A classification of attitudes and beliefs towards mathematics for secondary mathematics pre-service teachers and elementary pre-service teachers: An exploratory study using latent class analysis

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Abstract

This study describes a series of latent class analyses used to classify pre-service teachers based on their responses to questions on a survey regarding their attitudes and beliefs about mathematics. Results identified the pre-service teachers with the most positive attitudes and beliefs towards mathematics. Belief measures regarding confidence, enjoyment, and motivation were in the two least positive classes for elementary pre-service teachers. Recommendations for exposing pre-service teachers to positive attitudes and beliefs about mathematics are discussed.

Keywords: *latent class cluster analysis; mathematics attitudes and beliefs; pre-service teachers.*

Introduction

Today's society is inundated with technology of numerous forms. From our day-to-day lives of cell phones and computers, to medical advances and space exploration, the contribution of mathematics is steadfastly increasing. Never before has it been more important for students of all ages to comprehend and master concepts in mathematics. In order to do this, teachers of mathematics across all levels of the curriculum must be able to assist students in developing positive attitudes and beliefs towards mathematics.

For over thirty years, researchers have been investigating students' attitudes and beliefs towards mathematics. Fennema and Sherman are best known for the development the Fennema-Sherman Mathematics Attitude Scales, (Fennema & Sherman, 1976). These scales have been widely used to measure the attitudes and beliefs of students across all levels of the mathematics curriculum. In using various adaptations of this questionnaire, researchers have found a strong relationship between positive attitudes and beliefs towards mathematics and academic success in the subject. (i.e. Ashcraft & Kirk, 2001; Schenkel, 2009; Sherman & Christian, 1999; Tapia & Marsh, 2004; van der Sandt, 2007).

In light of these findings, it is important to identify where positive attitudes and beliefs towards mathematics are developed. Numerous studies have found that there is a connection between teachers' attitudes and their students' attitudes (Anderson, 2007; Ma & Xu, 2004; Relich, 1996). For instance, Relich (1996) found that teachers who had been identified as having low self-confidence in mathematics attributed this feeling to negative experiences in school mathematics, even when they had had a positive attitude towards mathematics previously. Also found in this study was that these teachers had low expectations of their own students, thus perpetuating their attitudes and beliefs about mathematics. The counterpoint belief was found as well, as teachers who had been identified as having high self-confidence in mathematics, attributed their success and enjoyment of mathematics to a previous teacher or teachers who had had a positive affect on them. As a consequence, these teachers tended to believe that anyone could do mathematics successfully and therefore had higher, more positive expectations of their students.

Anderson's (2007) research found that "[T]he most significant potential to influence students' identities exists in the mathematics classroom" (p. 12). When students, especially younger ones, are encouraged by teachers and find success in mathematics, their attitudes and beliefs can drastically improve (Ma & Xu, 2004). Similarly, Midgely, Feldlaufer, and Eccles (1989) found that mathematics teachers' beliefs in their efficacy to teach mathematics had an affect on their students. A significant relationship between teacher efficacy and students' confidence and beliefs in their ability to do mathematics was found. Specifically, students in the classes of teachers with a positive sense of efficacy in teaching were more likely to believe that they were performing better in mathematics than students in the class of teachers with a lower sense of efficacy in teaching mathematics. In addition, students of teachers with high efficacy believed mathematics to be less difficult than students of lower efficacy teachers. Overall, teachers' attitudes had a stronger relationship to the beliefs in mathematics of low-achieving students than to the beliefs in mathematics of high-achieving students.

This leads to the conclusion that it is important for teachers across all levels of mathematics instruction to exhibit positive attitudes and beliefs in order to allow their students to develop positive attitudes and beliefs towards mathematics. Unfortunately, this is not always the case. As researchers have discovered, teachers tend to shape their mathematics classroom practice based upon their own attitudes and beliefs, (i.e. Bolhuis & Voeten, 2004; Relich, 1996), and thus transferring their own attitudes and beliefs to their students. Furthermore, some studies have asserted that many pre-service teachers have been found to have negative attitudes towards mathematics that had developed when they were students, thus continuing a negative cycle (Arp, 1999).

While there have been numerous studies comparing the differing attitudes and beliefs of pre-service teachers towards mathematics, few studies compare the attitudes and beliefs of elementary pre-service teachers and secondary pre-service mathematics teachers. It is believed that teachers' behavior in the classroom is affected by their knowledge and comfort with the subject matter (van der Sandt, 2007). It might be assumed that when secondary teachers have chosen mathematics as the only subject they wish to teach, they must have a high sense of efficacy in teaching mathematics and therefore a more positive attitude towards mathematics than do elementary teachers who have chosen to teach all subjects. It has been found that many school administrators believe that as long as someone has the knowledge of mathematics and a relatively clear memory of how it was taught, that person is capable of teaching mathematics (Darling-Hammond, 2006). It is therefore worth investigating the attitudes and beliefs of pre-service teachers to determine how the attitudes of prospective secondary mathematics teachers compare with those of prospective elementary school teachers. Of particular interest is whether there are elementary teachers who have positive attitudes and beliefs towards mathematics as do secondary mathematics teachers, and if there are secondary mathematics teachers with negative attitudes and beliefs towards mathematics.

