# A Commentary on Elementary Teacher Preparation to Teach Statistics 

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#### Abstract

Since the prevalence of data and statistics in the media and workplace has greatly increased over the past few decades, the teaching of statistics in K-12 grades has also increased. This necessitates that teachers be ready to teach statistical concepts throughout the grade levels. The recently adopted Common Core State Standards for Mathematics (CCSSM) contain a large amount of statistics in the middle and high school grades and some at the elementary school level. This paper focuses on the preparation of elementary school teachers. The CCSSM and the Guidelines for assessment and instruction in statistics education (GAISE) report: A preK-12 curriculum framework are examined to illustrate the content covered in the elementary grades. Additionally, two examples of the type of content teachers are expected to know and teach are presented.


Key Words: Common Core State Standards; GAISE; Elementary Teacher Education

## Introduction

As the prevalence of statistics in today's society has greatly increased, it is imperative that K-12 education be prepared to educate students to become statistically literate. The recently adopted Common Core State Standards for Mathematics (CCSSM) (CCSS, 2010), a set of national standards that aim to unify K-12 education across the US for mathematics, place a large emphasis on statistics in the middle and high school grades; and also include some statistical content at the elementary school level. In this paper, we discuss the statistical content elementary teachers need to be prepared to deliver in their classrooms. In addition, we provide two specific examples of lessons that exemplify the type of elementary content that meets the CCSSM standards.
A gradual increase in the standards for teaching statistics can be seen in the United States (US) over the past 25 years (Franklin, et al., 2007; NCTM, 1989, 2000). Data-driven decision making and studies have drawn interest from the general population and policymakers as well as businesses and schools. Prominent people and news outlets are increasingly touting the importance of understanding and interpreting data. For example, Secretary of Education Arnie Duncan recently commented on the use of data in the evaluation of education: (Arnie Duncan, Speech August 22, 2012)

We didn't eliminate testing because we believe it is important to measure progress. We need to know who is ahead and who is behind - who is succeeding and who needs more support. In an ideal world, that data should also drive instruction and meaningful professional development.

Media outlets such as the Huffinton Post have also noted the importance of data in education: (Vineet Madan, Huff Post Education, posted on June 27, 2012)

Why is data so valuable in education? For one, it will advance us beyond the primarily aged-based system of educational advancement that has been in place for much of the past century. By generating and enabling access to consistent streams of data from different parts of the classroom, we are no longer limited to simply knowing what students know at a point in time -- we can now get a sense of when and how they learned it and how likely they are to retain and be able to apply that knowledge..... With data-driven insights in hand (or on tablet), educators are provided the information to tailor their instruction based on the knowledge, skills, and learning styles of students in each classroom.

Because of this new emphasis on the use of data to inform instruction (Reeves, 2007), it is crucial for us as educators to consider how we can prepare a statistically literate population. A statistically literate person, as defined by the Guidelines for assessment and instruction in statistics education (GAISE) report: A preK-12 curriculum framework, is a person who can engage in the statistical problem solving process by formulating statistical questions, collecting data, analyzing data, and interpreting data (Franklin et al., 2007). In general, statistical literacy refers to the ability to think and reason in the presence of uncertainty as well as make sense of statistical information (Garfield and Gal, 1999, Garfield, 2002)

Due to their increased role and use in society, statistics and probability have become an integral part of mathematics $\mathrm{K}-12$ education in the US, a change originally prompted by the inclusion of statistics in the National Council of Teachers of Mathematics (NCTM) recommendations for school mathematics curriculum (NCTM, 1989, 2000). These documents included a strand dedicated to the study of probability and statistics called Data Analysis and Probability. Although there is a large demand for improved education of teachers in all content areas, the Conference Board of the Mathematical Sciences (CBMS) reports titled The Mathematics Education of Teachers I and II (MET 1 and MET 2) Report have identified statistics as one of the largest areas of concern for teachers' preparation (CBMS, 2001, 2012).

The CCSSM unveiled in 2010 contain a very large amount of statistics at the middle school and high school levels. The K-5 standards also include some statistical content. As the elementary grades lay the foundation for success in the later grades, it is imperative that students are introduced to statistical topics in an accurate and appropriate manner during the elementary grades in order to be "statistically ready" for the large emphasis placed on statistics in the middle and high school grades (Franklin, et al., 2007). At this point in time, there is evidence that elementary teachers are not well versed in even seemingly simple elementary statistical topics. For example, Groth \& Bergner (2013) found that pre-service teachers through it made sense to use the mean as a summary of univariate categorical data. An important goal is therefore to illustrate what elementary teachers need to know about statistics in order to ensure students are prepared to learn statistics as described in the GAISE framework while meeting the CCSSM requirements. Because teachers are the vehicles through which statistical concepts are
presented to students, it is crucial to consider what teachers must know to be effective in increasing learning in the statistical content domain.

