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Pre-service Teachers' Ability to Identify and Implement Cognitive Levels in Mathematics Learning

Dr. Enrique Ortiz
Associate Professor
School of Teaching, Learning and Leadership
College of Education and Human Performance
University of Central Florida

Abstract

This study analyzed pre-service teachers' ability to identify and implement cognitive levels. The framework involved the use of the Concrete, Pictorial and Abstract (CPA) cognitive levels combined with the Virtual-level (CPVA-levels). The V-level involves applets and apps, and three digital-dynamic sublevels: virtual-Concrete, virtual-Pictorial and virtual-Abstract. The results of this study have provided a better understanding of pre-service teachers' ability to identify CPVA learning levels, and use them for instructional purposes. The participants' ability to implement the CPVA levels was affected by the availability of the physical and virtual manipulatives, and the guidance provided by the cooperating teachers during the internship.

Introduction

The goal of this study was to analyze pre-service teachers' ability to identify and implement cognitive levels using a framework expanded by the investigator. The framework involved the use of the *Concrete*, *Pictorial* and *Abstract* (*CPA*) *cognitive levels* with the addition of the *Virtual-level* (*V-level*, or *CPVA-levels*). The research question for the study was:

> How were pre-service teachers' able to identify the *CPVA-cognitive levels* for the implementation of two lesson plans and a case study after an introduction of these levels during a mathematics for elementary school methods course?

With the many advancements and accessibility of technology, it is necessary to analyze some of the ideas we know about the identification and implementation of cognitive levels during the learning process. Some examples of the available technology include laptops, tablets, cell phones and Interactive Whiteboards. The *CPVA-interpretive framework* presented in this article is a way to analyze the use of these technologies as part of the learning process.

First, the *C-level* involves the use of physical manipulatives or objects as a learning tool to model and explore mathematical ideas; for example, a student counts how many real pencils are included in the set of pencils. In a similarly manner, the *P-level* involves the use of drawings, pictures or images of the manipulative materials or objects; for example, a student draws a set of pencils to represent the cardinality of a set. The drawing could include digital images using drawing software. The *A-level* encompasses the symbolic or verbal representation of ideas; for example, a student reads or writes a numeral that represents a set of ten pencils.

The *CPA-levels* have received acceptance and support by mathematics education researchers (Clements, 1999; Naiser, Wright, & Capraro, 2004; Suydam, 1985; Suydam & Higgins, 1977). Research findings summarized by Clements (1999) indicate that students who use manipulatives to learn mathematics: outperform those students who do not (including retention of concepts learned and problem solving skills); benefit from this use no matter what grade level, ability level, or topic involved; and improve their attitudes toward mathematics as <u>knowledgeable</u> teachers use the concrete materials <u>appropriately</u>. For the current study, especial interest was placed on pre-service teachers' knowledge and appropriate use of the manipulatives.

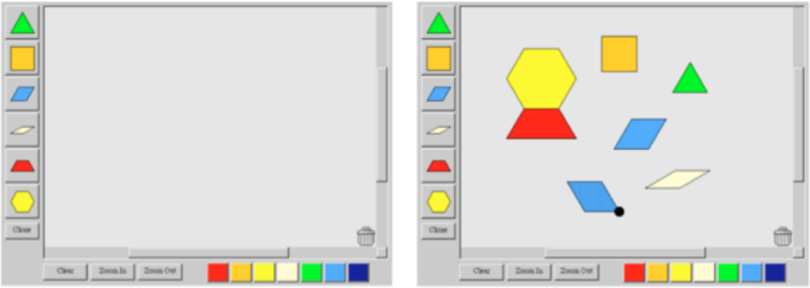
Finally, the *V-level* involves *virtual-learning tools*, and three sublevels, which require digital and dynamic (with movement) components. The sublevels are *virtual-Concrete* (*vC*), *virtual-Pictorial* (*vP*) and *virtual-Abstract* (*vA*). For example, a student moves ten digital color tiles as part of an app to represent the cardinality of a set. Notice that this task is considered to be at the *vC-level*, and includes the dynamic and digital components required for the *V-leve*l.