In addition to comparing elementary pre-service teacher attitudes and beliefs towards mathematics with the secondary mathematics pre-service teacher attitudes and beliefs towards mathematics, two other aspects were the focus of this study. Keeping in mind the interrelationship between teacher attitudes and beliefs and student attitudes and beliefs towards mathematics, this study was designed to investigate the attitudes and beliefs of teachers towards mathematics from a different perspective. In many studies on this topic (i.e. Allen, 2001; Clarke, Thomas, & Vidakovic, 2009; Sherman & Christian, 1999; White, Way, Perry, & Southwell, 2005-2006), pre-service elementary teachers have been examined as if they all had the same preparation and background in the process of becoming certified teachers. It has been found that attitudes and beliefs towards mathematics of elementary teachers differ, but possible identifiers that could be used to distinguish between teachers with positive attitudes and teachers with negative attitudes have not been explored. This study was undertaken in an attempt to discover if any characteristics of pre-service teacher background and preparation that may be related to more positive attitudes and beliefs towards mathematics, and thus to ultimately lead to effective teaching in mathematics. This study illustrates how latent class cluster analysis can be used to identify specific positive and/or negative attitude and belief items for pre-service teachers, and further describes how this can provide an opportunity to incorporate program changes that could provide pre-service teachers with an opportunity to gain more positive attitudes and beliefs towards mathematics.

Instrument Development

The original Fennema-Sherman Attitude Scales (1976) were designed to measure the attitudes and beliefs of secondary students. They consist of a group of nine instruments: (1) Attitude Toward Success in Mathematics Scale, (2) Mathematics as a Male Domain Scale, (3) and (4) Mother/Father Scale, (5) Teacher Scale, (6) Confidence in Learning Mathematics Scale, (7) Mathematics Anxiety Scale, (8) Effectance Motivation Scale in Mathematics, and (9) Mathematics Usefulness Scale.

The Attitudes Toward Mathematics Inventory, created by Tapia and Marsh (2004) was based on the Fennema-Sherman instrument, with some items eliminated in order to focus on only six factors (Confidence, Anxiety, Value, Enjoyment, Motivation, and Parent/teacher expectations). An adaptation of the Tapia and Marsh questionnaire was created for use in this study to be appropriate for college students who are elementary and secondary pre-service teachers. Questions relating to parent influence were eliminated and questions relating to confidence in teaching mathematics were included (see Appendix 1 for the instrument that was used).

The instrument presented in Appendix 1 was designed to measure the attitudes and beliefs of pre-service teachers. A total of 72-items were constructed to assess confidence, anxiety, value, enjoyment, motivation, and teacher expectations as identified by previous researchers (i.e. Allen, 2001; Clarke, et al., 2009; Ma & Kishor, 1997; Nicolidau & Philippou, 2004; Singh, Granville, & Dika, 2002; Tapia & Marsh, 2004). Appendix 2 provides a list of the questions that were used for each of the six categories of attitude and belief items.

The first category, the confidence category, attempts to measure students' confidence in approaching mathematical tasks and belief in their ability to successfully complete these tasks. An example of the statements on the survey that were designed to measure attitudes in this category is: “I generally have had difficulty relating new mathematical concepts to those I had previously learned.” A response of ‘Strongly Agree’ would indicate the lowest level of confidence, while a student’s response of ‘Strongly Disagree’ would indicate a high level of confidence. The second category, the anxiety category attempts to measure students’ level of anxiety and the effect of this anxiety on their performance in mathematics. An example of the statements that were used to measure anxiety is: “I get really uptight during math tests.” A student who responded by selecting ‘Strongly Agree’ would have a high level of anxiety about mathematics, whereas a student who responded by selecting ‘Strongly Disagree’ would have a low level of anxiety.

The third of the six categories, the value of mathematics category, attempts to measure students' beliefs on the importance and relevance of mathematics in their present and future daily lives. An example of how this was measured on the survey is the statement: “Math is needed in designing practically everything.” The fourth category, the enjoyment of mathematics category, attempts to measure the level of enjoyment that students experience when working on mathematical tasks. “Mathematics is enjoyable and stimulating to me” is an example of the statements that were used to measure responses in this category. The next to the last category, the motivation category, attempts to measure students’ interest in pursuing additional experiences in mathematics, and was measured by having students response to statements such as “I avoid taking math classes in college.” And finally, the teacher expectations category, attempts to measure the students’ perception of their teachers’ beliefs and expectations. An example of the statements on the survey that measured this category of attitudes is “I can recall math teachers who made me feel stupid in class.”

