens in the above problems if you use the command Int instead of int?

What happens if you use the command Diff instead of diff?

Section 2: Derivatives and Graphs

While the TI-86 can't do symbolic calculations its' GRAPH package does have some very nice features for demonstrating some calculus principles. With the TI-86 graph $y=x^2-x$ in the ZSTD. Now zoom in three times on the origin. The graph looks like a straight line. What do we call this line in calculus?

What is it's slope?

Now go back to the ZSTD picture and zoom in once. Choose MATH from the bar menu and then choose TANLN form the new bar menu. What do you think tanln stands for?

You should see the graph with the cursor positioned at the center of the screen (the origin in this case). Notice the left and right arrow keys allow you to move along the

graph. With the cursor at the origin press ENTER. What happened?

What is the number displayed in the lower left corner of the screen?

Press EXIT to return to the bar menu, choose TANLN again, move the cursor to some other point and draw the tangent line at that point. Do this a few more times on this and other graphs. What graph phenomena correspond to horizontal tangent lines?

Recall the point slope equation of a line is

$$y-y_1=m(x-x_1)$$

where (x_1, y_1) is a point on the line and m is the slope of the line.

Also recall that the derivative of a function y at $x=x_1$ is the slope of the tangent line to the graph of y at the point $(x_1, y(x_1))$.

Find the equation of the line tangent to the graph of the function $y=x^2-x$ at the point with $x=1$.

Use MAPLE to plot the graph of $y=x^2-x$ and its tangent line at $(1,0)$ on the same axes. Attach the MAPLE printout.

On the TI-86 graph the two functions $y(x)=x^2-x$ and $nDer(y(x),x)$ on the same axes. With MAPLE graph $y(x)=x^2-x$ and $diff(y(x),x)$ on the same axes. Attach the MAPLE printout. Look at these graphs and answer the following questions.

What is the relation between the graph of y and where the derivative of y equals 0?

What is the relation between the sign of the derivative of y and the graph of $y(x)$?

Section 3. Definite Integrals

Both MAPLE and the TI-86 can do definite integration (i.e. compute the net area between the graph of the function and the x -axis). Here is an example of how you do

it on the TI-86.

Sketch the graph of $y=x^2$ on your TI-86. Choose MATH from the bar menu then choose $\int f(x)$ from the GRAPH-MATH bar menu. To integrate x^2 from $x=0$ to $x=2$ move along graph until the x coordinate is 0 then press ENTER, now move cursor until x coordinate is 2 and press ENTER. The TI-86 will calculate the integral and display the result in the lower left corner. The result is

Why do you think the TI-86 shades in the region under the graph of x^2 and above the x-axis between $x=0$ and $x=2$? (Recall what this number represents with respect to the graph of x^2 .)

The MAPLE command for doing the same thing is **int(x^2,x=0..2)**; Try it, did you get the same answer? Of course you should be able to do this one by hand, how?

In each of the following first do the calculation by hand, then with the TI-86 succeeded by >Frac, and finally with MAPLE.

(by hand)	(TI-86) → (after >Frac)	(MAPLE)
<input type="text"/>	int(x^2,x=-2..2);	<input type="text"/>
<input type="text"/>	int(x^3,x=0..2);	<input type="text"/>
<input type="text"/>	int(x^3,x=-2..2);	<input type="text"/>
<input type="text"/>	int(exp(x^2),x=0..1);	<input type="text"/>
<input type="text"/>	evalf(%);	<input type="text"/>
<input type="text"/>	<input type="text"/>	<input type="text"/>

Exercise: Define the function p as below:

$$p:=(1/\sqrt{2\text{Pi}})\exp((-x^2)/2);$$

If you have had statistics you might recognize this as the standard normal distribution

function, or probability density function. The area of any portion of the of the region under its graph represents a percentage of the total population. Graph it from $x=-4$ to $x=4$ with both the TI-86 and with MAPLE. Attach a printout of the graph. It should be the standard bell shaped curve. (Recall the MAPLE command `plot(p,x=-4..4);`)

Use both the TI-86 and MAPLE to integrate p from -1 to 1 . The ZDECM window on the TI-86 will let you set these integration limits exactly. Do you remember why? To make the MAPLE result agree with the TI-86 result you will have to use the `evalf` command. What is the result?

