Pre-Service Teachers' Exposure to Using the History of Mathematics to Enhance Their Teaching of High School Mathematics

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Abstract

The history of mathematics is an important component in the learning of mathematics. This study examines how pre-service teachers view the role of history of mathematics in the high school curriculum. Quantitative and qualitative methods were used. Results showed significant changes in beliefs about how the history of mathematics should be integrated as well as preparedness to incorporate the history of mathematics in teaching.

Introduction

The National Council of Teachers of Mathematics (NCTM) emphasizes the value of teaching the history of mathematics in K-12 classrooms. This priority appears in the guidelines for college teacher preparation programs through the National Council for Accreditation of Teacher Education's (NCATE) mathematics Specialized Professional Association (SPA) program evaluation indicators, and college programs must meet these indicators to receive program approval from NCTM and NCATE. These indicators stem, in part, from the NCTM professional standards where it is outlined that mathematical tasks "should foster students' sense that mathematics is a changing and evolving domain, one in which ideas grow and develop over time and to which many cultural groups have contributed. Drawing on the history of mathematics can help teachers to portray this idea" (NCTM, 1991). This focus on the history of mathematics initiated this study to investigate pre-service teachers' exposure to mathematics and to determine their attitudes toward incorporating the history of mathematics in the high school mathematics curriculum.

"Mathematics is known as a discipline that is so difficult to teach that it has developed a specific educational sub-discipline with its own paradigms and characters (mathematics education)." (Furinghetti, 2000, p. 44). This difficulty may be the basis of the observation that many secondary mathematics teachers are severely lacking in their ability to provide meaningful explanations of content to their future students, and instead promote a rule-bound base to mathematics (Ball, 1990; Quinn, 2001).

Effective teaching involves much more than a teacher being mathematically competent (Begle, 1968; Eisenberg, 1977); mathematical knowledge alone does not translate into better teaching. According to Thompson (1992), desirable ways of teaching and learning mathematics are influenced by one's conception of mathematics. This conception of mathematics incorporates not only mathematics content knowledge (rules and algorithms) but a meaningful mathematics content base (knowing the why and how of mathematics) and a teacher's attitude toward mathematics.

> "If reform in mathematics education is to be successful, teachers of mathematics must have an adequate knowledge of meaningful mathematics content knowledge as well as a positive attitude toward the subject." (Quinn, 2001, p. 113).

According to Clark & Peterson (1986) and Romberg & Carpenter (1986), how teachers interpret and implement curricula is influenced significantly by their knowledge and beliefs. Phillippou & Chistou, (1998) noted evidence that prospective teachers may bring misconceptions and negative attitudes towards mathematics, but, it seems as if teacher-training programs are able to provide opportunity to influence attitudes positively. Ball (1990) contends that mathematics methods courses can change pre-service teachers' knowledge, assumptions, and feelings about mathematics, as well as their beliefs concerning their role as teachers in the classroom.

The history of mathematics is an important component in the learning of mathematics. Incorporating the history of mathematics in mathematics curricula can help students to learn that mathematics is, in fact, "a discipline that has undergone an evolution and not something that has arisen out of thin air" (Jankvist, 2009, p. 239). Using the history of mathematics in school curricula reveals a humanistic side of the subject and may enhance learners' motivation (Charalambous, Panaoura, & Philippou, 2009; Jankvist, 2010, NCTM, 1991). It is a judicious way to teach mathematics and has many benefits such as allowing students to make connections, sharpening problem-solving skills by offering diverse approaches and a variety of algorithms and techniques, laying a foundation for better understanding, and highlighting interactions between mathematics and society (Wilson & Chauvot, 2000).

"All too often, the formal written record of mathematics is what teachers study. The struggles, the false starts, and the informal investigations that lead to an elegant proof frequently are missing." (NCTM, 1991, p. 133). Using history of mathematics with pre-service teachers helps them become aware of the processes and struggles through which mathematics was developed, enhances their understanding of the content they expect to teach, equips them with methods and techniques for incorporating historical materials in their own teaching, and offers insights about the evolution of mathematics curricula (Charalambous, Panaoura, & Philippou, 2009).

The purpose of this study was to examine pre-service teachers' preconceptions of using the history of mathematics in teaching high school mathematics and to determine if these changed with exposure to topics from history of mathematics and methods that could be used to teach these topics. Pre-service teachers, themselves, have not had much exposure to the topic of history of mathematics in their own studies, yet they are expected to incorporate this topic in their high school teaching in the near future. Therefore, this study set out to determine any changes in (1) how pre-service teachers view the topic of history of mathematics, (2) the way pre-service teachers felt history of mathematics should be incorporated in the secondary mathematics curricula and (3) in the way pre-service teachers felt about being appropriately prepared to incorporate history of mathematics in their future secondary classrooms. Assignments designed to familiarize these pre-service teachers with some topics from history of mathematics were then completed and the results were examined to see if any changes occurred.

