Names: ____________________ / ____________________ / __________________________.

Chapter 5
Graphing

Section I: Polynomials

We begin this section with a question. “What do the roots of a polynomial have to do with its graph?”

The graph package on the TI-86 is accessed via **GRAPH** from the keyboard. Once in this package the various options are accessed via "F" keys under the appropriate item in a bar menu. Choosing **EXIT** from the keyboard always takes you back to the previous menu. The graphing options should be fairly obvious to you, and you will be expected to become familiar with most of them by experimentation and trial and error.

The MAPLE command for plotting a 2-dimensional graph of y = a function of x is

plot(the function, x=xmin..xmax, optional stuff);

This command automatically chooses a y-range that it thinks best. If you wish to specify the y-range then you must include it in the original command :

plot(the function,x=xmin..xmax,y=ymin..ymax, optional stuff);

On the TI-86 graph $y1=x^2-1$. (Go to the graph package and choose **y(x)=** from the bar menu. Define y1 being sure to use the x-VAR key to indicate that x is a variable, not just the letter “x”, then choose **GRAPH** from the bar menu.) What are the default ranges on x and y? (Choose WIND from the bar menu.)

xmin =_______	xmax =_______.
ymin =_______	ymax =_______.

Experiment with various values for the xScl and yScl. What do these determine on the graphs?

Choose **ZOOM** from the bar menu and experiment with the various ZOOM options.(The little > at the right indicates more stuff in the menu. To see it choose **MORE** from the keyboard.)

Explain in general terms what each of the following ZOOM options appears to do:

ZIN	
ZOUT	
ZSTD	
ZPREV	
ZFIT	
ZSQR	
ZTRIG	
ZDECM	
ZINT	

Next experiment with the TRACE option from the bar menu. Describe how it works.

Look at the graph after choosing ZINT, now choose TRACE and record the effect of moving along the graph. What is the change in x from one point on the graph to the next? What are the values of xmin and xmax in this case?

xmin=______ xmax= _______.

Use the above information to determine the number of pixels across the x-axis.

Number horizontal pixels = ___________________.

Now use MAPLE to draw the graph from xmin=-10 to xmax=10. What is MAPLE's choice for the range of y-values?

ymin= _______ and ymax =_______.

Now redraw the graph choosing ymin=-10 and ymax=10. Which is the best picture, the TI-86 with ZSTD, or the MAPLE picture?

Compare the MAPLE picture with the range (perspective) "constrained" (click on the graph the choose **1-1** from the graph toolbar) with the TI-86 picture using ZSQR. Experiment with the other options in the graph toolbar of MAPLE.

Back to our original question: Use both the TI-85 and MAPLE to draw the graphs of

$$y=x^2-2,\ y^2-2,\ \ y=x^2-1,\ \ y=x^2,\ \ y=x^2+1$$

What is the relation between the roots of these polynomials and their graphs? The TRACE feature on the TI-86 is very helpful (especially in ZDECM) in answering this question.

Explain why ZDECM on the TI-86 works better for finding the intercepts in this case that does ZSTD.

Notice in MAPLE you have to "position" the pointer at the x-intercept as best you can by "eye-balling" it, then clicking the mouse.

For purpose of demonstration it is nice to see all these graphs on the same axes. With the TI-86 this is automatic unless you use SELECT to turn off some of the graphs. However with MAPLE it seems to be more of a challenge, so here goes.

In MAPLE try the command

plot({x^2-2,x^2-1,x^2,x^2+1},x=-10..10,y=-10..10);

Actually this is more sensible than it first appears. The symbol **{ }** denotes a set of objects in MAPLE. So you just asked MAPLE to graph the set of functions, all on the same axes. A somewhat shorter version of the same MAPLE command is

plot({seq (x^2+i,i=-2..1)},x=-10..10,y=-10..10);

Try it. From the **<u>O</u>ptions** menu choosing **<u>P</u>lot Display** then **<u>W</u>indow** will cause all graphs to be plotted in a separate window, which then can be printed separately. Attach a printout of this graph. To title your graph insert **,title= “whatever”** as the

optional stuff.
The comparable TI-86 shortcut to defining all these functions at once is to define

y1=x^2+{-2,-1,0,1}

Try it.

MAPLE Hint: If you haven't already discovered it, you can often save yourself some time by defining an expression you will use several times as a letter, eg **f:=expression;**. Then instead of having to retype the expression every time you need it, you just type f.

Exercise: Using both the TI-86 and MAPLE plot the graphs of the functions

$yi=x^2-x+i$, for i=2,1,0,-1,-2.

Which of the yi have only complex roots? [ ]

Which of the yi have real roots? [ ]

Using TRACE on the TI-86 estimate the minimum value of each yi.

y2min=___ y1min=___ y0min=____ y(-1)min=____ y(-2)min=___.

Now do the same thing using the pointer with MAPLE.

y2min=___ y1min=___ y0min=____ y(-1)min=____ y(-2)min=_____.

