IUMPST: The Journal, Vol 2 (Pedagogy), January 2004. [www.k-12prep.math.ttu.edu]

Teacher Questioning with an Appropriate Manipulative May Make a Big Difference

Sinan OLKUN
Ankara University, School of Education
Cebeci Ankara, Turkey
olkun_s@ibu.edu.tr

Zulbiye TOLUK
Abant Izzet Baysal University
14280 Bolu Turkey
toluk_z@ibu.edu.tr

Abstract

The purpose of the present study was to present examples of the utilization of social processes such as teacher questioning and collective argumentation coupled with an appropriate use of a manipulative material to stimulate students' thinking in teaching geometry. We first present theoretical issues concerning the use of manipulatives, teacher questioning and collective argumentation. We then provide examples and describe the use of these components in teaching and learning geometry with pre-service elementary school teachers.

Mathematical meaning is not intrinsic but develops out of interaction with other people and environment and reveals itself as a new individual interpretation (Yackel, Cobb & Wood, 1999). In other words, it occurs in a social context. Therefore, designing learning environments in which sound mathematical learning occurs is an important task for teachers of mathematics. Martino and Maher (1999) argue that a very special combination of student, teacher, task, and environment furthers individuals' cognitive growth in mathematics classrooms. The main idea behind the present study, therefore, was the utilization of social processes such as teacher questioning and collective argumentation coupled with an appropriate use of a manipulative material to stimulate students' mathematical thought in teaching mathematics. In the following sections, we first present theoretical issues concerning the use of manipulatives, teacher questioning and collective argumentation. Then, we provide examples and describe the use of these components in teaching and learning geometry.

Manipulatives

The purpose of using manipulatives in mathematics classroom is the concrete modeling of abstract mathematical ideas. While it is virtually impossible to show a mathematical concept directly by means of a manipulative, it might be possible for a learner to construct a concept or discover a mathematical relationship through an appropriate use of a manipulative in a meaningful task environment. In addition to teaching new concepts, a manipulative can be used to identify the students' current conceptions of mathematical objects. Moreover, a manipulative can be used to develop or refine students' mathematical definitions.

Long-term use of manipulatives in mathematics classrooms shows that, as long as a manipulative makes sense for a topic it is beneficial, however, manipulatives are not sufficient to guarantee meaningful learning (Clements, 1999). The context in which a manipulative is used also needs careful consideration in terms of both content and delivery of the lesson.

Teacher questioning and collective argumentation

Questioning is a teacher tool that is used to guide or direct student attention toward the exploration and reinvention of mathematics (Martino & Maher, 1999). Through teacher questioning, students are invited to express their thinking in an inquiry-based classroom environment. Teachers, then make informed decisions about students' mathematical thinking to lead subsequent discussions. Individuals are challenged to consider their solutions through questions asked by the teacher and their classmates.

The content of questions is of importance. For instance, asking more open-ended questions aimed at conceptual knowledge and problem-solving strategies can contribute to the construction of more sophisticated mathematical knowledge by students (Martino & Maher, 1999). Therefore, in order to use teacher questioning effectively, the teacher must be knowledgeable about the content domain, and possess the ability to distinguish between student imitation and student reinvention of a mathematical idea. As a result, a task assigned by a teacher question should provide the opportunity for student reinvention of mathematical ideas through both exploration and the refining of earlier ideas (Martino & Maher, 1999; Middleton, Poynor, Toluk, Wolfe, & Bote, 1999).

The timing of the question is another concern with which a teacher should take great care. Teachers can contribute to the process of conceptual change through the vehicle of asking timely questions in support of student reinvention and extension (Martino & Maher, 1999). In sum, asking good and timely questions is an important task of teachers in the process of learning and teaching mathematics that requires knowledge about both mathematics and children's learning of mathematics.

Collective argumentation is another component of social processes in learning and teaching mathematics. Scientific argumentation rather than the manipulation of symbols and algorithms should be promoted in mathematics classrooms. Students should be encouraged to be actively involved in arguing about mathematical concepts so that learning how to support one's mathematical conclusions through the use of logical argumentation augment or even replace the often meaningless manipulation of symbols and algorithms in mathematics classrooms (Forman, Larreamendy-Joerns, Stein, & Brown, 1998). Therefore, questions should aim at revealing the level of student thinking. The reason for using manipulatives, questioning and argumentation at the same time in mathematics class is to create a "hands-on, minds-on math" environment.

