THE EVOLUTION OF A GRAPHING CALCULATOR COURSE FOR PRESERVICE MATHEMATICS TEACHERS

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Abstract

The article is about developing a course designed to present graphing calculator applications and assessment techniques to mathematics preservice teachers. Mathematics educators are faced with the task of not only preparing their students in the content but also training them to use the available technology. The article discusses why this course is taught, whom it is taught to, when and where it is taught, what topics are included, how it is organized, and what the requirements are. Selected questions are used to illustrate sample assessment and calculator applications. Pre and post tests statistical results from previous classes are discussed.

Introduction

The article is about the evolution of a course designed to present graphing calculator applications and assessment techniques to mathematics pre-service teachers. The article discusses why this course is taught, whom it is taught to, when and where it is taught, what topics are included, how it is organized, and what are the requirements.

Why?

Since the infusion of the graphing calculator into the education marketplace, mathematics educators have faced the task of not only preparing their students in the content area but also training them to use the available technology (Demana & Waits, 1990; Dunham & Dick, 1994; Fey & Good, 1985). The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) has advocated that all students have access to calculators at all times and at all levels of understanding. From the *Professional Standards for Teaching Mathematics* (NCTM, 1991, p. 1), the Council recommends that teachers become more proficient in "using and helping students use technology and other tools to pursue mathematical investigations." *The Standards 2000 Draft* (NCTM, 1998) stresses the importance of appropriate technology integration in the curriculum when they write:

> Teachers have the responsibility to make appropriate choices about the use of calculators in particular contexts, on the basis of how well the tool will help or hinder the intended mathematical development. Technology is an important part of our world, and schools must affirm the appropriate use of calculators at all levels as a legitimate and important tool for learning and doing mathematics.

Several research studies document the benefits of calculator use in the classroom (Campbell & Stewart, 1993; Carlson, 1996; Dunham, 1996; Dunham, 1993; Harvey, Waits, & Demana, 1995; Hembree & Dessart, 1986; Quesada, 1994; Smith 1997). These references to technology confirm the need for future mathematics educators to be proficient with current technology.

A search of the literature for curricular ideas for this course, found no curriculum that would meet the needs of the students. As the course evolved, ideas also appeared in research articles that validated the teaching methods being used. The teaching strategy of using activity-based learning, cooperative groups, and classroom exercises mirrored the ACE cycle (activities, class discussion, and exercises) advocated by Asiala, Brown, Devries, Dubinsky, Mathews and Thomas (1996). The technology related research previously cited supported the need for teachers to be trained to use technology in the classroom. However, this course was about integrating the technology with content. Again, as the course progressed, research became available to support this idea (Darling-Hammond and Ball, 1999). Over the years, the course has undergone some changes, but the underlying ideas of classroom activities, cooperative learning, and the careful integration of technology, pedagogy, and content has not changed and has eventually come to be an established and accepted practice of teaching undergraduate mathematics.

Some mathematics educators are fortunate enough to be in situations where calculators are used throughout the curriculum, and thus their students are already well versed in calculator use. However, there are many institutions where students proceed through all of their coursework and never have the opportunity to use a graphing calculator. If they have the opportunity to use the calculator, many times they have no instruction on proper assessment techniques using technology. It is the last two situations that led to the development of a course to address the technology needs of students about to enter the teaching field.

Whom?

Mathematics education students complete a rigorous sequence of mathematics courses. However, many feel that, while they are knowledgeable in the upper level mathematics content, they do not feel prepared to teach the content they will face in secondary school mathematics courses. Some students have an opportunity to take a methods course through either an education or a mathematics department, but still comment that the course did not adequately prepare them for classroom instruction. Therefore, a course designed to review content that the students will most likely be teaching once they are in the schools, to incorporate the use of the graphing calculator as it relates to that material, and to demonstrate assessment techniques was developed.

When and Where?