The goal of this paper is to present and discuss the content requirements elementary teachers are expected to teach. Furthermore, specific examples that illustrate the content in a way that is aligned with the GAISE framework are provided. Thus, this paper presents a concrete content-based justification for why elementary teachers should be prepared in statistics and how they should be prepared.

## Statistical Content Knowledge for Elementary School Teachers

Several sets of standards and recommendations have been put forth to outline what K12 students need to know (e.g., NCTM, CCSSM). These documents provide a set of minimum content requirements that a teacher must know at each grade level. In other words, if students need to know concept Z, then in order to teach it, teachers must at least know concept Z as well. Statistical content lessons that can be used directly in the K-5 classroom can provide excellent and ideal resources for teacher training - pre-service teachers not only will learn the content but they will also be well-versed in the exact type of lessons they will be expected to deliver in their classrooms.

With respect to statistics at the K-12 level, two influential documents are worthwhile to examine and compare: (1) the GAISE report, and (2) the CCSSM. The content geared towards elementary school students presented in each of these documents is illustrated and examined in the following subsections.

GAISE. In 2007, the GAISE framework emerged from the statistics community to provide an overarching outline for statistics education. The document identifies four components of the statistical problem solving process: formulate questions, collect data, analyze data, and interpret results. In order to characterize the natural advancement of statistical learning within each of these components, the framework presents a three-level developmental progression (levels A, B, and C) corresponding to the level of complexity within each component. The GAISE report thus highlights differences among the levels by the sophistication of the techniques employed at each level.

Although the GAISE framework does not outline statistical learning by grade, the report roughly suggests that students in the elementary grades should acquire level A depth, those in middle school should be around level B, and those in high school should achieve level C (Franklin \& Mewborn, 2006). Table 1 illustrates the GAISE framework for levels A and B, which are the most pertinent to elementary grades (pp. $23 \& 24$ for Level A and pp. 37 \& 38 or Level B).

Table 1. GAISE Levels A and B Coding

| Level A |  | Level B |
| :---: | :---: | :---: |
|  | 1. Formulate Questions | 1. Formulate Questions |
|  | a. Teachers help pose questions <br> b. Students distinguish between statistical <br> solution and fixed answer | a.Students begin to pose their own <br> questions <br> b.Students address questions involving <br> a group larger that their classroom <br> and begin to recognize the distinction <br> among a population, a census, and a |


|  | sample |
| :---: | :---: |
| 2. Collect Data | 2. Collect Data |
| a. Students conduct a census of the classroom <br> b. Students understand individual-toindividual natural variability <br> c. Students conduct simple experiments with nonrandom assignment of treatments <br> d. Students understand induced variability attributable to an experimental condition | a. Students conduct a census of two or more classrooms <br> b. Students design and conduct nonrandom sample surveys and begin to use random selection <br> c. Students design and conduct comparative experiments and begin to use random assignment |
| 3. Analyze Data | 3. Analyze Data |
| a. Students compare individual to individual <br> b. Students compare individual to a group <br> c. Students become aware of group to group comparisons <br> d. Students understand the idea of a distribution <br> e. Students describe a distribution <br> f. Students observe association between two variables <br> g. Student use tools for exploring distributions and association, including: bar graphs, dotplot, stem and leaf plot, scatterplot, tables (using counts), mean, median, mode, range, modal category | a. Students expand their understanding of a data distribution <br> b. Students quantify variability within a group <br> c. Students compare two or more distributions using graphical displays and numerical summaries <br> d. Student use more sophisticated tools for summarizing and comparing distributions, including: histograms, interquartile range, mean absolute deviation, five-number summaries and boxplots <br> e. Students acknowledge sampling error <br> f. Students quantify the strength of association between two variables, develop simple models for association between two numerical variables, and use expanded tools for exploring association, including: contingency tables for two categorical variables, time series plots, quadrant count ratio as a measure of strength of association, simple lines for modeling association between two numerical variables |
| 4. Interpret Results | 4. Interpret Results |
| a. Students infer to the classroom <br> b. Students acknowledge that results may be different in another class or group <br> c. Students recognize the limitation of scope of inference to the classroom | a. Students describe differences between two or more groups with respect to center, spread, and shape <br> b. Students acknowledge that a sample may not be representative of a larger population <br> c. Students understand basic interpretations of measures of |


|  | d.association <br> Students begin to distinguish <br> between an observational study and a <br> designed experiment |
| :--- | :--- | :--- |
|  | e.Students begin to distinguish <br> between "association" and "cause <br> and effect" |