The *V-level* adds a new dimension to *CPA-framework*. It provides "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373). Duebel (2010) stated that static and dynamic virtual models can be found on the Web, but static models are not considered "true" virtual manipulatives (*VM*). Static models are more like the *P-level*, which has traditionally been used in classrooms in digital and non-digital formats, but learners cannot actually "manipulate" them (Deubel). For this reason, in the present study, we use "*Virtual*" to mean both digital and dynamic. Examples of the *V-level* are available online: ABCya.com (2012), Annenberg Foundation (2014), Brainingcamp (2013), *Electronic Examples* (NCTM, 2004), Intel (2014), Little Bears Studio (2014), Math Playground (2014), *National Library of Virtual Manipulatives* (*NLVM*) (Utah State University, 2014), *NCTM Illuminations* (2014), and Ventura Educational Systems (2014).

The *vC-level* includes digital-dynamic (non-static) versions of *concrete* materials, which could be virtually manipulated in a manner similar to physical objects or manipulatives to represent quantities or patterns. *NLVM-Pattern Blocks applet* (Utah State University, 2014) provides an example of a possible use of *vC-level* (see Figure 1). The digital-dynamic pattern blocks can be manipulated and moved on the screen virtually like physical pattern blocks. Notice that the *vA-level* is involved if the students use the clear, zoom-in and zoom-out buttons; *P-level* is involved digitally with the static/non-dynamic images provided on the buttons, or if a screenshot or printout is made out of what you see on the screen; and *A-level* is involved digitally with the students read the words Clear, Zoom In and Zoom Out; and when the students verbally or symbolically name, identify, sort and classify shapes by any given attribute. The *vP-level* is not involved in this case because the images presented in the applet are not dynamic. An example of the *vP-level* will be presented later in the article.

Figure 1

Pattern Blocks applet before adding *Pattern blocks* (left side), and after adding *Pattern blocks* virtually ("black dot" or blue parallelogram indicates spot for rotation)



In general, the CPVA levels could be used in isolation or combined to communicate ideas. For most mathematics learning activities, the A-level is needed since we need to have words and/or symbols throughout the learning process. This framework could be used to help visualize concepts and skills, develop appropriate learning and assessment tasks, and analyze and implement research procedures and findings. Notice that the ideas of digital, non-digital (real, physical objects or manipulatives), dynamic (with movement) and non-dynamic (static or without movement) were used to describe the CPVA-levels, and provided a way to distinguish between possible characteristics of these learning sublevels. The digital component includes any electronic format used to present information (dynamic or non-dynamic). However, the dynamic component also needs to be present for the V-level. This components needs to allow movement, manipulation and transformation of objects or data (digitally). A non-dynamic format is static and does not allow for free movement of objects or data (digital or non-digital). Table 1 presents a summary of these of learning levels (C, P, A, vC, vP and vA) and component combinations (non-digital, digital, dynamic and non-dynamic).

Table 1.
Learning levels and component combinations

Level/Component	Non-Digital	Digital	Dynamic (with movement)	Non-dynamic (Static or without movement)
Concrete (*C*)	x		x	
Pictorial (*P*)	x	x		x
Abstract (*A*)	x	x		x
virtual-Concrete (*vC*)		x	x	
virtual-Pictorial (*vP*)		x	x	
virtual-Abstract (*vA*)		x	x	

Background

The research findings related to the use of the *V-level* in the classroom provide some indication of their possibilities. However, what we have is difficult to generalize across mathematics concepts and grade levels (Burns & Hamm, 2011). This limitation is in part caused by a lack of an accepted framework and terminology. In some cases, there is lack of clarity about the combination of *CPVA-levels* involved in a study. In most studies, the researchers compare the effectiveness of *C-level* (use of manipulatives) to the *vC-level* (use of virtual manipulatives).

Studies involving the *V-level* indicate positive results in student achievement, understanding and attitude toward mathematics (Bolyard, 2006, Bolyard & Moyer-Packenham, 2012; Char, 1989; Clements & Battista, 1989; Crawford & Brown, 2003; Kieran & Hillel, 1990; Lee & Chen, 2008a; Lee & Yuan, 2010; Moyer, Niezgoda, & Stanley, 2005; Moyer-Packenham & Suh, 2012; Reimer & Moyer, 2005; Steen, Brooks, & Lyon, 2006; Stellingwerf & Van Lieshout, 1999; Suh, 2005; Suh & Heo, 2005; Suh & Moyer, 2007; Thompson, 1992; Yuan, 2007; Yuan, Lee, & Huang, 2007). Other studies did not report differences in students' mathematics learning when comparing the *V-level* (Burns & Hamm, 2011). No study reported negative finding as a result of using *V-tools*. Other areas of research involving *vC-level* are the following: problem solving (Ainsa, 1999; Moyer-Packenham, Salkind, & Bolyard, 2008); fractions (Reimer & Moyer, 2005; Moyer-Packenham & Suh, 2012); geometry (Yuan, 2007; Steen, Brooks, & Lyon, 2006); integer computation (Bolyard & Moyer-Packenham, 2012).