Finally use the MAPLE command subs(x=1/2,yi); for each i and record your results:

y2(.5)=____ y1(.5)=____ y0(.5)=____ y(-1)(.5)=_____ y(-2)(.5)=______.

How did I know without ever looking at the graph that the minimum occurs at exactly x=1/2? (The answer lies in Calculus, do you remember?)

[ ]

Exercise: Graph $y=5x^3-5x+1$ using both machines. Use ZSTD to obtain a picture on the TI-86. Adjust the range if necessary to obtain a comparable picture with MAPLE. Now use ZBOX to obtain a TI-86 picture which isolates the interesting portion of the graph. (In ZBOX position the cursor and press ENTER to establish a corner of the desired viewing box. Then position the cursor to set the opposite corner and press ENTER.) Use these range settings to obtain the comparable MAPLE picture. Estimate the roots (x-intercepts).

TI-86 estimates are ______________ & ______________ & ______________.

MAPLE estimates are ______________ & ______________ & ______________.

How do these compare to the roots you find using the solve capabilities of each machine?

Can you find a number M so that the polynomial $5x^2-5x+M$ has no real roots? M=
Adding a positive M has the effect of raising the graph.) Explain.

Using both machines, graph each of the following polynomials. In each case estimate the real roots and if possible find a number M which added to the polynomial produces one with no real roots.(In other words, how high do you have to raise the graph so that it doesn't cross the x-axis?)

1. x^3-x^2

roots~____ M=_____.

2. x^4-x^2

roots~____ M=_____.

3. $2x^5+5x^4+6x^3+6x^2+4x+1$

roots~____ M=_____.

4. x^6-x^2+x

roots~____ M=_____.

Exercise: It appears that we can find such M for the even degree polynomials but we can not for the odd degree polynomials. Explain why you think this might happen.

Section II: Rational Functions

Use both machines to graph the rational function $f = \frac{x+1}{x-1}$. Attach a labeled printout of the MAPLE picture. You might need to play with the range to get a good MAPLE picture. Try ZSTD on the TI-86. What happens near x=1? Why?

Can you explain the presence of the vertical line at x=1 on the TI-86 graph?

Now redraw the TI-86 graph using the ZDECM. Now the vertical line is not there at x=1. Why?

To answer these questions experiment using TRACE in both pictures to record the x values a few pixels on either side of x = 1 and the corresponding y values.

ZSTD picture

x values	y values

ZDECM picture

x values	y values

What happens to the y values as the x values get very close x=1?

What happens at x=1?

What happens to the y values as the x values get very large?

We say f has a "vertical asymptote" at x=1 and a "horizontal asymptote" y=1.

If you simplify g = $\frac{x^2-1}{x-1}$ what do you get?

Is it true that $\frac{x^2-1}{x-1}$=x+1 for every value of x?

Use the TI-86 to graph g, making sure that x=1 is a sample point. (Use ZDECM.) Use trace to record the y value corresponding to x=1 on the graph of g.

x=1, y= ___.

Now do the same thing on the graph of x+1.

x=1, y=____.

Explain.

Now use both machines to graph g = $\frac{x^2+1}{x-1}$. Attach the labeled MAPLE picture. Note that g does not have a horizontal asymptote but does have something we call a "slant asymptote" (a line that approximates the y values for large x values). Experiment with the calculator to see if you can find the equation of this slant asymptote.

On both machines graph each of the following rational functions, attach the MAPLE picture. Determine the vertical and horizontal asymptotes if any.

1. y=(x^2-1)/(2x^3+3x-2)

vertical asymptotes at_______________&horizontal asymptote___________.

2. y=x^3/(x^2-1)

vertical asymptotes at_______________&horizontal asymptote____________.

This is as good a time as any to talk about how a machine draws a graph. Unless told to do otherwise the machine samples various values of x between xmin and xmax, computes the corresponding values of y and lights the appropriate pixel on the axes, then connects the dots for adjacent x's with a straight line, hence the presence of the vertical asymptotes drawn above. Sometimes it is instructive to turn off this feature and see only the data points. To do so on the TI-86 select FORMT from the GRAPH bar menu, then select DrawDot. Try this for the above graphs. As with the MAPLE graphs, click on the graph then click on the appropriate button in the graph toolbox. Try this for the above graphs. Attach printouts in point style for #1 and #2.

A careful look at these graphs in point style might point out a difference in the way the TI-86 chooses the x samples and the way MAPLE chooses them. The TI-86 chooses equally spaced values of x from xmin to, and including, xmax. There are 127 such points including both xmin and xmax, hence the uniform x step size of (xmax-xmin)/126. MAPLE on the other hand has an adaptive sampling procedure that computes more sample points at places where MAPLE deems the graph to exhibit more interesting phenomenon. What portions of the graphs of #1 and #2 above does MAPLE seem to find more interesting?