Method

Participants: Undergraduate and graduate pre-service teachers enrolled in the mathematics methods class in the adolescence mathematics education program at a private, religiously affiliated college in an urban area in the Northeast were offered the opportunity to participate in

this study. Three secondary mathematics methods classes were asked to participate, one in Spring 2008, one in Spring 2009, and one in Spring 2010. The 2008 cohort included 12 pre-service teachers: 10 undergraduates and 2 graduates, the 2009 cohort of 12 pre-service teachers was comprised of 8 undergraduates and 4 graduate pre-service teachers and the 2010 cohort of 8 pre-service teachers was comprised of 7 undergraduates and 1 graduate. While the assignments used were completed by all 32 members of the classes as part of their course work, the decision belonged to the participants as to whether or not they would allow their work to be recollected (anonymously) and examined for the purposes of this study. All participants agreed to allow their work to be included in this study.

Procedure: During the first class meeting of the secondary mathematics methods class, the participants completed a questionnaire designed to elicit their perceptions of the role of the history of mathematics in the high school classroom. They were asked to provide the last four digits of their social security number so the questionnaire could be matched with the same questionnaire completed at the end of the course approximately 16 weeks later. The results of the initial questionnaire were not made available to students nor discussed with them at any stage.

Likert style questions were asked addressing pre-service teachers' exposure to the history of mathematics in high school, college, and through their own reading; their opinion on whether or not history of mathematics should be integrated into the high school mathematics curriculum and their preparedness to teach the history of mathematics in their own high school classroom. This was followed up by free response questions exploring what the role of history of mathematics should be in the high school classroom, what they would need to improve preparation to integrate the history of mathematics in their classroom, and what they have experienced in their observations of high school mathematics in the area of history of mathematics. The final section listed mathematicians and asked participants to identify those with whom they were familiar as well as what contributions were attributed to them.

During the course of the semester, participants completed activities designed to enhance their exposure to the history of mathematics. They began by reading the book, *Fermat's Enigma* by Simon Singh. While reading, they were to keep track of mathematicians from the mathematical areas: number theory, algebra, geometry, calculus, discrete mathematics and statistics and probability. They were to keep a list of the mathematicians that included a description of the mathematics that made him/her famous, the events or challenges that led to the mathematics discovered, and the impact on that branch of mathematics or in other disciplines. Participants then chose their "favorite mathematician" and prepared a short, creative presentation about the mathematician.

They also developed a list of questions that they were able to use for a Book Talk. The class was divided into small groups and the groups discussed the book. Each groups' discussion was audio taped. The tapes were then transcribed and the results analyzed for common themes among the groups.

In addition, pre-service teachers also chose a mathematics course often taught in secondary schools. They chose a unit and described what history of mathematics would be able to be incorporated into that unit. Lastly, they each wrote a reflection based on Singh's book, the Book Talk and the assignments attached to this abbreviated study of the history of mathematics. Questions addressed included, "What did you learn?" and, "How might this series of assignments inform your further teaching?" In the final class session, after all of the assignments were completed, the questionnaire was re-administered.

Data Analysis

This study used a mixed method design, incorporating both quantitative and qualitative techniques. Data was analyzed quantitatively using paired samples *t*-tests performed with SPSS (SPSS, 2008). Data collected from the initial and final questionnaire were compared to determine significant differences. Effect sizes were also calculated.

Qualitative data from the open-ended question were initially considered by sorting the responses from the initial questionnaire into three groups: a minor/no role, a moderate role and a major role. Responses from the same question on the final questionnaire were also independently sorted into those same categories. The initial and final groups were then compared across cases, with the individual as the unit of analysis, to document changes.

Other qualitative data collected from the questionnaire and from the final reflection paper began with an open-coding approach whereby categories were developed (Miles & Huberman, 1994; Strauss & Corbin, 1998) and then a "grounded theory" approach was utilized (Miles & Huberman, 1994).