Another study by Anderson-Pence and Moyer-Packenham (2016) with a special connection with the current study involved several *VMs* and mathematical domains for fifth grade (division, geometry and fractions). Their exploratory study examined the influence of different *VM* types on the nature of student pairs' techno-mathematical discourse (*TMD*). The study included three pairs of fifth graders using *VMs*, and three *VM* types: 3 linked, 3 pictorial and 3 tutorial. The authors defined linked *VMs* "as open-ended *VMs* that present multiple representations of mathematical concepts (e.g., pictorial images, number line models, and numeric symbols) that change simultaneously as they are manipulated," and "reflect the user's actions and choices without dictating solution strategies" (p. 8), and *pictorial VMs* "as visual representations of mathematics concepts similar to physical manipulatives" and *"*reflect the user's actions and choices, but they do not include numeric symbols associated with the visual representation" (p. 8). These definitions also incorporated the need for a dynamic component in aspects of the *VMs*, but they are different than the definitions proposed in the current study. According to the definitions and examples provided by the authors and based on the *CPVA*-levels presented here, the *linked VMs* could involved a combination of *P*-, *A*-, *vC*-, *vP*-, and *vA-levels*, and the *pictorial VMs* could involve a combination of *P, A, vC*-, and *vP-levels*; and the *tutorial VMs* could involve a combination of *P*-, *A*-, *vC*-, *vP*-, and *vA-levels*. Three fifth grade student pairs participated in 9 sessions of mathematics instruction using *VMs*.

Their study compared three *VM* types: *linked, pictorial*, and *tutorial*. Students' levels of discourse in generalization, justification, and collaboration were measured while working with each *VM* type. They found significant differences in the quality of student discourse when using the different *VM types*. When working with *linked VMs*, students' discussions reflected consistently higher levels of discourse than when working with *pictorial* or *tutorial VMs*. However, *pictorial VMs* were associated with the largest amount of student-to-student discussion. The authors also indicated that the results of their study suggest that in order to encourage meaningful *TMD*, teachers should choose *VMs* with features that link multiple representations. The results of their study also indicated that for these pairs, tutorial *VMs* did not encourage meaningful student-to-student mathematical discourse. These findings emphasize to the importance of teachers' decision-making process as it relates to the *VMs*. Also, this idea is associated with the goals of the current study involving the implementation of *CPVA-levels*.

In general, these studies indicate that the use of the *V-level* could be beneficial alone or in combination with other *CRA-levels*. However, "[a]s evidenced by an examination of current literature, the choice between the *C*- and *vC-levels* in mathematics education is not very clear" (Burns & Hamm, 2011, p. 257).

Methodology

In the current study, the *CPVA framework* was introduced to 97 elementary school pre-service teachers (93 females and 4 males) enrolled in four different sections of a mathematics for elementary school methods course (29 section 1, 31 section 2, 25 section 3, and 12 section 4). The participants were in their second semester of their junior year or first semester of their senior year of their undergraduate bachelor's degree program. This course also involved the first of two internships, which was carried out concurrently with the course and involved whole day visits to elementary school classrooms twice a week (Mondays and Tuesdays) for 11 weeks and five times a week for two more weeks. This is the first of two internships required in the program. The second internship is part of their culminating experience in the program. They had previously taken a content course with an emphasis on using manipulative materials to learn mathematics. During the semester, other physical and virtual manipulatives involving whole numbers, fractions, decimals, integers, and geometry were presented and discussed in class using the *CPVA-framework*. The participants developed and implemented two lesson plans and one informal assessment during internship experiences. These reports were developed with the assistance of the cooperating teacher and the researcher who was also the instructor.