Section III: Transcendental Functions

The most common transcendental functions are the "trig" functions. All can be defined in terms of the sine (sin) and cosine (cos) functions. (By the way, what do you think the "sin" button stood for on the very first calculator? That's right: The Original Sin.)

Give the definitions in term of sin x and cos x of each of the following trig functions:

tan x =_______ cot x =________sec x=___________ csc x =____________.

Sketch the graphs of all these functions on the TI-86 and via MAPLE. (Use radian mode on the TI-86.) The important properties of the trig functions involve their symmetries. It is instructive to see how changing various parameters affects their graphs. To this end use the TI-86 and MAPLE to graph the functions y=sin x, y=2sin x, and y=3sin x   on the same axes.

Do the same for the functions   y=sin 2x, y=sin 2x, y=sin 3x

and for   y=sin(x), y=sin(x+Pi/2), y=sin(x+Pi).

Attach the MAPLE printouts in each case.

Sketch the graphs of y=cot x, y=tan x, y=sec x, and y=csc x using both the TI-86 and MAPLE.

In the elementary grades trig functions are most likely to be encountered in the context of degree measure of angles. Put you TI-86 in degree mode and redraw each of the above graphs. Is there a way to do this in MAPLE?

Question: Does cot x = 1/tan x? With the TI-86 graph   y=1/tan x   and record, using trace, the value of y corresponding to x = 90. What did you get?

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1/tan(90) = ________.
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Now graph   y = cos x / sin x   and record the value of y corresponding to x = 90. What did you get?

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cos(90)/sin(90) = ________.
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Explain.

Finally sketch the graphs of each of the inverse trig functions using both the TI-86 and MAPLE. Attach the MAPLE printouts.

Other common transcendental functions are the exponential and log functions. These are inverses of each other. Use both machines to sketch the graphs of $y=e^x$ (the exponential function in MAPLE is exp(x))and ln x on the same axes. Attach the MAPLE printout.

Use these functions to create some "wild" graphs. For example what does (e^x)sin x or sin(ln x) look like. Maybe x+sin x or sin x^2 are interesting. Try $(\sin x)^2 + (\cos x)^2$. You might also use the absolute value function, abs() in the TI-86 and MAPLE, to get interesting graphs. For example abs(sin x), sin(abs(x)), and cos(abs(x)) are sort of interesting. Attach some interesting MAPLE graphs.

Section IV: Implicit Graphs

Now for something the TI-86 can not do directly, sketch graphs of equations that are not functions. Such things are very common; for examples, ellipses, hyperbolas, and many parabolas are such. To graph such things with MAPLE we must first invoke a special graphing package, to do this execute the command

with(plots);

Notice all the plotting commands contained in this package. Now change the semicolon to a colon and see what happens when you press enter. The implicit plot works just the same as plot did above except the command is

implicitplot(equation in x and y, x=a..b, y=c..d, title='stuff');

Note that we must put limits on both the x values and the y values. Why do you think this is so?

Graph the unit circle $x^2+y^2=1$ with $-2\leq x \& y\leq 2$. (Using 1-1 perspective might give a better picture of what is going on.)

Now graph and attach the labeled printout for each of the following conics:

$x^2/4 + y^2 =1$ $\quad$ $x^2+ y^2/4 =1$

$x^2/4 - y^2 =1$ $\quad$ $-x^2 +y^2 =1$

$x^2 + y =1$ $\quad$ $x + y^2 =1$

Now graph the following conics on the same axes:

$i(x^2) +y^2 =1$ for i=-4,-3,-2,-1,0,1,2,3,4

Attach your printout with each graph labeled.

One of the nice features of the TI-86 is that we can actually watch the graphs being plotted, whereas with MAPLE they just all of a sudden appear. However, MAPLE does have a nice animation feature, which is useful for demonstrating the effects of changes of parameters on the graphs.

In the same package as above, that is to say **with(plots)**: execute the following commands and see what happens.

animate(t*x,x=-10..10,t=-1..1);
animate(x+t,x=-10..10,t=-2..2);
animate((x^2)+t,x=-2..2,t=-2..2);
animate((x+t)^2,x=-2..2,t=-2..2);
animate(t*sin(x),x=-6..6,t=1..3);
animate(sin(x+t),x=-6..6,t=0..3);
animate(sin(t*x),x=-6..6,t=1..3);

Exercise: Explore the on-line HELP to see if you can discover where I found the above commands. A little on-line help and experimentation is all it takes.

Section V: 3-d Graphing

The MAPLE command for plotting a 3-dimensional graph is
plot3d(function in x and y, x=xmin...xmax, y=ymin...ymax, optional stuff);

Use this command to plot the graph of $z=x^2-y^2$ for x and y from -3 to 3.

Click on the graph and drag it around to see different views. Also click on the graph and experiment with the different buttons on the graph tool bar to see their various effects. (This is really neat stuff!)

Do the same things for the function $z=(x^2-y^2)*\cos(x)$.

Attach a printout of a "neat" 3-d graph.

Reflection: What is the most confusing thing about graphing with MAPLE?