The purpose and setting of the present study

In this article, the researchers present episodes that took place in their mathematics methods classes for prospective classroom teachers. In all of the activities, a mason ruler was used as a manipulative. The researchers discuss how the use of a mason ruler and teacher questioning in their math method classes elicited the students' current conceptions about geometric shapes and how they were used to move the students toward more formal use of the concepts and higher level of thinking (van Hiele, 1986).

Students in our methods classes can have different mathematical backgrounds since math score is just one of several selection criteria at the entrance exam for the department. In the first year of the college, they take two 2-credit math courses called Basic Mathematics I and II. In the third year, they take two 4-credit Mathematics Methods courses. Both math and math methods classes are required for elementary school teacher candidates.

Although the experiences reflected in this paper are related to the education of pre-service elementary school teachers who are going to teach grades 1–5, the aim of these activities was to improve the geometry content knowledge of these students. In other words, the activities were prepared to remove the inadequacies of these students' geometry knowledge they brought from their secondary school geometry classes. We expected these pre-service elementary school teachers to be at the van Hiele geometric thinking level 3, because any student who completes secondary school geometry program must be at least the van Hiele geometric thinking level 3 (Teppo, 1991). That's why the activities could also be used at secondary school level.

In order to be a good model for prospective teachers, we usually try to use manipulatives while discussing mathematical concepts and relations. It is not, however, usually possible to provide a manipulative for each student since there are 50 to 70 students in our classes. Therefore, we use the manipulative and discuss with the students during presentation as in the following episodes. The episodes were taken from the three weeks (4 hours each week) of a mathematics methods course offered during the spring semester of 2001-2002 academic year.

Episode 1. Establishing relationships among the shapes: Rectangle, parallelogram, and square

Many students and even teachers think that squares, rectangles, and parallelograms are all distinct geometric shapes partly because they were taught that way (Clements & Battista, 1992) or because any relationship has never been established among these shapes in their secondary geometry classes. In the following episode, the mason ruler was used to elicit students' conceptions of a rectangle and a parallelogram.

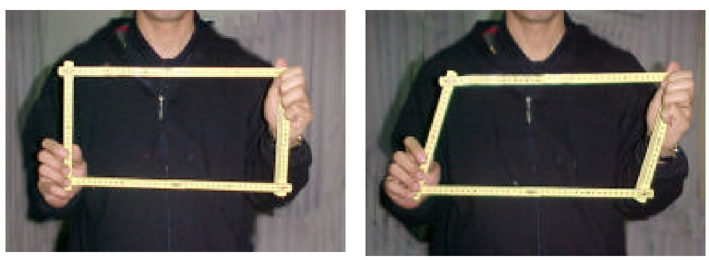

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Figure 1. Rectangle and parallelogram

T: What is this?" (To the class showing a shape made with the ruler that looked like Figure 1A)
Ss: A rectangle. (Many of the students responded promptly and the rest showed agreement by nodding).
T: Ok, what is this, then? (This time the ruler was tilted a little bit, like in Figure 1B).
Ss: Parallelogram. (Many students answered without hesitation).
T: Is this not a paralelogram? (showing something like in Figure 1A again by turning the shape into rectangle)
Ss: No, yes, no ...

There were students saying "yes" as well as "no", mostly "no." Some students were undecided. Many of them thought that a rectangle was not a paralelogram and that these were two different geometric shapes. This was partly due to the fact that some students were naming the shapes from their look rather than using their properties. After some careful questioning, they started to focus on the properties of the shapes, which made them establish relationships among the shapes.

T: What makes a quadrilateral a parallelogram?
S: It should be like tilted (a student said).
S: Opposite sides should be parallel (another student said, after awhile).
T: Does a rectangle have opposite sides parallel?
S: So, is that (implying Figure 1A) a rectangle or a paralelogram? (asked another student).
S: Both (a student answered).
T: What makes a quadrilateral a parallelogram, then?
S: Quadrilateral with opposite sides parallel.
T: Therefore, a rectangle is ...
S: a special kind of parallelogram.
T: because ...
S: it satisfies all of the properties of a parallelogram.
T: In other words, it is a special kind of parallelogram because it has two pairs of parallel sides. Is this sufficient for a quadrilateral to be a parallelogram?
S: Yes
T: So, when a parallelogram has four congruent or four right angles, it is called ...
S: A rectangle.