The first time the course was taught, Spring 1995 in New Jersey, it was taught as an elective course called "Selected Topics in Modern Mathematics" which integrated content and calculators. This author is now teaching at a university in Georgia and teaches the course once a year. Students seeking mathematics certification in Georgia must pass the Praxis Exam that requires the use of a graphing calculator. At the other author's Texas institution, mathematics majors who intend to teach can often graduate without touching a graphing calculator. Students graduating from Texas colleges and universities who seek certification must pass the Examination for the Certification of Educators in Texas (ExCET) which tests content and requires use of the graphing calculator. Therefore, those students have, in the past, been at an extreme disadvantage. Texas students began requesting technology assistance in the Fall 1994 so a mathematics special topics course was taught in summer 1995 by the Texas author using the other author's outline.

What calculators?

The first three times the course was taught several calculators were used including the Casio 7700 and 9700, Sharp 9300, Texas Instruments 82 and Hewlett-Packard 38G. The students were not required to purchase a calculator since the instructors would use loaner sets from the manufacturers. Because availability changed each time the course was taught, one course would begin with 82's and the next would begin with Sharps. The topics remained in the same order each time, but the graphing calculator used to demonstrate and present the topic would vary. As each loaner set came in, not only was it used in conjunction with the content, but special features of the calculator were demonstrated as well. For example, the lists on the 82, the solver feature on the 9300, or the dynamic graphing on the 9700 were all demonstrated whether it was related to the current content or not. Since Summer 1997, the TI-92 has been used during the last quarter of the semester. Special features of this calculator are demonstrated, assessment techniques are discussed, and the calculator is required on the final exam.

As calculator technology has continued to develop, many manufacturers have combined the same features on their calculators. Many public school districts furnish or require graphing calculators, but these differ in manufacturer. Advanced placement statistics is now taught using special statistics features on graphing calculators. For these and many other reasons, the authors continue to change which calculators are used in the class to stay current. The students are now required to purchase a calculator for their own use but the brand is not specified, only the capabilities. A calculator with lists, solver, and advanced statistics features is required, such as the TI 83, Sharp 9600, or Casio 9850. The CAS calculators by Casio, Hewlett-Packard, and Texas Instruments are not allowed until the last quarter of the semester when the TI-92's are furnished to all students.

What Topics?

The course uses *Advanced Mathematics: Precalculus with Discrete Mathematics and Data-Analysis* by Richard G. Brown as a textbook. Not all topics in the book are covered, but the book serves as an excellent reference for the students even after the course. Supplemental materials are provided by the instructors and change each semester. Selected topics follow:

* Introduction to calculator (basic operations, graphing, window setting, etc.)
* Linear Equations and Inequalities
* Quadratic Equations and Inequalities
* Polynomials
* Trigonometry
* Sequence Graphing, Modeling, and Functions
* General Results of Polynomial Equations and Linear Inequalities
* Polynomial Inequalities and Linear Programming
* Exponential and Logarithmic Functions
* Parametric Equations and Lab
* Descriptive Statistics and Graphing
* Inferential Statistics
* Curve Fitting

* Limits and Derivatives
* Integrals

From the above list it can be seen that there is a wide variety of topics. This is not an exhaustive list. One reason the course continues to evolve is that it changes based on the needs of the students enrolled in the particular semester it is being taught. The topics mostly stem from the six year evolution of the course, ExCET and Praxis test topics, changing calculator capabilities, NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989), *Standards 2000 Draft* (1998), and student suggestions. While covering the selected topics and teaching the calculator, every effort is made to demonstrate different teaching strategies. Discovery learning is emphasized and lecturing is de-emphasized. Cooperative learning, group discussions, student presentations, and lab activities are incorporated whenever appropriate. Alternative assessment techniques are used in the course. After all, the course is for future teachers who need to be exposed to as many teaching strategies as possible.

How is the course taught?

As indicated earlier, the course has been evolving since 1994. The topics and calculators change depending on the current technology and the mathematics preparation of the students, but the course requirements have undergone only one change. Homework, daily quizzes, reading and writing assignments, and tests have always been the core of the course. A project assignment was instituted later to broaden the course.

Homework is assigned at the end of every class and occasionally collected and graded. If it is not collected then two or three of the homework problems are given as a quiz at the beginning of the next class. One homework assignment that may be unique is the assignment of theorems from the text that the students are to “translate”. The students in the class are to take a seemingly complicated theorem and describe how they would convey the essence of the theorem to a high school student.