Data Collection

A sample of elementary and secondary pre-service teachers attending a large state university in the northeast United States completed this questionnaire (n = 293). Some of these college students, both elementary pre-service and secondary pre-service teachers, are just beginning their pedagogical preparation. Another group of elementary and secondary pre-service teachers are midway through their program, and the final group is comprised of future elementary and secondary educators who are at the final stage in their program before student teaching.

At this university, elementary pre-service teachers must declare a subject matter major. The elementary pre-service teachers in this study were divided into the following majors: mathematics (n = 84), mathematics and biology (n = 6), mathematics and earth science (n = 3), English (n = 55), English and Geography (n = 13), History (n = 25), English and History (n = 13), special education (n = 5), and K-12 in art, physical education, music or other (n = 33). For the purposes of this study, the only elementary majors considered to be mathematics majors were the pre-service teachers who only had a mathematics focus. In addition, there were 56 Secondary Mathematics pre-service teachers, who will be earning state certification to teach mathematics in grades 7 through 12.

The responses to the survey were measured on a Likert scale and ultimately coded as interval data such that a "1" represented more negative attitudes and beliefs about mathematics, whereas a "5" represented more positive attitudes and beliefs about mathematics. By coding the responses as interval data, this allowed for descriptive measures such as mean scores to be determined for multiple questions that were used in each of the six categories (Kelley, 1999).

Methodology

Clearly, mathematics teachers at all levels of the curriculum vary in their ability to be effective teachers. But one can argue that teacher effectiveness is a latent class that cannot easily be measured or observed. More specifically, teacher effectiveness can be represented by a discrete variable, representing either being an effective teacher or not. Furthermore, one could also suggest that being an effective mathematics teacher is based on having more positive attitudes and beliefs about mathematics. Therefore, by using measures of attitudes and beliefs about mathematics, this allows one to identify latent class memberships for teacher effectiveness, and these latent class memberships can be based on attitudes and beliefs about mathematics (Methén, 2001; UCLA Academic Technology Services).

Latent class cluster analysis (henceforth referred to as latent class analysis) is a model-based alternative to cluster analysis (Methén, 2001), and can be used to study the interrelationships between discrete or continuous observed variables and a discrete latent variable (McCutcheon, 1987). Although techniques such as cluster analysis have been extensively used to establish groups of classes or clusters, it is generally considered to be more of an ad hoc technique (Hair, Anderson, Tatham, & Black, 1998). This is because both hierarchical and nonhierarchical clustering procedures tend to rely on various stopping algorithms, along with other techniques that can be used to establish the optimal number of clusters (Hair, et al., 1998; Sugar & Gareth, 2003). However, this can pose a problem because the number and composition of the clusters often depends upon the choice of the stopping rules and the selection techniques that are used. Latent class analysis allows for a more systematic and model-based approach for both determining the number and description of the clusters (or classes), and thus allows for generalizations to be made to a larger population (Methén, 2001). The reason generalizations can be made with latent class analysis is because inferential techniques, such as maximum likelihood estimation, is used in latent class analyses, and thus various types of fit statistics can be used to determine the optimal number of classes that is based on the structure of the underlying data and the number of classes, where the number of classes nor the selection measures are pre-supposed by the researcher (Herman, Ostrander, & Walkup, 2007).

In order to discover any different patterns or classes in attitudes and beliefs of the pre-service teachers, the statistical software package M-Plus (version 5) was used to run a series of latent class analyses. To determine the optimum number of classes for each of the latent class analyses, four indicators of model fit were considered. They were the Bayesian Information Criterion (BIC), difference in BIC, entropy, and a likelihood difference test (Methén, 2001). The BIC statistic can be used to determine how much new information is gained with increasing numbers of classes, where smaller values of the BIC statistic suggest a better fit. Entropy was also used to determine how well the model classified the subjects, along with the Vuong-Lo-Mendall-Rubin likelihood difference test (Vuong, 1989) to determine if adding additional classes improved the model fit by more than just a small amount. Two separate latent class analyses were conducted; the first analysis included both secondary and elementary pre-service teachers, and the second analysis included only elementary pre-service teachers.

Results

The results of the first latent class analysis which included both secondary and elementary pre-service teachers, revealed a three-class model as can be seen in Table 1.

Table 1: Fit statistics for the first latent class analysis (n = 293).