As it is seen from the following episode, students were able to identify all of the figures correctly. However, they had difficulty in establishing a relationship between a square and a rectangle, and a square and a rhombus. This difficulty arised mainly from their inadequate definition of these shapes. For instance, having a pair of longer sides and a pair of shorter sides is an insufficient explanation for a rectangle because there are other shapes which also fits to this definition. Students failed to realize that the length of the sides was not a necessary condition for a quadrilateral to be a rectangle. However, after some questioning, they realized that it was sufficient for a quadrilateral to be a rectangle to have two pairs of parallel sides and four congruent angles.

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Figure 2. A square is ...

T: How about this one (showing a square, see Figure 2A).
S: It's a square.
T: Look at that shape (Figure 2B). What is this shape?
S: Rhombus.
T: What changed and what remained the same when I changed Figure 2A to Figure 2B?
S: The sides are still equal and no right angle.
T: What else? Are opposite sides parallel?
S: Yes. It is a parallelogram then.
T: What else? How many sides?
S: Four sides. Quadrilateral (long pause...)
T: How about a rectangle? What makes a quadrilateral a rectangle?
S: Two of the sides longer and two shorter.
T: Is it necessary?
Ss: No... yes...

T: A parallelogram is a rectangle if it has ...
S: four right angles.
T: Then, do you see these properties in this shape? (Showing Figure 2A).
S: Yes. It has four sides, opposite sides equal, and right angles.
S: A square is a rectangle, then? (a student asked.)
T: What do you think? (to the class).
S: Yes, 'cause it has all of the properties of a rectangle. It also has four congruent sides, which makes it a square and a rhombus too. (Another student replied.)

After these discussions, there were still a few students who were reluctant to accept these relationships among quadrilaterals and they continued to discuss with their classmates. These students needed a longer time to be convinced.

Episode 2. Incorrect Definition: Trapezoid

In the following episodes, the mason ruler was used to challenge the students' definitions of a trapezoid. Students' definitions contained uneccessary information about a trapezoid. But after some questioning, students modified their initial definitions by replacing uneccessary information with the necessary and sufficient one.

Figure 3. Quadrilateral

T: What is this shape? (showing Figure 3)
Ss: Trapezoid, quadrilateral ... (many of them remained slient, possibly undecided)
T: Ok, students. Who said trapezoid? Why did you think that way?
S: I said trapezoid because it has no regular shape.

In Turkish, Trapezoid means "Yamuk," which has other meanings in daily language. It means inclined, oblique, crooked, bent, or something that has no regular shape. Therefore, students might be confused with the daily use of the term "yamuk".

T: What makes a quadrilateral a trapezoid?
S: Four sides.
T: What else?
S: Parallel top and bottom sides.
T: Like this? (see Figure 4A)
S: Yes

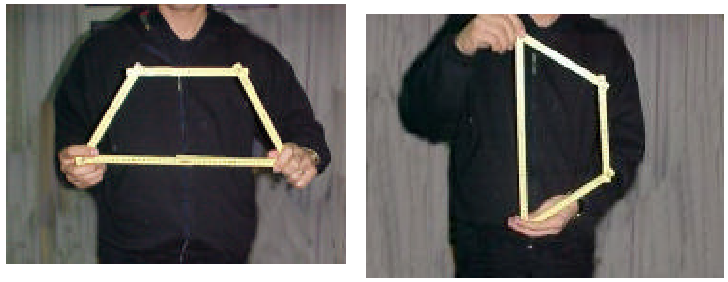

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Figure 4. An equilateral Trapezoid

T: How about this one? Is this not a trapezoid anymore? (see Figure 4B)
S: It is still a trapezoid but turned to the side (smiling faces).

This response shows that the student has a very static mental picture of a trapezoid, which is placed long side at the bottom and parallel sides horizontally. This caused students to produce an incorrect definition based on visual judgements easily distracted by the shape's orientation.

