Names: ______________________ / ______________________ / ______________________.

Chapter 4
Roots of Polynomials

Section I: Linear Polynomials

Consider the polynomial equation ax + b = 0, symbolically solve for the value of x.

X=

Of course the value of x is called the root of the linear polynomial ax + b. The TI-86 has a built in polynomial solver package accessed via **2nd POLY** on the key board. To find the roots of a polynomial using MAPLE we can use the command

solve(our polynomial=0);

Find the roots of the polynomials in each of three ways: first "by hand", then via the TI-86, and lastly using MAPLE. Record the answers in order left to right.

	P&B	SGC	CAS	Comments
1. x + 1				
2. 2x + 1				
3. $(\sqrt{2})x + 1$				
4. $\pi x + 1$				
5. ax + b				

Try #5 using the command **solve(a*x + b=0, x);** in MAPLE.
Why do you think the information ",x" must be put into the MAPLE command in #5 when we didn't need it in the others?

What conclusion do you make with regard to using the TI-86 directly to solve linear polynomials?

In using MAPLE in #5 we have encountered our first instance of using the amazing feature that distinguishes a Computer Algebra System (CAS) from the previous generation of mathematics software packages: it can do Symbolic Manipulation!

Section II. Quadratic Polynomials

Next let's solve some quadratics; that is, find the roots of polynomials of the form ax^2+bx+c. You have done this once, but do it again. The quadratic formula tells us the two roots of ax^2+bx+c are

x =	and	x=

As above, find the roots of the following "by hand", with the TI-86,(it should work here) and with MAPLE.(Can you guess which problems will require insertion of ,x into the solve command, and which will not?)

	P&B	SGC	CAS	Comments
6. x^2 -1				
7. x^2				
8. x^2 + 1				
9. x^2 - 2				
10. x^2 + 2				

What's special about #8 and #10?

Notice the difference in the way the TI-85 expresses complex numbers and the way MAPLE expresses them.

	P&B	SGC	CAS	Comments
11. $2x^2 + 3x - 2$				
12. $x^2 + x + 1$				
13. $x^2 + (\sqrt{2} - 2)x - 2\sqrt{2}$				
14. $x^2 + (1-\pi)x - \pi$				
15. $x^2 + (a+b)x + ab$				

Now see what MAPLE gives when you ask it to solve for the roots of
16. $ax^2 + bx + c$.

X =	& X =

This should look familiar to you, what is it?

Exercise: In each of the above, 6-15, try to use the **evalf** command in MAPLE to obtain the TI-86 approximate answer. You will first need to define something to be the roots, eg,

r:=expression;

defines r to be whatever is the expression. So

r:=solve(2x^2-3x+2=0,x);

represents the two roots of 2x^2-3x+2=0.
Since r represents two roots **evalf(r[1]);** will evaluate the floating point approximation to the first root and **evalf(r[2]);** will approximate the second. So try this for each of the above. See if you can guess in advance which work and which don't.

MAPLE ans $\xrightarrow[\text{evalf}]{}$ Approximation TI ans

	MAPLE ans	Approximation	TI ans
6.			
7.			
8.			
9.			
10.			
11.			
12.			
13.			
14.			
15.			

Next we will play with two MAPLE commands that further illustrate the nice features of symbolic manipulation:

expand(algebraic expression); and **factor(algebraic expression);**

In each of the following first do the required calculation "by hand", then let MAPLE do it.

by hand		MAPLE
	16. expand((x+3)*(x-4));	
	17. factor(%);	
	18. factor(2x^2 +3x -2);	

19. expand((ax+1)*(bx+1));

20. factor(x^2 +bx +ax +ab);

21. expand((x+I)*(x-I));

22. factor(%);

You should now be able to "build" quadratics with any roots you like using expand and factor. Make some interesting ones.

Chapter 4, Section III: Higher Degree Polynomials

In each of the following first find the roots with the TI-86, then with MAPLE, and finally factor the polynomial using MAPLE.

SGC	CAS	(Factored form)
	23. x^3-x^2-2x	
	24. $6x^3+11x^2+6x+1$	
	25. x^3+x^2+x+1	

26. x^4-1

27. x^4+1

28. $3x^4+13x^3+7x^2-17x-6$

29. $6x^5-5x^4+4x^3-4x^2-2x+1$

Exercise: We saw some quadratics and degree 4 polynomials which have only complex roots. Which numbers are they?