Number of classes	**BIC**	**Difference in BIC**	**Adjusted BIC**	**VLMR *p*-value**	**Adjusted VLMR *p*-value**	**Entropy**
2	5086.693	---	4997.898	---	---	0.952
3	**4681.445**	**405.248**	**4557.766**	**0.0093**	**0.0100**	**0.949**
4	4488.118	193.327	4329.555	0.1436	0.1481	0.932
5	4470.407	17.711	4276.960	0.5524	0.5543	0.932
6	4324.552	145.055	4096.222	0.5676	0.5733	0.942
7	4287.902	36.650	4024.688	0.1011	0.1044	0.933

The four- and five-class models showed only a modest drop in BIC that would indicate that model parsimony only slightly improved by considering the four- or the five-class model instead of the three-class model. Furthermore, the entropy for the three-class model was higher than the four-class or the five-class model. The Vuong-Lo-Mendall-Rubin likelihood difference test and the adjusted Vuong-Lo-Mendall-Rubin likelihood difference test, also suggest that the four-class model does not fit any better than the three-class model ($p > 0.05$). Therefore, by considering all these fit statistics together, it was decided that the three-class model was the best fitting model when all pre-service teachers were considered. Based on the three-class model, Table 2 gives the counts and proportion of elementary and secondary pre-service teachers that fall into each of the three latent classes.

Table 2: Counts and proportions of elementary and secondary pre-service teachers that fall into each of the three latent classes in the first analysis (n = 293).

Latent Class	**Count**	**Proportion**
1 – Math Negative	64	21.84
2 – Math Neutral	83	28.33
3 – Math Positive	146	49.83

Table 3 gives the probabilities of females, minorities, and secondary majors that fall into the three latent classes. Table 4 gives the mean scores on the attitude and belief items that are based on the three latent classes.

Using similar measures of fit to determine the optimal number of latent classes, the results of this second latent class cluster analysis, in which only elementary pre-service teachers were considered, revealed a five-class model that can be seen in Table 5.

Table 3: Probabilities for the categorical measures of gender, minority status, and secondary versus elementary track by latent class for the first analysis (*n* = 293).

	Class 1 Math Negative	**Class 2 Math Neutral**	**Class 3 Math Positive**
Female	0.854	0.822	0.775
Minority	0.109	0.107	0.103

Table 4: Mean scores on attitude and belief items based on latent class for secondary and elementary pre-service teachers (*n* = 293).

	Class 1 Math Negative	**Class 2 Math Neutral**	**Class 3 Math Positive**
Confidence	2.171	2.974	3.707
Anxiety	2.337	3.421	4.122
Value	3.105	3.627	4.309
Enjoyment	2.248	3.252	4.181
Motivation	2.207	3.244	4.242
Teacher	3.058	3.268	3.535

Table 5: Fit statistics for the second latent class analysis (*n* = 237).

Number of classes	**BIC**	**Difference in BIC**	**Adjusted BIC**	**VLMR *p*-value**	**Adjusted VLMR *p*-value**	**Entropy**
2	4107.149	---	4018.398	---	---	0.950
3	3746.030	361.119	3622.413	0.0055	0.0060	0.949
4	3604.788	141.242	3446.306	0.0637	0.0675	0.940
5	**3515.352**	**89.436**	**3322.003**	**0.0487**	**0.0518**	**0.945**
6	3630.942	-115.590	3402.728	0.8156	0.8138	0.941
7	3665.241	-34.299	3402.160	0.8836	0.8801	0.949

Discussion

As is seen in Table 2, the three classes found in the first latent class analysis consisting of both secondary and elementary pre-service teachers, with percentages given parenthetically will be referred to as Math Negative (21.84%), Math Neutral (28.33%), and Math Positive (49.83%). Table 3 illustrates the composition of each class based on gender, minority status, and secondary major status. As can be seen in Table 4, the mean Math Negative scores were the lowest of the three classes in every single category of attitudes and beliefs. The mean Math Positive class scores were the highest of the three classes in every single category. The mean Math Neutral class scores were between the low and high scores in every single category, with all scores of approximately 3.

The mean Math Negative class scores in the six categories were between 2.171 and 2.337 for confidence, anxiety, enjoyment, and motivation. The mean scores for value and teacher expectations were slightly higher with 3.105 and 3.058 respectively. While the students in this latent class believe that mathematics is somewhat useful, they may not enjoy it, and lack confidence in their ability to do it.

The mean Math Positive class scores in the six categories were between 3.707 and 4.309 for confidence, anxiety, value, enjoyment, and motivation. The mean score for teacher expectations was slightly lower at 3.532. The students in this class believe that mathematics is useful and are relatively confident in their ability to do it.

The mean Math Neutral class scores ranged from 2.974 (confidence) to 3.627 (value). The mean score for teacher expectations was within this range at 3.268. These students do not have strong opinions in these categories, having neither positive nor negative responses, on average.