T: So, what should we say about its parallel sides?
S: A trapezoid is a quadrilateral with a pair of parallel sides. It does not change if you turn the shape.
T: Therefore, a quadrilateral is a trapezoid if it has ...
S: A pair of parallel sides
T: If I say "a trapezoid is a quadrilateral with at least one pair of parallel sides?" Do you think that a rectangle or a square is a trapezoid?
Ss:... (confused)

There were again some students saying "yes" and some "no" as well as some remaining silent. The instructor repeated the definition emphasizing "at least one pair" of parallel sides. Some texts give different definitions for a trapezoid. Some define a trapezoid as a quadrilateral with exactly one pair of parallel sides (Drooyan & Rosen, 1986; Bassarear, 1997) others define a trapezoid as a quadrilateral with at least one pair of parallel sides (Billstein, Libeskind, & Lott, 1993; Schmid & Schweizer, 1983). We stick to the second definition since it seemed more appropriate for a hierarchical classification of 2D geometric shapes.

S: A rectangle is a trapezoid because it has at least one pair of parallel sides. It has an additional pair of parallel sides, though. It is a quadrilateral and a parallelogram since it has four sides and opposite sides parallel.
T: So you say a rectangle is a trapezoid. How about a parallelogram? Can we say that a parallelogram is also a trapezoid?
Ss: …(no clear response at all).
T: We said, "A trapezoid is a quadrilateral with at least one pair of parallel sides?" Does a parallelogram have a pair of parallel sides?
Ss: Yes
T: So what do you think? Is a parallelogram a trapezoid or not?
Ss: yes… no…
T: Think about it.

After these discussions, students were assigned to classify quadrilaterals (without deltoids) on the chalkboard in the classroom. They made a logical classification of quadrilaterals guided with teacher questioning and an effective use of a manipulative.

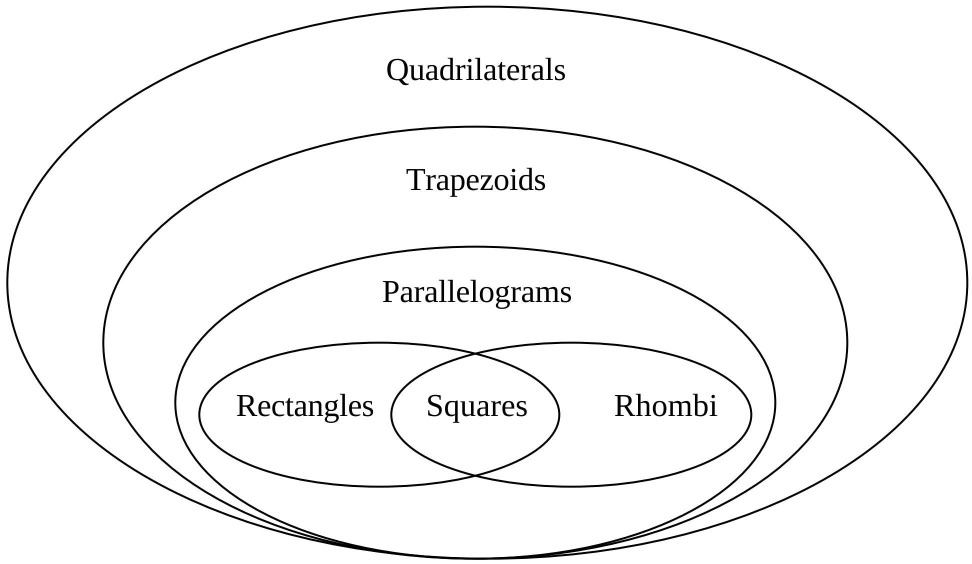

Figure 5. A logical classification of quadrilaterals

Here are some of the conjectures students produced from the Figure 5: (1) All squares are rhombi, and all squares are rectangles. (2) Some rectangles and some rhombi are squares. (3) All squares, rectangles, and rhombi are parallelograms. (4) All parallelograms are trapezoids, (5) All trapezoids are quadrilaterals but not all quadrilaterals are trapezoids.

Episode 2. What is a deltoid?