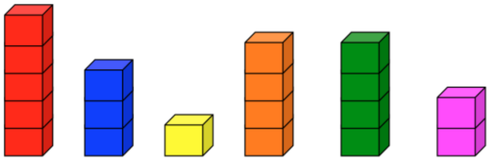
In terms of the participants' background with using different materials to learn mathematics, they indicated that they had prior experiences with base-ten blocks (83%), color cubes (67%), fraction tiles (38%), calculators (97%), tablets (such as iPads) (30%), attribute blocks (2%), color chips (31%), hands-on equations (33%), graphing calculators (90%), Interactive Whiteboards (49%), pattern blocks (32%), tangrams (37%), Cuisenaire rods (2%), virtual manipulatives (16%), spreadsheets (40%), two-pan balances (11%), algebra tiles (10%), Geoboards (67%), apps/applets (17%), and others (3%: computer games, counting bears, and flash cards). In terms of their background with using different materials to teach mathematics, they indicated that they had prior experiences with base-ten blocks (51%), color cubes (30%), fraction tiles (21%), calculators (41%), tablets (such as iPads) (16%), attribute blocks (1%), color chips (15%), hands-on equations (18%), graphing calculators (10%), Interactive Whiteboards (29%), pattern blocks (15%), tangrams (15%), Cuisenaire rods (1%), virtual manipulatives (11%), spreadsheets (7%), two-pan balances (3%), algebra tiles (2%), Geoboards (28%), apps/applets (9%), and others (3%: play money, food, board games, measuring cups, and beans). This background seems to indicate that they have some level of prior experiences with using physical and virtual manipulatives to learn mathematics, but naturally, have a lower level of experience with using them for teaching mathematics (50% or lower).

Treatment

The students were introduced to the *CPVA framework* during the second class of the mathematics for elementary school methods course. The different levels were discussed and examples were introduced. This introduction and discussion lasted about 45 minutes. The students were ask to identify and discussed their preferences of different learning situations:

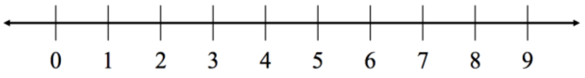
- What level or levels are represented by this graph made of <u>real cubes</u> to represent number of students who prefer each color (see Figure 2)?
  Expected answer: *Concrete* (real cubes), and *Abstract* (color names, and values for columns of cubes).

Figure 2
Graph made of real cubes representing preferences



- What level or levels are represented by a number line (printed on paper) (see Figure 3)?

Figure 3
Number line printed on paper

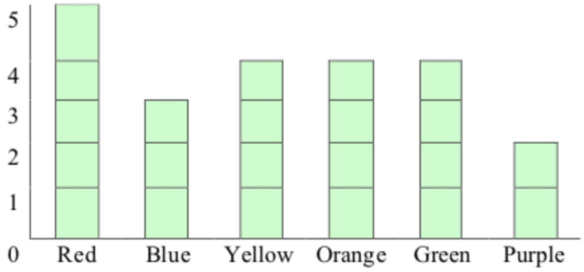


Expected answer: *Pictorial* (printed line with tick marks), and *Abstract* (written or printed number names or numerals).

- What level or levels are represented by this graph (printed on paper) (see Figure 4)?

Figure 4

Bar graph printed on paper



Expected answer: *Pictorial* (squares and bars), and *Abstract* (number symbols or numerals, and color names).

- What level or levels are represented by the frequency table below (assume it is printed on paper) (see Table 2)?

Table 2

Frequency Table

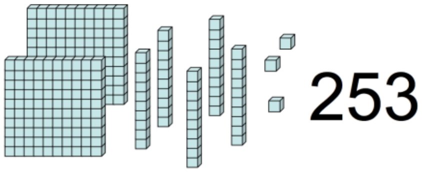
Color	Frequency
Red	5
Blue	3
Yellow	1
Orange	4
Green	4
Purple	2

Expected answer: *Pictorial-Abstract* (number symbols or numerals, and color names). The table is an organizing tool in this case, and could be considered semi-concrete or pictorial representation.

- What level or levels are represented by the Base-ten blocks below (real objects) and numeral written on the right side? Flats = hundreds, Longs = tens, Units = ones) (see Figure 5).