Interestingly, all three classes had similar mean scores for teacher expectations, with Math Negative, 3.058, Math Neutral, 3.268, and Math Positive, 3.532. As previously stated, studies have indicated that mathematics teachers transfer their own attitudes to their students. This begs the question as to why all three classes of students have similar perceptions of teacher expectations while having varied personal attitudes and beliefs towards mathematics. This may be explained by the fact that these students have taken mathematics classes from mathematics specialists for at least the previous seven years in middle school, high school, and college. It is likely that students answering the questionnaire were reflecting on the expectations and behaviors of those teachers, and not their elementary school teachers. Therefore, if teachers do affect students' attitudes, it may be that their elementary teachers had the greatest impact, and by the time they entered secondary school, their attitudes were already ingrained.

When looking at the make-up of students in each latent class it became increasingly clear that the secondary mathematics majors had, indeed, the most positive attitudes towards mathematics. In fact, the Math Negative class was made up entirely of elementary pre-service teachers. The Math Neutral class was 97.5% elementary pre-service teachers. Almost all of the secondary mathematics majors who completed the questionnaire were in the Math Positive class. The Math Positive class, however, was 62.8% elementary pre-service teachers, representing 92 of the subjects in this study. Since 38.8% of the elementary pre-service teachers' attitude ratings were comparable to almost all of the secondary mathematics majors, it became apparent that more information about elementary pre-service teachers could be found by running a second latent class analysis for the elementary pre-service teachers alone.

Table 5 presents the results of the second latent class analysis that included only elementary pre-service teachers, and this revealed a five-class model as optimal. In this analysis, the "mathematics major" category was now used to differentiate between elementary mathematics majors and elementary non-mathematics majors. In this study, students that had a mathematics/earth science or mathematics/biology were not counted in the mathematics major category. Table 6 gives the counts and proportion of elementary pre-service teachers that fall into each of the five latent classes, and these are referred to as Most Math Negative (13.50%), Math Negative (22.37%), Math Neutral (24.47%), Math Positive (28.27%), and Most Math Positive (11.39%). Table 7 gives the probabilities of the latent class composition by gender, minority status, and elementary mathematics major status. Table 8 gives the mean scores on the attitude and belief items for each of the five latent classes for the elementary pre-service teachers.

As Table 8 illustrates, the Most Math Negative students' responses resulted in mean scores between 1.891 and 2.040 in the categories of confidence, anxiety, enjoyment, and motivation. The mean scores for value and teacher expectations were quite a bit higher with 2.975 and 2.924 respectively. The Math Negative students scored, on average, between 2.538 and 2.861 in the categories of confidence, anxiety, enjoyment, and motivation. The mean scores for value and teacher expectations were higher with 3.306 and 3.206 respectively. Students in both of these classes have negative attitudes and beliefs about the way in which they relate to mathematics, while still believing that mathematics has some importance in the world, and having a neutral view of teacher expectations.

Table 6: Counts and proportions of elementary pre-service teachers that fall into each of the five latent classes in the second analysis (*n* = 237).

Latent Class	Count	Proportion
1- Most Math Negative	32	0.1350
2 – Math Negative	53	0.2237
3 – Math Neutral	58	0.2447
4- Math Positive	67	0.2827
5- Most Math Positive	27	0.1139

Table 7: Probabilities for the categorical measures of gender, minority status, and major (mathematics versus non-mathematics for elementary pre-service teachers) by latent class for the second analysis (*n* = 237).

	Class 1 Most Math Negative	Class 2 Math Negative	Class 3 Math Neutral	Class 4 Math Positive	Class 5 Most Math Positive
Female	0.860	0.874	0.794	0.822	0.925
Minority	0.152	0.060	0.121	0.089	0.000
Elementary Math Major	0.000	0.000	0.145	0.846	0.757

Table 8: Mean scores on attitude and belief items based on latent class for elementary pre-service teachers (*n* = 237).

	Class 1 Most Math Negative	Class 2 Math Negative	Class 3 Math Neutral	Class 4 Math Positive	Class 5 Most Math Positive
Confidence	1.891	2.538	3.041	3.613	4.000
Anxiety	2.040	2.861	3.481	4.049	4.481
Value	2.975	3.306	3.694	4.132	4.539
Enjoyment	1.964	2.645	3.370	3.949	4.481
Motivation	1.951	2.581	3.372	4.008	4.512
Teacher	2.924	3.206	3.296	3.543	3.795

The Math Neutral mean scores ranged from 2.538 (confidence) to 3.306 (value). These students do not appear to have strong opinions in these categories, having neither positive nor negative responses, on average, yet still placing the value of mathematics of highest positive attitude.

The Math Positive mean scores in the six categories were between 3.613 and 4.132 for confidence, anxiety, value, enjoyment, and motivation. The mean score for teacher expectations was slightly lower at 3.542. The Most Math Positive mean scores in the six categories were between 4.000 and 4.512 for confidence, anxiety, value, enjoyment, and motivation. The mean score for teacher expectations was slightly lower at 3.795. The students in both of these classes believe that mathematics is useful and are relatively confident in their ability to do it.

