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Experiencing the Power of Learning Mathematics through Writing

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Abstract

As part of the Writing Across the Curriculum movement, teachers are asked to integrate writing into their teaching of mathematics; however, this can be a difficult task given that most elementary school teachers have had little experience using writing as a tool to learn and communicate their understanding of mathematics. To give students in my mathematics content courses the valuable experience of writing mathematical explanations, writing has become an integral part of my courses for pre-service teachers. The paper that follows focuses on how I support strong written explanations in my mathematics content courses for elementary school teachers.

Introduction

As part of the Writing Across the Curriculum movement, teachers are asked to integrate writing into their teaching of mathematics; however, this can be a difficult task given that most teachers have had little experience using writing as a tool to learn and communicate their understanding of mathematics (Totten 2005). To give students in my mathematics content courses the valuable experience of writing mathematical explanations, writing has become an integral part of my courses for pre-service elementary school teachers. Moreover, making writing a fundamental part of my courses supports the overarching goal of the courses I teach, which is to provide prospective elementary teachers with a deeper understanding of the mathematics they will be called upon to teach. Being able to explain the mathematics and master the language and tools of the discipline are vital skills for teachers. Through writing, these skills are developed and strengthened. Writing mathematical explanations is also a powerful tool to emphasize the essential role the Process Standards (NCTM 2000) – Problem Solving, Reasoning and Proof, Communication, Connections, and Representation— play in learning and in doing mathematics. Therefore, the pre-service teachers in my classes are expected to solve complex mathematical problems, make sense of the mathematics, and present clear, logical explanations. My task as their instructor is to support them in their learning; this includes their ability to communicate in writing about the concepts and processes they are learning. The paper that follows focuses on how I support strong written explanations in my mathematics content courses for elementary school teachers.

Building the Foundation

Just as elementary school students need a great deal of guidance and support to become comfortable writing in mathematics class (Burns 1995), so do pre-service teachers. At the start of each semester, I dedicate a good amount of time to build the foundation and support for my students' writing. During the first few classes, I find it beneficial to discuss the goals of the course and how writing is fundamental to meeting these goals. I often pose the following questions: "What does it mean to possess a deep understanding of mathematics? What does it mean to make sense of mathematics? What does it mean to explain your reasoning and the reasoning behind a mathematical concept or procedure?" As a class, we answer these questions; we also discuss why time spent explaining students' reasoning is valuable to both the students in a class as well as the teacher and why writing is such a powerful mode to do this. I believe it is important to explain my instructional goals and choices to my students, especially to those students who are pre-service teachers because they will soon be making similar instructional decisions in their own classrooms. Moreover, Totten (2005) noted that pre-service teachers need to construct a strong rationale for incorporating writing into teaching mathematics; this rationale should include specifics about how writing in mathematics facilitates learning, comprehension, and retention. Totten noted that for pre-service teachers to incorporate writing into their own teaching, they must be convinced that writing helps students learn content more thoroughly and comprehend it more deeply, and I believe that the best way to do this is to have them experience the power of learning through writing.

One way that students can begin to practice solving complex mathematical problems, making sense of the mathematics underlying the problem, and presenting clear, logical explanations is by working in small groups on their problem solving, reasoning, and communication skills. Working in groups and as a class to solve interesting mathematics problems and model the skills of presenting clear, logical oral explanations provides students with the opportunity to practice these skills and observe what it means to makes sense of mathematics and clearly explain one's reasoning. This also provides students with more immediate feedback on their thinking and explanations. However, as Liebars (1999) and Dougherty (1996) note, oral communication requires a different amount of elaboration than is necessary for written work. The process of writing emphasizes gathering, organizing, revising, and clarifying thoughts, which are all skills that can readily be applied to solving mathematical problems (Burns 1995, p. 5). Moreover, writing not only helps us organize information and procedures, it also helps us to learn more about our own thinking processes (McIntosh & Draper 2001).