The students are asked to complete reading assignments from current (less than two years old) journals. The assignments are broken up so that some students turn in written reports and some present oral reports each class meeting. The articles deal with the use of the graphing calculator (their choice of articles), technology in a specific content area (instructor choice of articles), and probability and statistics with a technology focus (student choice). The students must use the World Wide Web to obtain one of their articles. The articles chosen by the instructor are usually research based. These assignments provide students with the opportunity to read what others are doing with technology in the classroom and also to become familiar with journals, mathematics education research, and the importance of these as resources when they are in the classroom.

Designing test questions have become easier over the years, but still provide a challenge. Test questions that the authors had used in the past have become less viable as the technology is constantly changing. Over time, an excellent collection of test questions has been assembled. These will be discussed in the assessment and applications section.

Writing is essential in the course. The theorem assignment and the article summaries are just two examples. Most test questions require some type of written explanation along with the mathematics. Students must learn that, in order to teach mathematics, they must know how to communicate mathematics. Many students feel they can communicate their mathematical knowledge only by working the mathematics in a problem. The course attempts to change that notion and improve their communication skills.

Students are required to complete a project in the course. Simply learning to push buttons on the calculator and review content does not guarantee that the students can develop their own ideas and lessons once they are in the classroom. The projects are not designed to fit the mold of the education lesson plans that most students have written. Instead, they are to develop three classroom ready activities that they can use. The format is very informal. They only have to tell what content they are teaching, student prerequisite knowledge, materials needed, a student worksheet, and the way they will teach the topic. They must present one of their lessons in the class as part of their project grade. After the presentation of the lesson, every other student in the class evaluates the presentation. Most students find this project to be one of the most valuable aspects of the class.

Because this course is taught in the mathematics department, the authors cannot teach pedagogy or "education" ideas such as constructivism or inquiry-based learning. However, many teaching strategies are modeled in order to present a variety of techniques to students. Hopefully, by viewing and participating in varied teaching strategies, these future teachers will incorporate them into their classrooms. So many teachers, authors, and probably many readers, teach as they were taught. This often has a negative impact on a new teacher's classroom. This course aims to have a positive impact in the way it models teaching.

Assessment and Applications

Assessment takes on many forms in this course, such as classroom activities, group projects, article summaries, writing assignments, and peer evaluation. Students also have practice writing assessment items and a selection of the student items is included on the final exam. Multiple representation (Rule of Three) is stressed during the course and used on assessment items. The following are selected questions that have been used as sample assessment and calculator application items.

1. (a) Sketch the graphs of the following functions on the same set of axes:

   $y = 3x^2 - 2x - 5$ and $3x - 2y = -2$

   (b) Determine the point(s) (if any) of intersection algebraically

   (c) Using the graphing calculator, discuss four methods for finding the intersection point(s) (if any) of the above problem and include the answers.

2. Suppose you correctly enter an expression for the variable y on a graphing calculator; however, no graph appears on the display when you graph the equation. Give a possible explanation and the steps you could take to remedy the problem. Illustrate your explanation with an example.

3. The function $y = 0.03x^2 + 254.50$, $0 < x < 100$

   approximates the exhaust temperature y of a diesel engine in degrees Fahrenheit, where x is the percent load on the engine.

   (a) Determine the inverse of the function. What does the inverse function mean in words?

   (b) Use a calculator to graph the function and its inverse.

(c) Determine the percent load interval if the exhaust temperature must not exceed 500^{o} F.

4. Use the graph to find the solution to $[\ (2x - 5)^{2}\ ] \div [\ x^{2} - 3x - 28\ ] \geq 0$.
Be sure to indicate what you entered as Y=, your window settings, and any points of interest on the graph. Give your answer in standard or interval notation.

5. Find the $\lim_{x \to 0}\ [\ \sin(x) / x\ ]$
a) Graphically
b) Numerically (table)

6. The world population y (in billions) for the years 1983 to 1994 is given in the table, where $x = 3$ corresponds to 1983.

x	3	4	5	6	7	8	9	10	11	12	13	14
y	4.68	4.77	4.85	4.94	5.02	5.11	5.20	5.29	5.38	5.48	5.55	5.64

(a) Use the regression capabilities of the calculator to fit a linear model to the data.
(b) Use the regression capabilities of the calculator to fit an exponential model to the data.
(c) Population growth is often exponential. For the twelve years of data given, is the exponential model better than the linear? Explain
(d) Use both models to predict when the population will be double the 1983 population. Which of the answers do you think is more accurate and why?
(e) If you take the integral from 3 to 14 of the above function, what will your answer mean in terms of the problem?