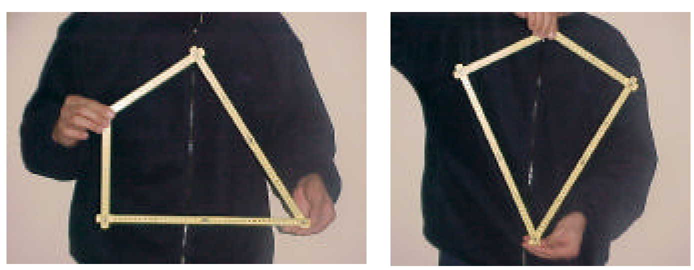

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Figure 6. Deltoid in two different orientations

T: What is this shape? (showing Figure 6A)
S: Quadrilateral (promptly).
T: Right. How about that? (turning the same figure a little bit like in Figure 6B)
S: Deltoid (same student answered)

The student was using visual judgements in deciding the names of Figure 6A and 6B. Even though both of the shaps were deltoids, the student only called 6B a deltoid. In fact, it was more obvious from Figure 6B that the shape is a deltoid. This may be because students usually see deltoids in that orientation only.

T: How do you describe that shape? (showing Figure 6A and B).
S: Four sides and ...
S: Two of the sides equal. I mean two equal and the other two equal.
T: How about if I say "A quadrilateral with two distinct pairs of congruent, adjacent sides." How could we place the deltoid in this figure (Figure 5)?

S: Outside trapezoids but inside quadrilaterals.
T: Why?
Ss:…
T: Do you think that a rectangle has these properties?
S: No. It has two pairs of congruent sides but they are not adjacent.
T: Right. How about a square and a rhombus? Do they have "two distinct pairs of congruent, adjacent sides?"
S: I guess so.

After these discussions, students tried to insert deltoids into the set-model classification of quadrilaterals (see Figure 5). However, it was not that easy. Based on the definitions of geometric shapes, the following classification was produced (see Figure 7).

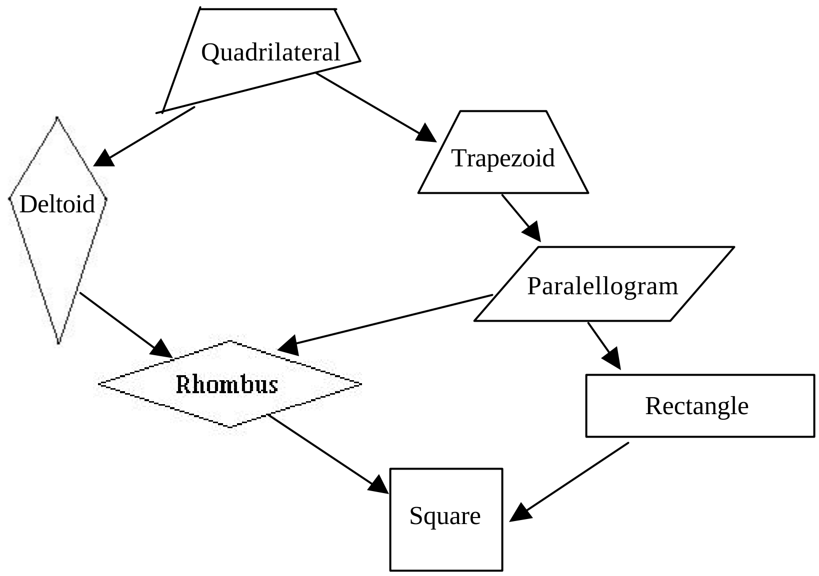

Figure 7. A more detailed classification of quadrilaterals

Episode 4. Area of a parallelogram

Many students rotely assume that the areas of a square and a rhombus are equal if the lengths of their sides are equal (Hewit, 2001). Similarly, some students make the assumption that the area of a rectangle does not change if it is tilted without changing the lenght of its sides. Yet all of the students correctly calculate the area of a given figure if it has the necessary dimensions on it.