Experiment to see if you can find a cubic (degree 3) polynomial with only complex roots.

What is the relation between factoring a polynomial and finding its roots?

Is there an analog to the quadratic formula for cubic polynomials? To answer this use MAPLE to solve for the zeros of ax^3+bx^2+cx. They are

Why do you think students are never asked to memorize the general formula for finding the roots of a cubic polynomial?

Exercise: Complete the following set of MAPLE instructions. At each step explain just what it is you told MAPLE, or asked MAPLE to do.

```
>p:=5*x^3-5*x+1;
```

```
>solve(%);
```

```
>r:=%;
```

```
>simplify(r[1]);simplify(r[2]);simplify(r[3]);
```

```
>subs(x=r[1],p);subs(x=r[2],p);subs(x=r[3],p);
```

```
>evalf(r[1]);evalf(r[2];evalf(r[3]);
```

```
>fsolv(p);
```

Compare the last two results with what you obtain from the TI-86.

What do you think the "f" in **fsolve** refers to?

Find all the roots of each of the following polynomials:

	roots
p[1]:= x^2+x+1;	
p[2]:= x^3+x^2+1;	
p[3]:= $x^4+x^3+x^2+x+1$;	
p[3]:= $x^4+x^3+x^2+x+2$;	

Just for fun simplify (factor the numerator and denominator and cancel the common factors) the following rational expression. Define f as follows:

f:= $(2x^5+5x^4+6x^3+6x^2+4x+1)/(2x^5+9x^4+12x^3+12x^2+10x+3)$

Use MAPLE to pick off and define the numerator and denominator of f, factor them, and simplify f using the following command sequence:

```
>n:=numer(f); d:=denom(f); factor(n); factor(f); simplify(f);
```

Numerator factors are

Denominator factors are

Simplified expression is

Look at the factors of n and d and see if you can guess what you will get from the MAPLE commands gcd(n,d); and lcm(n,d);. What did you guess?

factor(gcd(num,den));=

factor(lcm(num,den));=

(gcd and lcm in MAPLE work on more general things than just integers. The commands igcd and ilcm are reserved just for integer calculations and sometimes work faster than the gcd and lcm commands.)

Finally try the following MAPLE operations:

expand($(x+y)^3$);=

expand($(n+1)^4$);=

Where have you seen these before?

Addendum 1 to Chapter 4

We and MAPLE know the quadratic formula. We've seen that MAPLE also knows a formula for finding roots of cubic polynomials. Does MAPLE appear to know such a formula for general 4th degree polynomials? In fact no such formula, involving radicals, exists for 5th or higher degree polynomials. Knowing this let's see if we can stump MAPLE. In each of the following solve using the TI-86, then using MAPLE, then use evalf on the MAPLE solutions to compare them with the TI-86.

x^2+x+1

x^3+x^2+x+1

$x^4+x^3+x^2+x+1$

$x^5+x^4+x^3+x^2+x+1$

$x^6+x^5+x^4+x^3+x^2+x+1$

Reflection: Some people think the skill to factor polynomials and simplify rational expressions (A.K.A. Algebra II) is becoming unnecessary because of CAS's like MAPLE. What is your opinion and why?

Addendum 2 to Chapter 4

AN ALTERNATE WAY TO DEFINE A FUNCTION OR POLYNOMIAL.

q:=5*x^5-5*x+1; defines the symbol q to represent the symbol 5*x^3-5*x+1 and MAPLE treats p as the polynomial symbolically.

If we want MAPLE to treat p as a function of x, thinking more like p(x) we define it as follows:

p:=x->5*x^3-5*x+1; defines the function p acting on the variable x.

This has some advantages as demonstrated in the following sequence of commands similar to that done previously. Indicate the MAPLE response and what you think MAPLE did each time.

p(t);

p(u+1);

p(3);

p(0);

solve(p(x)=0);

r:=%;

evalf(r);

p(r[1]);

simplify(%);

What should p(r[2]) and p(r[3]) both equal?   Why?

Notice how much easier substitution is if you have defined the expression as a function sending x to an expression in x.. What do the roots of p(x) have to do with its graph?