Names: ________________/______________________________/______________________.

Chapter 3
Elementary Number Theory

The expression **lcm(m,n)** stands for the least integer which is a multiple of both the integers m and n. The expression **gcd(m,n)** stands for the biggest integer that divides both m and n.

Exercise. Find **lcm** and **gcd** in the TI-86 CATALOG and place them into your custom catalog.

Following the procedures of the first Chapter 1, compute each of the following integers.

	B&P	SGC	CAS	Comments
1. gcd(6,8)				
2. lcm(6,8)				
3. gcd(6,8)lcm(6,8)				
4. gcd(140,429)				
5. lcm(140,429)				
6. #4 * #5				
7. 140*429				

Exercise. Experiment with several different pairs of integers m and n computing gcd(m,n)*lcm(m,n) and m*n. What relationship is suggested by your experiment?

Let's drop the pencil calculations for the time being, OK?

	SGC	CAS	Comments
8. $\gcd(101+45,36^2)$			
9. $\operatorname{lcm}(5(1+4^3),2^5)$			
10. 36/gcd(36,16)			
11. lcm(38,100)/gcd(38,100)			
13. lcm(1524,5587)			
14. lcm(1524,5588)			

Notice that #13 and #14 show us that bigger integers don't necessarily yield bigger lcm's.

15. $\gcd(10^{13},1050)$			
16. $\operatorname{lcm}(14^6,15^{15})$			
17. gcd(your ss#,a friends' ss#			
18. lcm(your ss#,same friends' ss#)			

Two integers are said to be <u>relatively prime</u> if the biggest integer that divides both of them is 1.

Exercise. List all pairs of integers from 100 to 105 that are relatively prime.

An integer is said to be prime if its only divisors are 1 and itself, otherwise it is called composite. For example 2,3,5,&7 are the only primes less than 10. What are the primes between 10 and 20?

The MAPLE command "isprime" determines whether or not a given integer is prime. Use it to see if the following numbers are prime. In each case circle P if prime and C if composite.

29	35	1537	your ss#	10!+1	329891.
P C	P C	P C	P C	P C	P C

* You might find the programs and the end of this chapter to be interesting.
The **FUNDAMENTAL THEOREM OF ARITHMETIC** states that every integer can be factored in a unique way into a product of powers of primes.

Exercise. Use paper and pencil, aided by the TI-86 if necessary, to write each of the following numbers as a product of powers of primes:

i. 10!=

ii. 340704=

The MAPLE command "ifactor" does this automatically. Now use it to compute the prime factorization for each of the following:

i. 10!

ii. 340704

iii. 10!+1=

iv. 100!=

iv. your ss# =

(Notice how just adding 1 to 10! reduced the number of prime factors.)

For two integers m & n with $n \geq m$ we say the remainder of n divided by m is r if n = qm + r, for integers q &r with $0 \leq r < m$. In MAPLE this remainder is denoted by "n mod m". In the TI-86 CATALOG it is denoted by "mod(n,m)".

Exercise Use P&B, SGC, and CAS to compute the following remainders:

	P&B	SGC	CAS
24 mod 5 =			
156 mod 7 =			
32 mod 2 =			
57 mod 2 =			

Compute each of the following remainders using the TI-86 first and MAPLE next, as usual.

	SGC	CAS
19. 13200 mod 134		
20. (12^3+53) mod 26		
21. 400*23 mod 19		
22. 1321 mod (5*7)		
23. 45^{16} mod 2		

Exercise. Which answer in number 23 do you think is correct?

Explain

Exercise. Explain why an integer n is even exactly when n mod 2 =0, and n is odd exactly when n mod 2 =1.

Recall the statement of the **BINOMIAL THEOREM.**

$(m + n)^k =$

Exercise. Use the binomial theorem to explicitly expand, (ie. compute the binomial coefficients)

i. $(x + y)^3 =$

and

ii. $(n + 1)^4 =$

Exercise. Explain why every power of an odd integer must be an odd integer also.

You should be able to compute each of the following **using brain only**. Do so and then check your answer using both technologies. Record your results in the appropriate places.

	Brain	SGC	CAS	Comments
24. 1342 mod 2				
25. (10!+1) mod 2				
26. 17^9 mod 2				
27. 17^{11} mod 2				
28. 17^{12} mod 2				

Exercise. Explain why the TI-86 thinks 17^{12} is even, when we, and MAPLE, know better.

Reflection:

1. What are the main things you learned from Chapter 3?

2. What's the hardest thing to understand from Chapter 3?

* Look at the following MAPLE programs. Try to determine what they should do and then execute them to see if you were correct. (To move from a line of MAPLE input to another line without MAPLE wanting to execute something, hold down the shift key as you press the enter key.)

Program I

```
> for i from 1 to 100
do
 if
  isprime(i)=true
    then print(i)
 fi;
od;
```

Program II

```
> for i from 100 to 105
  do
    for j from i to 105
       do
         if
           gcd(i,j)=1
            then print(i,j);
         fi;
       od;
   od;
```