This area from -1 to 1 represents the percentage of the total normal population between -1 and 1 . This is the percentage of the total population within one standard deviation of the mean. With MAPLE compute the integral from $-\infty$ to ∞ . Explain the result. (What does it mean?)

Finally execute

`Int(p,x=-infinity..infinity)= int(p,x=-infinity..infinity);`

to get a good picture of the integral notation.

Section 4: Rectangles to approximate area

The point of this section is to see how to use rectangles to approximate the area of regions with curved edges. The (net) area between the graph of f and the x -axes from $x=a$ to $x=b$ is **the** definite integral, thus the MAPLE command is **`int(f, x=a..b);`** This command followed by **`evalf(%)`**; would give a floating point approximation to the area. MAPLE has a package for exhibiting how we can use rectangles to approximate this area. It is invoked via the command

`with(student);`

Define the function $f := 16 - x^2$ and print out the graph of f from $x= -6$ to $x= 6$. What is the actual area from $x= -4$ to $x= 4$, between the graph of f and the x -axes?

Try the MAPLE command

`middlebox(f,x=-4..4,4);`

Attach a printout and use it to estimate the area using the rectangles drawn.

Try the MAPLE command

`leftbox(f,x=-4..4,4);`

Attach a printout and use it to estimate the area using the rectangles drawn.

Try the MAPLE command

rightbox(f,x=-4..4,4);

Attach a printout and use it to estimate the area using the rectangles drawn.

A~

Try the MAPLE command

leftbox(f,x=-4..4,8);

Attach a printout and use it to estimate the area using the rectangles drawn.

A~

Which approximation is closer to the actual area, the one using `leftbox(f,x=-4..4,4);` or `leftbox(f,x=-4..4,8);` and why?

Now fill in the boxes indicating the results of each of the following command sequences:

middlesum(f,x=-4..4,4)=evalf(middlesum(f,x=-4..4,4));

leftsum(f,x=-4..4,4)=evalf(leftsum(f,x=-4..4,4=));

leftsum(f,x=-4..4,8)=evalf(leftsum(f,x=-4..4,8=));

Finally attach a printout of the result of the command **middlebox(f,x=-4..4,200);**

What are the results of the following command sequence:

middlesum(f,x=-4..4,200)=evalf(middlesum(f,x=-4..4,200));

How does this compare to the actual value of the area?

Section 5. Limits

You know that calculus instructors are all the time going on about limits, continuity, and differentiability. Use the TI-86 and MAPLE to sketch the graphs and describe the behavior of the following functions in the vicinity of $x=0$ (Be sure your TI-86 is in radian mode. This may also be a good place to use the ZFIT zoom option.):

$$y=1/x^2$$

$$y=\sin(1/x)$$

$$y=x\sin(1/x)$$

$$y=x^2\sin(1/x)$$

$$y=\sin(x)/x$$

MAPLE can easily compute limits, the calling sequence is $\lim(f(x),x=a)$, where $f(x)$ is the function and a is the value of x at which the limit is to be taken. Compute the following limits, and compare these limits with the above graphs. Do the limits concur with the graphical view near 0? Explain.

1. $\lim(\sin(x)/x,x=0)$;

2. $\lim(\sin(1/x),x=0)$;

3. $\lim(x*\sin(1/x),x=0)$;

4. $\text{limit}(1/x^2, x=0)$;

Section 6. Sequences and Series

Another important concept from Calculus is that of sequences (lists of numbers) and series (infinite sums). You probably know that the geometric series converges and the harmonic series diverges. Let's check this out with MAPLE. Compute the following summations:

1. (Harmonic Series) $\text{sum}(1/i, i=1..10)$;

2. $\text{sum}(1/i, i=1..n)$;

Look at the answer in #2 and decide what you think will happen as n gets bigger and bigger.

3. $\text{sum}(1/i, i=1..infinity)$;

4. (Geometric series) $\text{sum}((2)^i, i=0..10)$;

5. $\text{sum}((2)^i, i=0..n)$;

Look at #5 and decide what you think will happen as n gets bigger and bigger.

6. $\text{sum}((2)^i, i=0..infinity)$;

Look at the following command sequences:

$\text{Sum}(1/i, i=1..infinity) = \text{sum}(1/i, i=1..infinity)$;

$\text{Sum}((2)^i, i=0..infinity) = \text{sum}((2)^i, i=0..infinity)$;

@Some educators think it is no longer necessary to stress techniques of integration and differentiation in Calculus classes. What is your opinion and why?