Results

Quantitative Results: Responses to the Likert scale items were scored as follows: Strongly Agree = 4, Agree = 3, Disagree = 2, and Strongly Disagree = 1. Paired samples *t*-tests were used to compare means and effect sizes were calculated. Table 1 shows the mean total scores along with the results of paired samples *t*-tests of the differences between the two administrations of the instrument for the following prompts:

1) History of mathematics should be integrated into the high school mathematics curriculum (integrating history)
2) I am prepared to incorporate the history of mathematics in my high school teaching (prepared to incorporate)
3) I have been exposed to the history of mathematics in my own high school experience (exposed high school)
4) I have been exposed to the history of mathematics in my college experience in other courses (exposed college)
5) I have been exposed to the history of mathematics through my own reading/study (exposed on own).

Table 1
Mean Totals for Prompts

Items	**Mean Totals (n = 32)**		**Mean Diff**	**Std. Deviation**	***t* (2-tailed)**	**Sig.**	**Effect Size**
	Pre	**Post**					
Integrating history	3.31	3.63	0.313	0.592	2.985	.005	.529
Prepared to incorporate	2.06	2.88	0.813	0.998	4.605	.000	.814
Exposed high school	2.38	1.94	-0.438	0.801	-3.091	.004	.547
Exposed college	3.09	3.22	0.125	1.070	0.661	.514	.117
Exposed on own	2.48	2.77	0.290	1.006	1.606	.119	.289

The means of the items have been adjusted such that a higher score indicates a more desirable (positive) response. Paired samples *t*-tests indicated that the difference was significant ($t = 2.985$, $df = 31$, $p < 0.05$) for the prompt "integrating history of mathematics" indicating that at the time of the final questionnaire, respondents were more agreeable to the integration of the history of mathematics in the high school mathematics curriculum than they were at the time of the initial questionnaire. There was also a significant finding ($t = 4.605$, $df = 31$, $p < 0.05$) for the prompt "prepared to incorporate" indicating that pre-service teachers felt more comfortable with their ability to integrate the history of mathematics in their high school teaching by the end of the unit on history of mathematics than they were at the beginning of the unit. There was also a significant finding ($t = -3.091$, $df = 31$, $p < 0.05$) for the prompt "exposed high school". This finding illustrated that pre-service teachers initially felt that they had been exposed to the history of mathematics but that changed to them feeling that they did not have adequate exposure to the history of mathematics by the time of the final questionnaire. There were no significant differences for the two questions dealing with exposure to the history of mathematics in college experience (outside the current course) or on their own. The effect size in relation to the first and third prompt was medium, 0.529 and 0.547, respectively, and a large effect size of 0.814 was realized for the second prompt.

Qualitative Results: For the open-ended question, "What should the role of 'history of mathematics' play in the high school mathematics curriculum?", a similar number of pre-service teachers favored a moderate role as did a minor role for the history of mathematics. Only 3 responses fell into the category of "major role" with the initial questionnaire data. Responses included the following: "When students are introduced to a new topic they should also be introduced to the history of that topic. So basically I feel like history of math should be 'sprinkled' on top of the lesson."; "The history of mathematics should be used as an introduction of interesting 'extra' knowledge."; and " I feel that it should be slightly involved when applicable to give students math related knowledge seen in a different light. Historical facts or names of mathematicians can give students the opportunity to make connections with concepts."

At the final questionnaire, most pre-service teacher responses fell to a more favorable role than at the initial questionnaire. This time, only 3 responses were in the minor role category while the majority of the remainder fell into the major role category. In the initial data, one pre-service teacher wrote, "I feel that it should be slightly involved when applicable to give students math related knowledge seen in a different light. Historical facts or names of mathematicians can give students the opportunity to make connections with concepts", and by the final questionnaire had changed to, "It should be incorporated in units/lessons in which it relates and would enhance/further develop. It is important to engage students and stimulate interest." Another student's initial response was "Familiarizing the students with the origin of the material learned", which changed to "It should be integrated into the curriculum for those students who struggle with math and excel in other subjects", by the time of the final questionnaire.

Examination of the final reflection papers submitted by the participants provided further clarification of the perceived value of the history of mathematics assignments in improving the participants' own knowledge and providing them with reasons to include history of mathematics in high school mathematics. The sections below expand upon these two benefits.

Improved own knowledge: Participants indicated that they enjoyed the assignments that were part of this unit. Reading the book, participating in the book talk and developing ways to

incorporate history of mathematics into their own lessons encouraged participants to do some of their own research on mathematicians and the "whys" and "hows" of some of the mathematical symbols and theorems they have used. Their own knowledge was improved because many of them had never been exposed to, or had limited exposure, to the history of mathematics. One participant indicated that the series of assignments "felt like a mathematical journey". Others indicated how they learned about the history of mathematics and how "different fields of mathematics relate to each other" as well as getting a "greater understanding of math as a whole" and "learning mathematical logic". Another indicated that this assignment "forced me to learn more about the people that are working in the field of mathematics now" as opposed to what most people think about the history of mathematics being about "ancient times".