In examining the Table 1, we notice that Level A is, in general, more teacher-directed while Level B shows slight shifts towards being student directed. At Level B, students begin to pose their own statistical questions and design appropriate data collection strategies to answer their questions. Level A learning is typically contained within the individual classroom environment while Level B begins to expand past the student's classroom to make assertions about other classrooms or outside of the school. Both Levels A and B focus on data displays and interpretation. At Level A, students are expected to use the following types of displays: bar graphs, count tables, dot-plots (also referred to as line plots), and scatterplots. Presumably, the more complex displays would be covered in the upper elementary grades. Because the GAISE Level A and B guidelines outline the type of statistical content that is recommended for elementary and middle school students to know and learn, elementary teachers should be made aware of the report and be prepared to teach the content described. The GAISE framework may also be used as a teaching resource. It provides several examples of sample questions and problems for each level. A teacher may therefore be able to incorporate the report directly into his/her teaching.

CCSSM. The CCSSM offer grade-by-grade benchmarks for grades K-8 and a content outline for high school. Since the CCSSM has currently been adopted by 45 states, it has great potential to be very influential in mathematics and statistics education in K-12. Due to the CCSSM emphasizing statistics in middle and high school, elementary school teachers may be required to teach much less statistics than what is recommended by the GAISE framework.

In the elementary grades, the CCSSM contains a strand entitled "Measurement and Data" under which a few statistical concepts are described. Table 2 outlines the concepts covered within measurement and data by grade level.

Table 2. Common Core Measurement and Data Strand

| Grade 1 | Grade 2 | Grade 3 | Grade 4 | Grade 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Organize, <br> represent, and <br> interpret data <br> with up to three <br> categories | Generate <br> measurement data <br> by measuring <br> lengths of several <br> objects or by <br> making repeated <br> measurements of | Draw a scaled <br> picture graph and <br> a scaled bar graph <br> to represent a data <br> set with several <br> categories | Make a line plot to <br> display a data set <br> of measurements <br> in fractions of a <br> unit | Make a line plot to <br> display a data set of <br> measurements in <br> fractions of a unit |


|  | these same object |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Ask and answer <br> questions about <br> the total number <br> of data points, <br> how many in <br> each category, <br> and how many <br> more or less are <br> in one category <br> than in another | Show the <br> measurements by <br> making a line plot, <br> where the horizontal <br> scale is marked off <br> in whole-number <br> unit | Solve one and two <br> step "how many <br> more" and "how <br> many less" <br> problems using <br> information <br> presented in scaled <br> bar graphs | Solve problems <br> involving addition <br> and subtraction of <br> fractions by using <br> information <br> presented in line <br> plots | Use operations on <br> fractions for this <br> grade to solve <br> problems involving <br> information <br> presented in line <br> plots |
|  | Draw a picture <br> graph and a bar <br> graph to represent a <br> data set with up to <br> four categories | Generate <br> measurement data <br> by measuring <br> lengths using rules <br> marked with <br> halves and fourths <br> of an inch |  |  |
|  | Solve simple put <br> together, take-apart, <br> and compare <br> problems using <br> information <br> presented in a bar <br> graph | Show the data by <br> making a line plot, <br> where the <br> horizontal scale is <br> marked off in <br> appropriate units- <br> whole numbers, <br> halves, or quarters |  |  |

As seen in the Table 2, the CCSSM focuses the Measurement and Data strand on the following central ideas for the elementary grades - generating bar graphs for categorical data, taking measurements of objects, sorting, and drawing line plots. Grades 1, 2, and 3 have students display data using bar graphs. Using these graphs, students must answer questions related to comparing the different categories and total number of data points represented. Grades 4 and 5 shift to drawing line plots (dotplots). Students are expected to generate data by taking measurements of objects and then represent the data using line plots. At the very least, elementary teachers must be prepared to teach the content outlined in the CCSSM.

Teaching both the CCSSM and GAISE. As states and districts strive to implement the CCSSM, it is important to have effective resources and examples as to what teachers should be prepared to teach in the classroom. The importance of this is emphasized by several studies which have shown that many elementary teachers do not have the content knowledge necessary to teach statistics at the level suggested in the GAISE framework and the CCSSM (Callingham, 1997; Groth \& Bergner, 2006; AUTHOR 2, 2007, 2008; AUTHOR 2, 2010; Leavy \& O’Loughlin, 2006; Makar \& Confrey, 2004; Mickelson \& Heaton, 2004). The content in the curriculum obviously needs to be aligned with the CCSSM but also, with regards to statistics, must be connected to the GAISE
recommendations. As noted in AUTHOR 1 (2012), the GAISE framework actually can be used to support the implementation of the CCSSM in the classroom in an effective manner.