7. Novocain is injected as an anesthetic for minor surgical and dental procedures. It is eliminated from the body primarily by the kidneys. In one hour, a person with normal renal functions removes 20% of the Novocain from the blood. If the initial dose was 500 mg. then:
(a) Write the sequential formula for the calculator.
(b) What other equation could be used to model this problem?

8. Solve $x^{3} = 3^{x}$. Describe the process you used to find the solution.

9. Have each student bring a one-variable data set with approximately 20 values to the test which if of personal interest. Use the data set to answer the following questions:
(a) Use your calculator to find the mean and standard deviation. Explain each in words for the data set.
(b) Show how you verify the calculator values for the median, lower quartile, and upper quartile.
(c) Draw and label the box-and-whisker plot.
(d) Are there any outliers? How do you know?

10. The temperature of a just poured cup of hot tea is recorded to be 158^{o} F. The tea slowly cools over time t (in minutes) and its temperature T is recorded.

t	*T*
0	158

10	132.8
30	105.8
50	92.3
70	84.2
90	79.2
110	76.1
120	75
125	74.7
130	74.5

(a) Graph the data.
(b) Calculate the exponential regression for the data and write the equation (accurate to two decimal places) and the r^2 value (accurate to four decimal places).
(c) Subtract 74 from each T value and re-calculate the exponential regression equation and r^2 value (accurate to two and four decimal places respectively).
(d) Graph both regression equations on the above scatterplot.
(e) Why would the second equation (part c) have a better r^2 when the graph is not a better fit?
(f) Adjust the second equation so that it does fit the data best.
(g) Verify that your adjusted equation for part (f) is better than the equation in part (b) by graphing the residuals.

Discussion and Conclusions

Each semester this course has been taught, pre and post tests have been administered solely for instructor self-evaluation and reflection. The results have always shown statistically significant improvement ($p < 0.0001$), but the courses were not designed as part of a research study. The items were scored holistically on a 0-5 scale and the problems on the pre and post tests were very similar. Pre-test results from the fall 1998 course included scores ranging from 6 to 30 with mean 13.4 and median 13 ($n = 25$). Corresponding post-test results ranged from 54 to 96 with mean 81.3 and median 85 ($n = 25$). The test questions were designed to test conceptual understanding rather than content so the impact of the calculator could be assessed. However, after analyzing the answers given, it became apparent that the differences in scores appeared to be due to both the weak background of the students and the inclusion of the calculator in the course. The scope and sequence of the course is now becoming equally focused on the content and the calculator. The students who take the class have glaring weaknesses in both areas. The course is improving student content knowledge, their conceptual understanding of mathematics, their ability to use and incorporate a graphing calculator effectively, and their understanding of the assessment process needed in a technology enhanced mathematics course.

After five years, the authors are still enjoying teaching the course. However, it is a challenge to teach. Preparation time is at least one and one-half times any other course. Having to know a wide variety of calculators can become confusing not to mention the fact that all the calculators and their overhead projecting devices must be in working order. Assessment techniques have to be continually revised. But for all the work involved, both authors are still teaching the course at the request of students. Such marked improvements in student skills provide justification for continuing the course.

A major concern now becomes the students who are not taking this course (since it is an elective and not a requirement) and their ability to pass certification tests and conduct themselves in a classroom. Recently this was brought to one author's attention when a certification pre-test was given to mathematics student teachers and the results were dismal with no students (n = 12) passing. After examining the pre-test, it was determined that approximately 57% of the 150 questions needed to be solved using the calculator. Mathematics departments need to understand that they are training some future teachers and not all research mathematicians. Likewise, education departments need to realize that teaching mathematics (as well as many other subjects) does not always fit into the mold used in curriculum classes. A concerted effort must be made by everyone involved in teacher preparation to send properly trained mathematics teachers into the classroom.

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