Perhaps the most striking result can be found in Table 7, which shows that in the two lowest classes (Most Negative Math and Math Negative), there were no elementary mathematics majors. The Most Negative Math latent class and the Negative Math latent class were completely comprised of elementary pre-service teachers with majors other than solely mathematics. The Neutral Math latent class was composed of 85.5% elementary non-mathematics majors and 14.5% elementary mathematics majors. The two positive mathematics attitude latent classes were 15.4% (Positive Math) and 24.3% (Most Positive Math) non-mathematics majors. Clearly the overwhelming majority of these pre-service students who have positive attitudes towards mathematics are elementary mathematics majors with 84.6% of the Positive Math latent class, and 75.7% of the Most Positive Math latent class.

Summary

The first latent class analysis identified 146 pre-service teachers in the Positive Math class. Of these, 54 were secondary pre-service teachers. As there were only 56 secondary pre-service teachers who participated in this study, clearly the vast majority of these students have positive attitudes towards their ability to do mathematics. This result was not unexpected.

It became clear though, that there were a large number of elementary pre-service teachers (n = 92) who were also in the Positive Math latent class. Many studies have concluded that elementary pre-service teachers do not have positive attitudes towards mathematics (Arp, 1999). Other studies reported that not all elementary teachers had negative attitudes, but no research has described any identifying variables. This study attempted to, and found, a significant identifying variable, namely subject matter focus.

For the university considered in this study, students who are preparing to become elementary school teachers must identify a subject matter focus. In this study, the pre-service Elementary Mathematics majors had more positive attitudes towards mathematics than did Elementary Education majors with a subject focus that did not solely include mathematics. While this result may not be surprising, it appears to have been overlooked in previous studies. Although elementary pre-service teachers with a mathematics focus had more positive attitudes, not all of them were in the Most Positive latent class. Previously reported as percentages, the Neutral Math latent class was composed of eight Elementary Mathematics majors, while the Positive Math attitude latent classes and the Most Positive Math attitude latent class contained 56 and 20 Elementary Mathematics majors respectively. Furthermore, as can be seen in Table 8, the lowest mean scores in the Most Math Negative and the Math Negative classes represent confidence, enjoyment, and motivation.

Now that this information has been uncovered, consideration should be given as to the circumstances in which it can be used productively. Two particular avenues present themselves at this time. The first is in consideration of the pre-service teachers whose responses to the questionnaire suggested negative attitudes towards mathematics. The second is in consideration of how the elementary mathematics majors in the different levels of more positive attitudes can be distinguished from one another.

The teacher preparation at this institution recognizes the importance of providing all elementary pre-service teachers opportunities to improve their attitudes and beliefs towards mathematics as they take required classes in their programs. At this time, non-mathematics elementary majors are only required to take three mathematics methods courses, while the elementary mathematics majors take a total of five mathematics methods classes and six mathematics classes. In addition, the mathematics methods classes are taught by a number of different professors, some full-time and some part-time, leading to the possibility that not all students are receiving the same positive encouragement in these courses that might improve their attitudes and beliefs towards mathematics. The results of this study indicate that it is imperative that the non-mathematics majors have positive experiences in their mathematics classes. In order to accomplish this, professors of the mathematics methods courses can use knowledge of the students' majors to differentiate instruction in an effort to give the non-mathematics majors the kinds of experiences that will improve their confidence and beliefs, and ultimately, their attitudes towards mathematics.

As the second latent class analysis revealed, there were three attitude and belief items that were the lowest for the elementary pre-service teachers that were in the Most Math Negative and Math Negative classes (Table 8). Given that these three attitude and belief items were the lowest in these two classes, consideration needs to be given to how the elementary pre-service curriculum can be revised to incorporate ways to help students who may be in these two latent classes to experience greater confidence with mathematics, more enjoyment of mathematics, and become more motivated in doing mathematics. As identified by Allen (2001), negative experiences in the mathematics classroom can lower the self-concept of students. Professors of these pre-service students therefore must focus on ways in which to make mathematics class a more positive environment. Professors should strive to present material in a way that will help students develop their own skills while they are learning how to teach the skills to children. Research exists that indicates that attitudes towards mathematics can be linked to different learning styles (Peker, 2009). Teaching students in ways that best coincide with their learning styles can help them process mathematical concepts, thus giving them more success in mathematics (Knisley, 2002). It has been found to be the case that positive experiences can contribute to the development of positive attitudes. This can be created by professors with positive attitudes towards pre-service students, giving them individual attention and encouragement, and demonstrating to them a strong commitment to help them learn (Allen, 2001; Peker & Mirasyedioğlu, 2008).