The CCSSM essentially covers a subset of the concepts outlined by GAISE, however, differences between the two ultimately lie in the manner in which students are supposed to acquire and be guided through statistical learning. GAISE focuses on student's overall statistical literacy - this means enabling students to think statistically from the beginning of a statistical problem to the end and emphasizing the statistical process. Encompassed in this idea is a student's ability to pose a statistical question and then subsequently investigate the answer using appropriate data collection and data analysis methods. On the other hand, the CCSSM focuses on ensuring that students can perform specific tasks related to statistics. In the elementary grades, those tasks are focused around the conceptual themes outlined above. Using GAISE as a guide, the teaching of these themes can be approached and taught in a way that promotes students' statistical understanding and better prepares students for future grades.

## Example Lessons for Teacher Training and K-5 Classrooms

A few outlets exist that provide lessons that can be used to prepare teachers as well as lessons that can be used directly in the K-5 classroom. For example, the ASA sponsors the STatistics Education Web (STEW), a website dedicated to collecting lesson plans aligned with the GAISE report. The review process for publishing a lesson on the website is double blind thus ensuring the high quality of work being posted. Another example resource for teacher training is Bridging the Gap between Common Core State Standards and Teaching Statistics (AUTHOR 2, 2012). This publication offers 21 investigations that effectively illustrate how to present statistical material for students in grades K-8. Of these 21 investigations, 9 are dedicated to level A understanding and are very appropriate for training elementary school teachers and the K-5 classroom.

One such example is Investigation 2.4, "How can we sort our junk?" (pp. 53-62 in Bridging the Gap Between Common Core State Standards and Teaching Statistics). This investigation has students organize objects according to different attributes and then create a graphical display. Using graphical displays, students describe the patterns they see and begin to understand the concept of variability. While variability is not directly mentioned in the K-5 CCSSM, it is a concept that takes time to develop and many things K-5 students learn about statistics directly compliment the learning of variability. This lesson is aligned with the following CCSSM standards:
K.MD3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.
1.MD4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or fewer are in one category than in another.
2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, takeapart, and compare problems using information presented in a bar graph.
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Small groups of students are each given identical bags of "junk" from their classroom containing for example, beads, buttons, charms, cubes, blocks, fabric swatches, crayons, catalog pictures of clothes. Teachers can then ask students to empty the bag and organize all of the material using teacher prompts such as "What kind of objects do we have?" "What type of attributes should we use as a sorting factor" "How will the choice of the attribute affect the outcome of the sorting process?" In small groups, once the students have decided how they are going to sort the objects, they then complete a data collection sheet. Examples of filled out data collection sheets are pictured in Figure 1.

Figure 1. Sample data collection sheets


Students are then asked to either make a pictograph or a bar graph (see Figure 2 for example results) to display their results. Once the groups have completed their graphical displays, they are asked to describe what sorting factor was used, what categories were used for this attribute, and which category had the most, least, and equal number.

Figure 2. Sample pictograph and bar graphs


| Beads Soteded by Shape |  |  |
| :---: | :---: | :---: |
| 9 | 图 | (a) |
| 6 | 图 | (6) |
| $\stackrel{1}{9}$ |  | 0 |
| \% |  | (0) |
| ond | Remank | found |

Up to this point, the students have explored three of the four GAISE components: Formulate Questions, Collect Data, and Analyze Results. For the remaining GAISE component, Interpret Results, students are asked to draw conclusions from what they did. For example, groups discuss the following (pg. 58 in Bridging the Gap Between Common Core State Standards and Teaching Statistics):

1. What other attributes did your group discuss? Why did your group choose this attribute?
2. How did you determine the categories for your attribute? For example, what would you change if instead of using the actual color of the object, we considered categories, for example, "red" and "not red"?
3. Which groups used the same attributes? Are their displays similar? How are they similar? How are they different?
4. In the context of the original question "How can we sort our junk?", write a summary of their analysis based on how they displayed their data, what graphs they created, and what they learned about the type of junk they have through the process of sorting.

As seen in this example lesson, the approach with which the CCSSM themes are presented is done in a way that is supported by the GAISE framework recommendations. The investigation not only covers the CCSSM content identified in three specific standards but also promotes statistical learning in the spirit of the GAISE framework.