Figure 5

Base-ten blocks with numeral written on the right side

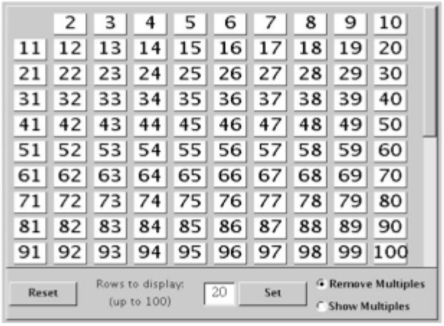


Expected answer: *Concrete* and *Abstract* (number symbols or numerals).

- What level or levels are represented by <u>*Sieve of Eratosthenes*</u> (see Figure 6) (Utah State University, 2014)? This virtual manipulative displays a grid containing numbers from 2 to 200. You can use it to explore patterns and relationships involving multiples. Using this virtual manipulative you may:
    - Remove multiples of a number

  - Show multiples of numbers
  - Reset the workspace
  - Choose how many rows to display

Figure 6
Sieve of Eratosthenes app



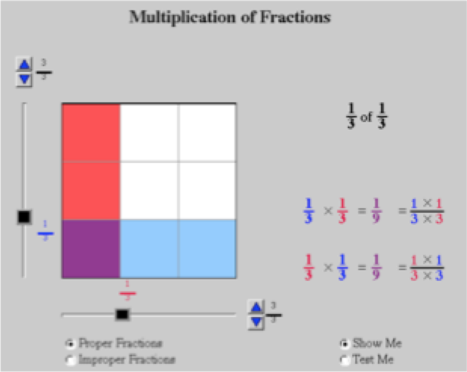
	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Rows to display: (up to 100) 20
● Remove Multiples
○ Show Multiples

Expected answer: *Pictorial* (Digital or Non-digital, Non-Dynamic or Static): Table format, *Abstract* (written or printed number names or numerals) (Digital or Non-digital, Non-Dynamic or Static), and *vAbstract* (digital, and dynamic and non-static manipulation of numbers to get factors and multiples).

- What level or levels are represented by *Rectangle Multiplication of Fractions* (see Figure 7) (Utah State University, 2014)? Use this virtual manipulate to graphically demonstrate, explore, and practice multiplying fractions. The grid shows two fractions multiplied together by showing one fraction in red on the left and another in blue on the bottom of a grid. The area of the overlapping region shown in purple is the product (result of multiplying) the fractions.

Figure 7
Rectangle Multiplication of Fractions app



Expected answer: *Pictorial* (number line, and printed line with tick marks), Abstract (written or printed number names or numerals), *vPictorial* (ability to manipulate columns and rows with colors, but without movement of pieces), and *vAbstract* (arrows that can change fractional parts). The sliders used in apps or applets provide specific *digital-dynamic* actions, which could be related to the *vC*-, *vP*- and/or *vA-levels*. The sliders, like the ones used in this app, could also be part of line segments, lines, and circles. For example, a point on a triangular path may be used as a sliding point to transform the size of the triangle. In this case, the sliders control the number of horizontal and vertical lines dividing a rectangle equally to form fractional parts.

- What level or levels are represented by <u>Fraction Pieces</u> (see Figure 8) (Utah State University, 2014)?
  This virtual manipulative can be used to work with fractional parts of a circle or a square. Using this virtual manipulative you may:
  - Add blocks to the workspace
  - See a fraction name for a block
  - Remove blocks from the workspace.

Figure 8

Fraction Pieces app (left side), and added fractional parts (right side)

Expected answer: *Pictorial* (circle and pieces) Non-dynamic-Digital, *Abstract* (written or printed number names or numerals): Non-Dynamic-Digital, *vConcrete* (Dynamic-Digital pieces), and *vPictorial* (movable colors, can change colors): Dynamic-Digital.