Providing Meaningful Feedback

For many students this is the first time they have had to explain mathematics in writing. Obtaining my goal of strengthening their ability to communicate through writing about the mathematical concepts and processes they are learning requires that I provide meaningful feedback on their work. Early each semester, I use an activity developed to construct or refine a rubric for providing feedback on their written explanations. To make the rubric more meaningful to my students, the pre-service teachers participate in the process of refining the rubric, so they have more ownership of it. After the students write their first problem explanations and share their explanations in small groups, I post the following question, "What makes a written explanation to a mathematics problem exemplary?" I then ask them to describe the characteristics that make a written explanation exceptional and give each student several small sheets of stick-on notes to record their responses, writing one response per sheet. After having sufficient time to reflect, the students share their responses in groups of three to five, which provides a platform where all students can share their ideas. The stick-on notes allow them to join those similar or shared responses. The groups then report out, and as a class, we use the responses to discuss and refine the rubric, which will be used to provide feedback on their work (see Appendix A for an example of the rubric). Then at the following class, the students apply the rubric to their own work and the work of several peers. Through this process, they develop a clear understanding of what is expected of their work and how they will be evaluated.

The students have noted that the most meaningful feedback comes from the individual responses I include about the quality of their work. I always begin by stating at least one specific strength reinforcing what they did well since a major part of my role as educator is to support and encourage my students in their learning. I also include clear statements about how they can improve their explanations and correct any misconceptions that I find. For example, I included the following feedback on one students work:

> I like how you used the meaning of multiplication to help you solve this problem. You also do a nice job using an array to show why Ted's method was wrong and then how he could correctly calculate 12 x 15 using an array and the distributive property. Your explanation is insightful and correct. On your last page, can you arrange the smaller rectangles within the larger rectangle so that the dimensions of both are aligned, which will make your diagram even clearer?

Though it is time-consuming, other educators (Liebars 1999) and I have found that our effort is rewarded by their growth. I also have found that this is the best way to learn about my students' misconceptions and to formatively assess their understandings.

Because I think learning is a process, the students are asked to improve upon their written explanations by resubmitting them and then meeting with me to discuss their work. Given that I choose problems that are important for elementary pre-service to investigate and understand, I don't place any limit on the number of times that students can resubmit an assignment. However, each time they resubmit an assignment, they must meet with me again to discuss their newly revised work. Most students only meet with me a few times throughout the semester, but a few students take full advantage of the extra teaching time. This is one way that I differentiate and provide more support to those students who need it in the college classroom.

Reading through their explanations together gives me the opportunity to question their thinking, challenge weaker arguments, and push them to include more details as well as discuss the strengths of their work in person. The students appreciate the feedback and the opportunity to improve upon their work. For example, while meeting with a pre-service teacher in my class who was also an English major, he mentioned that writing in math class is different than the writing that he did for his English classes. He stated that writing in math class was more difficult for him, but that he was thankful to have the opportunity to do it and have the opportunity to also be able to get feedback and continue to revise his work. He noted that this was a valuable learning experience for him and that he was going to be making it part of his teaching. He also stated how important it is for someone who is going to require his students to write in a math class to have the opportunity to do the same.

Another effective tool that I have used to support my students' writing and provide them with meaningful feedback is through the use of writing circles, which are a great opportunity for peer interaction in learning. Students respond to each other's work with valuable comments that support each other's writing. Most students in my courses have previously participated in writing circles, but not in a mathematics class. The first time or two I use writing circles in class, we begin by reviewing what makes a mathematical explanation strong and how to give valuable feedback (for example, begin by stating something positive and give specific, clear suggestions). The students then share drafts of their work and thoughtfully respond to each other's explanations. This gives them immediate feedback in language that they can readily understand. Also by viewing the work of their peers, the strengths and weaknesses of their own explanations often become clearer. Moreover, they get the opportunity to practice giving feedback while observing the different ways people think about the same mathematical problem, which is a valuable experience for prospective teachers.