Example lessons that follow the spirit of the GAISE report and exceed the requirements of the CCSSM can also be found in curricula currently being used in classrooms across the US. More specifically, the reform curricula Mathematics Trailblazers, Investigations, and Everyday Mathematics have all been found to align well with the GAISE guidelines (AUTHOR 1, 2012). All three of these curricula are widely used. For example, Trailblazers is used by approximately 500,000 students and Everyday Mathematics by over 3,000,000 students.

The third grade lesson "Comparing with Bar Graphs" from Investigations exemplifies a lesson that may be used for teacher training purposes because it illustrates the type of statistical content a teacher must be ready to teach in their classroom. This investigation has students pose a statistical question that will form a 2-category bar chart, take a classroom survey, make a bar chart, and answer a set of questions about the results. This lesson is aligned with the following CCSSM standards:
K.MD3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.
1.MD. 4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or fewer are in one category than in another.
2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, takeapart, and compare problems using information presented in a bar graph.
3.MD. 3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

Students begin the lesson by tallying how many students from their classroom survey are wearing red and how many are not. These data then are displayed in a bar graph as a class. Students are asked comprehension questions about the graph such as what the numbers mean on the $y$-axis of the graph and how they would draw a bar if there were 7 students wearing red.
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The next portion of the lesson spills into GAISE level B thinking by asking students to compare groups. This portion of the lesson is in fact beyond the K-5 CCSSM standards. However, because currently used curricula require teachers to be able to teach such lessons, it is imperative that teacher preparation cover the content. It is also important to note that although reform curricula such as Investigations may have content in the elementary grades that is not included in the CCSSM for the elementary grades, the content does appear in the CCSSM middle grade standards.

After the introductory wearing red activity, the students are given a double bar graph displaying the way students get to school for two different schools. Again, teachers ask students graph comprehension questions such as what do the black lines represent. Then, students are asked to interpret the graph by answering: (1) how many students from each school walked? (2) what was the most popular way to get to school for each school? (3) what are some hypotheses about why there are differences between the two schools?


The Investigations example aligns with both the Formulate Questions and GAISE guideline by having teachers pose questions as well as having students formulate their own question (e.g., Are you wearing red today?). The students Collect Data by conducting a classroom survey, Analyze Data by graphically displaying two types of bar graphs, and Interpret Results by drawing comparisons about the different schools.
Through this example, a pre-service teacher can build up their understanding of bar charts representing one variable to bar charts representing two variables. In the two variable case, the content becomes focused on making comparisons.

These examples provide illustrations of how and what elementary teachers should know and be ready to teach in the classroom. Examples such as these can directly be used
in the teacher preparation courses to teach teachers the content as well as inform them of the statistical content they will be teaching in their own classrooms.

## Conclusion

Due to the large emphasis on data and data analysis in today's society, it is crucial for statistical educators to consider how to prepare a statistically literate population. The Conference Board of Mathematical Sciences (CBMS) includes statisticians in their most recent call to action.

Although most statistics courses for future teachers are taught by mathematicians or statisticians in mathematics departments, on campuses where there is a separate department of statistics, statistics courses for teachers is seldom a department priority. This needs to change. (CBMS, 2012, p. 9)

This paper has made the case for teachers to be trained to teach statistics and it would seem that statisticians and statistics educators need to work diligently to ensure statistics is a vital component of all teacher preparation programs. At the elementary teacher preparation level, statistics typically appears as one unit in a mathematics course dedicated to pre-service teachers. During this one unit, lessons such as the ones presented in this paper can be used to cover the content as well as expose pre-service teachers to exactly the type of lessons they will be expected to deliver in the classroom. Because statistics preparation is a weak point for teachers and teacher preparation programs, using actual K-12 statistics lessons that have been reviewed and written by statistics education experts in pre-service courses can provide an initial step in the right direction for teacher training in the statistical content domain.

Positive experiences with statistics during elementary school years will provide the necessary foundation for students to learn statistics at the level suggested in the GAISE framework and CCSSM during middle and high school. The manner in which statistical content is presented plays a key role in preparing elementary students for the emphasis on statistics in grades 6-12 of the CCSSM. Using the spirit of the GAISE framework to teach the little statistics present in the K-5 CCSSM, will ensure that students enter the later grades with some statistical preparation. Elementary school teachers thus must have an understanding of how the GAISE framework fits with the CCSSM. In general, teachers must have a general understanding of the process of thinking statistically and be able to choose example lessons such as the ones provided in this paper that not only cover the CCSSM but also elicit students' statistical thinking.

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