As a final activity, the students were asked to describe an effective mathematics activity that they have experienced as a student or teacher, match the activity with of a *CPVA-level* or combination of levels, and share with the class to figure out the levels represented in the activity. Throughout the semester, the framework was connected to other topics using the *CPVA-levels*:

- rational counting ideas: cubes, Cuisenaire rods, Color Tiles App (Brainingcamp, LLC, 2013), and Number Rods App (Brainingcamp, LLC, 2013)
- sorting, classifying, logic, patterns: pattern blocks, attribute blocks, Attribute Blocks App (Ventura Educational Systems, 2014), and Pattern Blocks App (Brainingcamp, LLC, 2013)
- number operations concepts: cubes, Cuisenaire rods, Color Tiles App (Brainingcamp, LLC, 2013), and Number Rods App (Brainingcamp, LLC, 2013)
- place value involving whole numbers and decimals: base-ten blocks, and Place Value MAB App (Kondys, 2014)
- fraction concepts: fraction tiles, Tangrams, and TanZen Lite App (<u>Little Bears Studio</u>, 2014)
- whole number, decimal and fraction computation: base-ten blocks, fraction tiles, and Place Value MAB App (Kondys, 2014)
- area, perimeter ideas: Geoboard, Tangrams, Geoboard App (<u>Clarity Innovations</u>, 2014), and TanZen Lite App (<u>Little Bears Studio</u>, 2014)

Lesson Plan and Assessment Reports

The participants' ability to identify and implement the *CPVA-learning levels* was assessed as they prepared and carried out two lesson plans, and completed an informal assessment (case study) involving a student with an identified weakness in a target mathematics skill or concept during their the internship. The participants were also asked to identify and explain the *CPVA-levels* involved in these reports.

Data Analysis

Data were collected qualitatively from participants' work with two lesson plans and the informal assessment. These data were analyzed for themes, and triangulated by looking at the two sources of information using the *CPVA-levels* as a framework. The main data from the lessons and informal assessments were analyzed for implementation of the *CPVA-levels*. The researcher looked for knowledgeable and appropriate use and correct identification of *CPVA-levels.*

Findings

Out of 194 possible lesson plans, 174 (92%) were available at the end of the study. Of the available lesson plans, 88 (51%) included the correct selection of *CPVA-learning levels* by the participants. The frequency of levels used in the lesson plans by grade level is presented in Table 3. The most frequent combination of learning levels used in the lesson plans was *CPA-levels*, followed by *PA-levels*, and *CA-levels*. The use of the *V-tools* was correctly identified and used in three lesson plans: virtual cubes used for counting and operation activities, virtual counters of different types used for operation concepts, and virtual two pan balance used to demonstrate equations, but was incorrectly identified 53 times in the available lesson plans (30%).

Out of 97 possible informal assessment reports, 88 (91%) were available at the end of the study. Of these available lesson plans, 66 (about 75%) included the correct selection of learning levels by the students. The distribution of levels used in the informal assessment reports is presented in Table 4. Similar to the distribution of selection for the lesson plans, the most frequent combination of learning levels used in the lesson plans was *CPA-levels*, followed by *PA-levels*, and *CA-levels*. The *V-tools* were not used in any of the informal assessment reports, but incorrectly identified 5 times (out of 97 reports, about 5%).

Conclusion

The results of the study related to participants' ability to identify and implement the *CPVA-levels* in lesson plan and informal assessment reports during their internship experiences provided valuable insights into this process. The students were more successful with the *CPA-levels* as part of lesson plans and informal assessment. However, they did have some confusion related to identifying some learning materials or activities in terms of *CPA-levels*. For example, indicating that tally marks are at the concrete level instead of the pictorial level, flash cards with numerals printed on them as concrete level instead of abstract level are misconceptions some participants had.

The results of this study also provided a better understanding of pre-service teachers' ability to identify *V-sublevels* (*vC*, *vP* and *vA*) as part of the reports. The participants demonstrated several misconceptions related to the identification of these learning levels in learning activities. Some participants incorrectly identified static-digital images or projections of PowerPoint presentations, websites and handouts as *V-level*. They did not take into account the dynamic component of the *V-level*. Using the *CPVA-framework*, the presence of the digital component is not enough to make these situations *VMs*.

Table 3

Number of available lesson plans the students identified correctly and expected correct identification by learning and grade levels

Levels/ Grades		CPA	PA	CA	A	PA-vC	PA-vPA	CPA-vCP	Total
K	Identified Correctly	11	-	2	-	1	-	-	14
	Expected Identification	19	3	3	-	1	-	-	
1	Identified Correctly	12	3	2	2	-	-	1	20
	Expected Identification	19	14	-	2	-	-	-	
2	Identified Correctly	5	2	2	-	-	-	-	9
	Expected Identification	15	7	1	-	-	-	-	
3	Identified Correctly	8	5	2	-	-	-	-	15
	Expected Identification	23	9	2	2	-	-	-	
4	Identified Correctly	8	9	2	1	-	-	-	20
	Expected Identification	11	15	3	2	-	1	1	
5	Identified Correctly	4	2	3	1	-	-	-	10
	Expected Identification	7	5	6	2	1	-	-	
	Total Identified Correctly	48	21	13	4	1	-	1	88
	Total Expected Identification	94	53	15	8	2	1	1	174
	% Identified Correctly	27.5	12	7.5	2	0.5	0.5	0.5	50.5