Emphasizing the Process Standards

When choosing problems for my students to solve and clearly explain in writing, my major goal is finding problems that deepen their understanding of the concepts we are studying by emphasizing the Process Standards (NCTM 2000): Problem Solving, Reasoning and Proof, Communication, Connections, and Representations. That is, I select problems that engage students in developing new ideas, techniques, and mathematical relationships. Good problems inspire the exploration of significant mathematical ideas, cultivate persistence, and reinforce the need to understand, use, and explain various strategies, properties, and relationships (NCTM 2000). They encourage students to express, develop, investigate, and justify conjectures about mathematical relationships. I choose tasks that help my students recognize and use connections among mathematical ideas we have studied. These connections help students understand how mathematical ideas interconnect and build on each other to produce a coherent body of knowledge rather than as a set of complex, discrete concepts, procedures, and processes (NCTM 2000). I also select problems that lend themselves to being represented and solved in multiple ways. To be able to support their students in the future, pre-service teachers need experience developing and using a variety of representations of mathematical ideas to model problem situations, to investigate mathematical relationships, and to justify or disprove conjectures. They need experience using informal representations, such as drawings, to highlight various features of problems and to serve as tools for thinking about and solving problems. Since the way in which mathematical ideas are represented is vital to how one learns, understands, and applies those ideas, representations should be viewed as essential ingredients in supporting the development of a deep understanding (NCTM 2000). Moreover, to further emphasize the important role the Process Standards play in learning and doing mathematics, I use them to categorize the essential components of students' work on the problem write-up rubric (see Appendix A).

Reasoning and Proof: Most elementary school teachers think of proof as a foreign concept they encountered in their high school geometry course. Because proof and proving are fundamental to knowing, doing, and understanding mathematics, the mathematics education community now recognizes that they are central to students' mathematical experiences in all grades and in all content areas. Research suggests that teachers' knowledge of proof plays an important role in managing and building their students' proving activity and experiences (Stylianides, 2007). I found writing is a powerful means for teaching pre-service teachers about important aspects at the heart of proof and proving in mathematics. For example, I assign the following problem:

> Adam has made up his own method of rounding. Starting at the rightmost place in a decimal number, he keeps rounding to the value of the next place to the left until he reaches the place to which the decimal number was to be rounded. For example, Adam would use the following steps to round 11.3524 to the nearest tenth: 11.3524 -> 11.352 -> 11.35 -> 11.4. Is Adam's method a valid way to round? Explain why or why not (Beckman, 2008, 62).

Many students give numerous examples as support for why Adam's method works (a few even admit that this is how they were taught to round). This problem provides a great platform to discuss when something is valid in mathematics, what makes an argument valid, and why empirical arguments – arguments based on the use of examples—aren't sufficient evidence for a claim to be true in mathematics. For example, I provided one student with the following feedback:

> Sam, Your write-up is well-written. You clearly explain how to use Adam's rounding method to round 11.3524, and I like that you use the number line in your explanation as a visual representation. However, you just provided several examples using Adam's method as support for your answer. To answer the question, "Is Adam's method a valid way to round?", then his method should always work with any number "up and down" the number line. The second part of the question is to "Explain why or why or not." To explain why Adam's method is valid, you need to provide an explanation of why his method will work for all Real numbers. If you find one example when Adam's method of rounding doesn't produce the correct answer, then this counterexample provides sufficient evidence that his method isn't valid.

Because the students are given multiple opportunities to revise their work, many not only deepen their understanding of rounding but also uncover the reason why empirical arguments aren't sufficient proof that a statement is valid.