Another observation made of the results of the study is that even though elementary mathematics majors are more likely to have positive attitudes and beliefs towards mathematics, not all of these pre-service teachers have the same level of positivity. If attitudes and beliefs towards mathematics of elementary teachers should be one of the criteria considered among the administrators in any school district that are charged with the responsibility of hiring of teachers, the assumption that a candidate who was an elementary mathematics major will have a very positive attitude towards mathematics is faulty. Even though this study found that for pre-service teachers, mathematics majors are more likely to have more positive attitudes and beliefs towards mathematics than non-mathematics majors, the results indicate that mathematics majors have attitudes that range from Math Neutral to Most Math Positive. This suggests that further investigation of differences among these students that might lead to determining if objective data exists that can differentiate the elementary mathematics majors in the Math Most Positive latent class from those in the Neutral or Math Positive latent classes

Clearly this study sets the stage for further investigations of how to discern the attitudes and beliefs of prospective teachers by using objective data. School administrators may have access to candidates' academic records that describe the courses taken, overall grade point average, and success in mathematics courses. This information includes the student's subject matter major, but if there are additional identifiers within this data that can set apart students with the most positive attitudes towards mathematics, this information could be extremely useful to school administrators in setting hiring policies for elementary teachers.

Limitations

Since this study was conducted at a single university, generalizations to other universities, even those with similar programs, may not be externally valid. Furthermore, there were a relatively small number of secondary majors in this study, as well as small numbers of elementary majors whose focus was not mathematics, and thus we were not able to make more refined distinctions between the different majors.

Although latent class analysis is a non-parametric technique that does not make any assumptions regarding any underlying distributions, it is assumed that the observations are independent within each class. Although it seems reasonable to suggest that the observations are independent within each class for this analysis, this assumption could have been violated if some of the members of a given class shared some type of dependent relationship. While some researchers have found that this assumption is a crucial aspect of latent class analysis (i.e. Song & Fox, 2005), the analysis conducted for this paper was more exploratory and primarily used to identify a set of specific attitudes and beliefs that may lead to effective teaching, and also to suggest program changes to incorporate opportunities for students to be exposed to such attitudes and beliefs in a positive way in their elementary pre-service teacher classes.

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Appendix 1: The adapted Fennema-Sherman instrument (1976).

SD = Strongly Disagree D = Disagree U = Undecided A = Agree SA = Strongly Agree

1	I have usually been at ease during math tests.	SD	D	U	A	SA
2	I struggled with many concepts in mathematics.	SD	D	U	A	SA
3	My teachers relied on overhead projectors or chalkboards as tools to present information.	SD	D	U	A	SA
4	My teachers spent the necessary amount of time helping me to understand math concepts.	SD	D	U	A	SA
5	I do not want to teach mathematics in the future.	SD	D	U	A	SA
6	I had many competent math teachers.	SD	D	U	A	SA
7	I have often helped others with their math homework.	SD	D	U	A	SA
8	My teachers emphasized understanding and not just memorization.	SD	D	U	A	SA
9	I elected to take part in mathematical competitions.	SD	D	U	A	SA
10	During my math classes I was expected to sit quietly and listen.	SD	D	U	A	SA
11	I usually comprehended math content well and seldom got lost.	SD	D	U	A	SA
12	I did not feel comfortable seeking help from my math teachers outside of class.	SD	D	U	A	SA
13	I did not like being introduced to new mathematical content.	SD	D	U	A	SA

14	Mathematics makes me feel uncomfortable and nervous.	SD	D	U	A	SA

SD = Strongly Disagree D = Disagree U = Undecided A = Agree SA = Strongly Agree						
15	I get really uptight during math tests.	SD	D	U	A	SA
16	My teachers focused mainly on memorization facts and procedures.	SD	D	U	A	SA
17	My math teachers were supportive in my efforts to learn mathematics.	SD	D	U	A	SA
18	My teachers assigned several homework problems each night.	SD	D	U	A	SA
19	1 almost never get uptight while taking math tests.	SD	D	U	A	SA
20	My teachers had confidence in me as a student of mathematics.	SD	D	U	A	SA
21	I learned best when my teachers took the time to connect new concepts to that which I had already learned.	SD	D	U	A	SA
22	I have usually been at ease during math courses.	SD	D	U	A	SA
23	I chose a major that did not require too many math courses.	SD	D	U	A	SA
24	I have taken math classes even though they were not required.	SD	D	U	A	SA
25	I have dropped math courses because they became too difficult.	SD	D	U	A	SA
26	I usually don't worry about my ability to solve math problems.	SD	D	U	A	SA
27	New math content has usually been easy for me to understand.	SD	D	U	A	SA

28	I did not take a math class my senior year in high school.	SD	D	U	A	SA
29	It wouldn't bother me at all to take more math courses.	SD	D	U	A	SA