In a similar misconception related to *V-sublevels*, five students indicated that videos from YouTube Education (n.d.), TeacherTube.com (2014), or *BrainPOP - Animated Educational Site for Kids* (2014) presented by using computer or tablet screens, or interactive whiteboards as being *V-level* learning activities. These participants misunderstood the dynamic component of the *V-level*. In this case, the dynamic nature of videos is not the one we want to have in *V-level tools*. Videos are digitally animated, but do not allow for the manipulations and interactivity necessary for *VMs*.

The minimal presence of *V-levels* in lesson plans and no presence in the informal assessments need close attention when presenting this framework to participants. The pre-service teachers had access to interactive white boards, but when used for instruction, it was mainly to show videos and projection of handouts, PowerPoint presentations or websites. This use of the technology was regularly identified as the digital and dynamic components necessary for the *V-level*, which was a common misunderstanding. In these cases, the participants did not see the need for the virtual movement that should be present when using *V-level* apps or applets.

Table 4
Number of available Informal Assessment Reports the students identified correctly and expected correct identification by learning and grade levels

Levels /Grades		CPA	PA	CA	A	Total
K	Identified Correctly	5	1	1	-	7
	Expected Identification	7	1	1	-	9
1	Identified Correctly	12	2	2	2	18
	Expected Identification	19	2	2	2	25
2	Identified Correctly	6	2	-	-	8
	Expected Identification	7	2	-	-	9
3	Identified Correctly	5	3	2	-	10
	Expected Identification	7	3	2	2	14
4	Identified Correctly	6	5	-	-	11
	Expected Identification	8	5	2	1	16
5	Identified Correctly	5	4	2	1	12
	Expected Identification	7	5	2	1	15
	Total Identified Correctly	39	17	7	3	66
	Total Expected Identification	55	18	9	6	88
	% Identified Correctly	44	19	8	4	75

Similarly, some participants indicated that static images on the computer screen were at the *vP-level* instead of the *P-level*. This type of images contains the digital component, but not the dynamic component needed for *virtual tools*. The participants might need to be exposed to more examples and more practice to clarify and avoid this misunderstanding.

In terms of the teaching experiences, it is possible that participants' ability to identify the cognitive levels is related to their ability to properly use them to meet the needs of all students, in particular children with special needs, and this ability could impact the effectiveness of the learning activities they implement. In most cases, the participants' ability to implement the *CPVA-levels* was affected by the availability of the physical and virtual manipulatives, and the guidance provided by the cooperating teachers during the internship activities. It was hard for the participants to use the *CPVA-levels* when they were not being use during their internship experiences. Some cooperating teacher for one reason or another did not include physical or virtual manipulatives as part of their regular learning opportunities. This is an issue that deserves more investigation.

One of the many instructional decisions teachers make is the selection of instructional materials. If teachers are not able to understand and select the appropriate cognitive level or combination of levels that facilitates learning and meet a student's need at a given time, then the teacher's effectiveness and impact on a student's learning could be diminished. As expected, the participants had different levels of understanding and misconceptions of the definition and uses of *CPVA-cognitive levels*. After analyzing the data, we have a better idea of how participants understood these levels, possible misconceptions they might had, and how to help them avoid or overcome these misconceptions. As a result, participants could be more effective in their selection and implementation of the *CPVA-levels* to meet students' needs in the area of mathematics.

In general, the *V-level* provides additional help for students at all ability levels to learn mathematics and "to develop their relational thinking and to generalize mathematical ideas" (Moyer-Packenham, Salkind, & Bolyard, 2008, p. 204). We need to take appropriate advantage of the many apps and applets available for free or very affordable prices from different sources. We should not blindly incorporate an applet or app just because it is believed to be "virtual." We need to identify which virtual resources can help with a specific concept or skill.

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