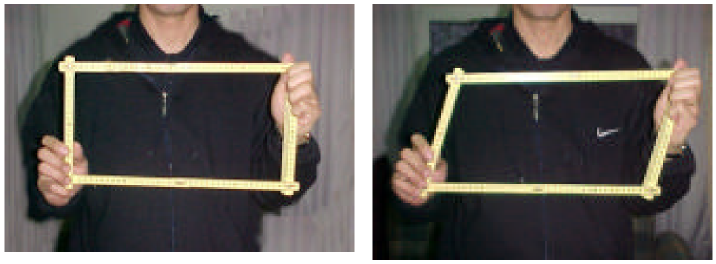

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Figure 8. Area of a rectangle and parallelogram

T: Look at the area of this shape (showing a rectangle, like the one in Figure 8A)
T: Look at that shape (Figure 8B, just a little bit tilted rectangle). Did the area change?

There were students saying "yes" and "no" but mostly no. Some students remained undecided at the time.

T: Ok, you either said yes or no, think a little bit more about why did you think that way?
S: I said no because nothing changed. You just tilted it.
S: I said no because the sides did not change.

We continued to tilt the shape little by little like the one in Figure 9 until there was no area at all. In other words, the figure became a horizontal line. We were seeing the changes in students' faces. When they realized that something was changing they started to smile. At just that time, we asked "What do you think is changing and what remains the same?" A student said "Although the length of the sides were not changing, the height was getting smaller"

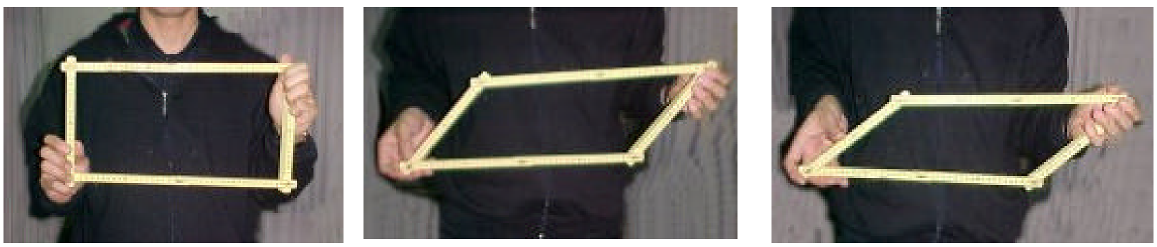

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Figure 9. Change in the height of a parallelogram

T: What difference does it (a change in height) make in the area of the shape?
S: The area of a parallelogram equals to the base times height. Since the height is getting smaller so does the area.

As it is seen from the conversation, students were initially ignoring the role of height in thinking about the area of a parallelogram. Even though a parallelogram and a rectangle were related, the formula for the area of each was different in the way that height of a parallelogram became a side of the rectangle. This actually makes both formulas equivalent. Since a rectangle is a special kind of parallelogram, its one side is also its height.

Conclusions

The use of teacher questioning to direct the student attention to the important aspects of geometric shapes made with an aproppriate use of a manipulative, mason ruler, triggered a relational understanding of plane geometric shapes in a hands-on minds-on environment.

The mason ruler is a very cheap and yet powerful manipulative in constructing certain geometric shapes. To our experience, it was also found enjoyable by the students of various ages. The dynamic and flexible nature of the manipulative, mason ruler, made it possible to experiment Dienes' mathematical variability principle (Post, 1992) by focusing on the properties of each geometric shape and transitioning from one shape to the other. This way, students were able to discover some properties of and relationships between geometric shapes. Especially, making the transitions from one shape to the other provided us with an opportunity to observe the properties that changed and the properties that did not change.

Presenting examples and non-examples of geometric shapes was also possible with this manipulative. During the questioning and subsequent discussions, it was possible to see the changes in students' conceptions about some geometric shapes and their properties as well as the relationships between them. This evidence shows that many students increased their level of geometric thinking to a higher level (van Hiele, 1986). The long lasting effect of this instruction was also evident in their exam papers in that almost all the students made acceptable arguments about the logical classification of geometric shapes with reference to the mason ruler.

The focus on only formal definitions and lack of variations on concepts in geometry classes may have led pre-service teachers to construct incorrect definitions of these concepts, or caused them to fail to establish relationships among the concepts during their secondary school and early college years. As a result, pre-service teachers may have a collection of isolated bits of knowledge about geometric shapes even in their third year of college education. If geometry teaching is supported with visual aids, coupled with appropriate teacher questioning, it should be much easier for students to construct robust conceptions of geometry concepts and to establish relationships between them.

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