Challenges and Meeting Those Challenges

As previously stated, writing has become an integral part of my teaching; however, it is still one of the most challenging aspects of my teaching. I struggle with teaching students how and why to be concise when writing their explanations and how explaining the reasoning behind their steps is different than just explaining their steps. Since most of my experience and training is with teaching mathematics, I struggle with how to teach writing – the actual grammar and semantics. Though I continue to push myself to learn more about teaching writing and gain tremendous support and knowledge from my colleagues, I find that utilizing the knowledge and strengths of my students is often one of the best solutions for dealing with such challenges. To help model good writing practices, I believe that I need to spend even more class time highlighting and deconstructing exemplar explanations and discussing what makes them effective. This however can lead to another challenge, which is how to spend more class time on writing while still having sufficient time to spend on the courses' mathematics topics. I found that ultimately this problem solves itself. As the semester progresses, I can assign increasing more complex problems for students to write about, which extend far deeper into the mathematics than class time allows. Subsequently, setting the stage for good problem solving and writing skills eventually pays off in terms of the in-class time.

Concluding Thoughts

Mathematics teacher educators have a responsibility to model the types of teaching practices we believe our pre-service teachers should one day use. Supporting pre-service teachers' writing in a mathematics class means promoting an invaluable tool. Explaining their solutions to important mathematical problems in writing not only deepens students' understanding of the mathematics, but it also strengthens their ability to communicate about the mathematics and learn the vocabulary and tools of the discipline. The thinking involved in justifying a strategy or explaining a solution is different from that needed to merely solve a problem; written explanations in mathematics are more about *what* is being done and *why* it works (Kenney 2005). They provide a plethora of information about students' knowledge and misconceptions and accordingly are an extremely powerful assessment tool.

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Appendix A. Problem Write-Up Rubric

Problem Write-Up Rubric					
Your work is…	**Problem Solving**	**Reasoning and Proof**	**Communication**	**Representations**	**Connections**
Exemplary	**- Your work demonstrates that you clearly understand all parts of the problem. - You use a correct strategy (or strategies) and carry the work out accurately and efficiently. - You verify conjectures and the solution and extend or generalize your findings when applicable.**	**- Your work shows that your method and solution are valid and why they are valid. - All steps are justified and organized in a logical and insightful way.**	**- Your work is clear, concise, and easy to follow. - It is professional in nature – neat and grammatically and semantically correct. - You use formal mathematics vocabulary and symbols accurately and effectively.**	**- Your work includes appropriate and accurate representations (symbols and pictures, diagrams, graphs, and/or charts with labels that clarify) that support the problem-solving method(s) used and highlight or clarify important information about the problem, mathematical relationships, and/or the solution.**	**-You make clear connections between different approaches and representations used to solve the problem and/or different concepts and problems in mathematics.**
Good	**- Your work demonstrates that you have a good understanding of the problem. - You address all parts of the problem. - Your strategy is mathematically correct; however, your work may contain a minor error.**	**- Your work shows that your method is valid. - The majority of steps are justified and organized in a logical way.**	**- Your work is clear and easy to follow. - You have few (if any) errors in grammar and/or semantics. - You use mathematics vocabulary correctly.**	**- Your work includes appropriate and accurate representations (symbols and pictures, diagrams, graphs, and/or charts).**	**- You make connections between different approaches or representations used to solve the problem or different topics or problems in mathematics.**
In Progress	**-Your work demonstrates that you have some understanding of the problem. - Your strategy and work are largely correct without serious shortcoming. - Your work may contain minor flaws.**	**- You justify some of the steps used to solve the problem. - However, some of your reasoning is incomplete and/or unclear.**	**- You work is comprehensible, but could be clearer. - Your writing contains a number of errors. - And/or, you use every day (not formal mathematical) language to communicate your ideas, or you use a mathematical term incorrectly.**	**- You attempt to use mathematical representations to solve the problem or highlight aspects of the problem-solving method(s) used.**	**- You attempt to make connections between different approaches or representations used to solve the problem or different concepts or problems in mathematics.**

Poor	**- Your write-up shows little understanding of the problem situation. - Your strategy and work contain serious shortcomings.**	**- Your reasoning is lacking, flawed, and unclear.**	**- Your writing is unclear and/or difficult to follow. - It contains numerous errors, which may include those using grammar, semantics, or mathematical language inappropriately.**	**- Your work contains no mathematical representations.**	**- You make no effort to make connections between different approaches, representations, or problems.**
Additional Feedback:					