SD = Strongly Disagree D = Disagree U = Undecided A = Agree SA = Strongly Agree						
30	When confronted with a difficult math concept, I generally worked until I understood the concept.	SD	D	U	A	SA
31	I look forward to teaching mathematics.	SD	D	U	A	SA
32	I can't recall many mathematical concepts that were hard for me to understand.	SD	D	U	A	SA
33	My math teachers were very patient with me.	SD	D	U	A	SA
34	Many of my math teachers were incompetent.	SD	D	U	A	SA
35	My teachers did not believe I was capable of learning mathematics.	SD	D	U	A	SA
36	When I had trouble with a concept I usually gave up and stopped trying.	SD	D	U	A	SA
37	I get a sinking feeling when I think of trying hard math problems.	SD	D	U	A	SA
38	My teachers often applied their math lessons to real world situations.	SD	D	U	A	SA
39	Mathematics makes me feel uneasy and confused.	SD	D	U	A	SA
40	My teachers used a combination of manipulatives, visual aids, and cooperative learning.	SD	D	U	A	SA
41	1 was frequently lost and had trouble keeping up in my math classes.	SD	D	U	A	SA
42	My teachers used math games to reinforce my understanding of concepts.	SD	D	U	A	SA
43	My mind goes blank and 1 am unable to think clearly when doing mathematics.	SD	D	U	A	SA

44	1 can recall math teachers who made me feel stupid in class.	SD	D	U	A	SA
45	I have selected mathematics as my area of emphasis.	SD	D	U	A	SA

SD = Strongly Disagree **D = Disagree** **U = Undecided** **A = Agree** **SA = Strongly Agree**

46	1 have generally considered math as a related, sequential, progression of ideas.	SD	D	U	A	SA
47	I generally have had difficulty relating new mathematical concepts to those I had previously learned.	SD	D	U	A	SA
48	I am avoiding taking math classes in college.	SD	D	U	A	SA
49	My math teachers often became frustrated with me.	SD	D	U	A	SA
50	My math teachers frequently used a lecture format.	SD	D	U	A	SA
51	I enjoy going beyond the assigned work and trying to solve new problems in math.	SD	D	U	A	SA
52	Mathematics is enjoyable and stimulating to me.	SD	D	U	A	SA
53	Math makes me feel uneasy and confused.	SD	D	U	A	SA
54	I am interested and willing to use math outside school and on the job.	SD	D	U	A	SA
55	I have never liked mathematics, and it is my most dreaded subject.	SD	D	U	A	SA
56	I have always enjoyed studying math in school.	SD	D	U	A	SA
57	I would like to develop my mathematical skill and study this subject more.	SD	D	U	A	SA
58	Mathematics makes me uncomfortable and nervous.	SD	D	U	A	SA
59	Mathematics is dull and boring because it leaves no room for personal opinion.	SD	D	U	A	SA
60	Math is very interesting, and I have usually enjoyed	SD	D	U	A	SA

	courses in this subject.	
61	I am interested and willing to acquire further knowledge of mathematics.	SD D U A SA

SD = Strongly Disagree D = Disagree U = Undecided A = Agree SA = Strongly Agree

62	Math has contributed greatly to science and other fields of knowledge.	SD D U A SA
63	Math is less important to people than art or literature.	SD D U A SA
64	Math is not important for the advance of civilization and society.	SD D U A SA
65	Math is very worthwhile and necessary subject.	SD D U A SA
66	It is important for artists and writers to understand math as well as scientists.	SD D U A SA
67	Mathematics is not important in everyday life.	SD D U A SA
68	Math helps develop a person's mind and teaches him to think.	SD D U A SA
69	Math is needed in designing practically everything.	SD D U A SA
70	Mathematics is needed in order to keep the world running.	SD D U A SA
71	There is nothing creative about mathematics, it's just memorizing formulas and things.	SD D U A SA
72	I don't use mathematics in my everyday life.	SD D U A SA

Appendix 2: Categories and questions.

Confidence	Anxiety	Value	Enjoyment	Motivation	Teacher
2, 3, 9, 11, 19, 22, 26, 27, 30, 32, 35, 36, 41, 45, 47, 51, 52, 56,	1, 12, 13, 14, 15, 19, 22, 37, 39, 41, 43, 44, 47, 49, 53, 55, 58,	23, 24, 28, 38, 45, 46, 48, 54, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72	24, 25, 31, 36, 43, 44, 45, 46, 48, 51, 52, 54, 55, 56, 59, 60, 71,	5, 24, 28, 29, 31, 36, 45, 46, 48, 51, 52, 55, 57, 59, 61, 71,	3, 4, 6, 8, 10, 16, 17, 18, 20, 21, 33, 34, 35, 38, 40, 42, 44, 49, 50,

There are some questions are listed in more than one category, as researchers have found that some questions measured more than one attribute.