*Reasons to include history of mathematics in high school mathematics: M*any participants indicated that they felt it was important to expose their high school students to the history of mathematics. They felt that "more students will be motivated in math if they learn about the history of math" and "students will gain a deeper appreciation for mathematics by learning the origins". They also addressed pedagogical reasons for including the history of mathematics in high school mathematics such as the following:

- "Teaching the history of math is important because it expands to more styles of learning. Some students will learn through the use of stories, and by teaching the history behind some rules and theorems, students will start to make connections;"
- "Incorporation of math history brings in interdisciplinary activity opportunities helping the students make connections;"
- "Learning about the history of math, learning about mathematicians, and having students do projects/assignments will allow students who do not excel in math to 'shine' and show their other abilities…It will catch the students' attention and they will look forward to coming to math class;
- "Teaching the history of mathematics is also an excellent way of incorporating literacy into the classroom."

Discussion

Both the quantitative and qualitative results converged in showing pre-service teachers' views on integrating the history of mathematics and being prepared to incorporate the history of mathematics in their own teaching where both changed significantly after their exposure to this series of assignments. This positive change could be due to a better understanding of what is involved in incorporating the history of mathematics into the curriculum. Pre-service teachers had the opportunity to research some of the mathematicians, and they planned at least one lesson where the history of mathematics was incorporated into the lesson. In addition, they gave and watched presentations on famous mathematicians which took on very creative measures. These mini lessons showed the pre-service teachers that there were many interesting ways to incorporate the history of mathematics in a relatively short amount of time. This may have dispelled some previous misconceptions consistent with findings from Fauvel (1991) and Wilson & Chauvot (2000) that incorporating the history of mathematics would be challenging and time consuming.

Another significant finding from this study was that some pre-service teachers initially felt that they had been exposed to the history of mathematics in high school but after their exposure in the methods class, they felt that they had not previously had adequate exposure to the history

of mathematics, as indicated by the final questionnaire. It is possible that these pre-service teachers didn't understand the component of the history of mathematics at the initial questionnaire and thought that they had been exposed. The results from the final questionnaire indicated that there had been a significant change in the negative direction. Participants no longer felt as strongly that they had been exposed to the history of mathematics in high school.

The prompts, "exposure to the history of mathematics in my college experience" and "through my own reading" yielded non-significant results in respect to change over time. It should be noted here that participants were asked to disregard the current course and set of assignments along with any of their own reading that occurred within the confines of this assignment when responding on the final questionnaire.

The qualitative data from the final reflection overlapped in the two categories that emerged. What the pre-service teachers found interesting and what they learned about the history of mathematics transferred to their comments of how they hoped to teach and motivate their students to learn the history of mathematics. This is consistent with Ball's (1990) findings that mathematics methods courses can have an impact on pre-service teachers and how they view mathematics as well as how they view their role as a mathematics teacher.

Final reflection comments also included a number of benefits that the teaching of the history of mathematics would have. These comments were consistent with the literature, even though the participants did not consult the literature in responding. Bidwell (1993) discusses making connections and Wilson & Chauvot (2000) include making mathematical connections, laying a foundation for better understanding and highlighting the interaction between mathematics and society.

While all of this translates to a very positive reception of this attempt to expose pre-service teachers to the history of mathematics, more needs to be done to develop a deeper understanding of this topic before students get to a methods course. In a methods class, more time should be spent on designing lessons and sharing ways that the content can be taught NOT on teaching the content itself! Methods course professors can model the integration of the history of mathematics in lessons so the pre-service teachers will think about integrating history as they teach mathematics.

Future research could be conducted to determine if this motivation to use the history of mathematics translates into the actual high school classrooms. It is quite possible that when these pre-service teachers are student teaching or inservice teachers, they may teach the way they were taught which, for most, does not include the history of mathematics.

Conclusion

Through this project, participants were beginning to satisfy the NCATE SPA indicators by their exposure to the historical development of different content areas of mathematics. They were also gaining an appreciation of how they may be able to incorporate this in their teaching. While more research needs to be done about incorporating the history of mathematics into college methods courses, this project provides one example of a meaningful introduction to this topic that can be used by other higher education faculty with their pre-service teachers. Hopefully, this will translate into the integration of the history of mathematics into secondary classrooms in the future!

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