PROSPECTIVE TEACHERS READING RESEARCH ARTICLES: EXAMINING THE POTENTIALLY EMPOWERING AND DEBILITATING EFFECTS

David E. Meel
Bowling Green State University
Department of Mathematics and Statistics
Bowling Green, OH 43403
meel@bgnet.bgsu.edu

Abstract

The current mathematics education reform efforts regarding teacher preparation emphasize the development of content and pedagogical knowledge. In particular, the adequacy of teachers' mathematical knowledge receives considerable attention along with the effects of content knowledge on pedagogical practices. This paper illustrates how reading a particular research article designed to draw personal relevance for the investigation into the division of fractions can have both empowering and debilitating effects. The responses of 23 prospective elementary teachers specializing in mathematics portray the variegated efficacy of using the reading of the Borko et al. (1992) article to pique the prospective teachers' interests and draw relevancy for the content under discussion. Additionally, the study indicates the need for attention to emotional upheavals which result from such an instructional intervention.

Teachers' subject matter and pedagogical content knowledge have received greater attention as some researchers turn their focus from studying elementary school students' understanding of mathematics content to examining the content and pedagogical knowledge held by inservice and prospective teachers. This research has resulted in an increased awareness that some teachers have difficulty explaining basic concepts to students and these difficulties affect instruction and students' developing understandings. In particular, Fennema and Franke (1992) found that teacher knowledge influences instruction since classroom discourse partially depends on teacher knowledge. In effect, teacher subject-matter knowledge influences the richness of class discussion and presented material. Pedagogical content knowledge, on the other hand, effects a teacher's instructional style, selected activities, and student learning (Fennema & Franke, 1992).

Pedagogical content knowledge links with subject-matter knowledge to guide the sequence of concept presentation and with general pedagogical knowledge to draw on global techniques of teaching (Marks, 1990). Additionally, a teacher's understanding of the difficulties students encounter during mathematical investigations influences the decisions and the presented classroom learning opportunities (Carpenter, Fennema, Peterson, Chiang & Loef, 1989). These elements affect the choices a teacher makes about what to teach, how to teach it, how to organize the classroom, what techniques to use, how to individualize instruction, and what modifications will be made. All of these decisions are guided by a teacher's pedagogical content knowledge in concert with subject-matter understandings, perceptions of pedagogical practices, student difficulties, and expected roles of the teacher and student as well as the role of the subject matter.

This article focuses on the use of reading research articles to engender changes in prospective teachers subject-matter knowledge and pedagogical content knowledge regarding situations involving the division of fractions. An extensive body of research has identified that children, adolescents, and prospective and inservice teachers have difficulties with fractions (Azim, 1995; Ball, 1990, 1993; Behr et al., 1983; Behr et al, 1984; Behr, Wachsmuth & Post, 1985; D'Ambrosio & Campos, 1992; Hunting, 1983, 1986; Johnson, 1999; Katzman, 1997; Khoury & Zazkis, 1994; Kieren, 1988; Lehrer & Franke, 1992; Leinhardt & Smith, 1985; Lester, 1984; Ma, 1999; Mack, 1990; Piel & Green, 1994; Schifter, 1997; Simon, 1993; Thipkong & Davis, 1991; Tzur & Timmerman, 1997). Many of these researchers focused their attention on the difficulties students and teachers have in explaining concepts relating to fractions and the division of fractions. According to Ball (1990), one of the reasons that both students and prospective teachers have difficulty explaining the division of fractions algorithm is that "Division of fractions is rarely taught conceptually in school; most of the prospective teachers probably learned to divide with fractions without necessarily thinking about what the problems meant" (p. 141). As a result, the lack of conceptual focus causes prospective teachers, who eventually become practicing teachers, to teach the division of fractions from an exclusively procedural prospective (Ball, 1990; Piel & Green, 1994; Simon, 1993; Tzur & Timmerman, 1997). This turns into a loop of insufficient instruction revisiting students decade after decade. In fact, Thompson (1985) reported that even after taking a methods course little or no change occurred. The perceptions and beliefs acquired through a prospective teacher's previous experiences predominate even after taking courses on the methods of teaching.

However, the current mathematics education reform effort envisioned in the NCTM Standards requires both altered and richer understandings of mathematics currently held by prospective teachers (Ball, 1989, 1996; Even & Lappan, 1994; Thompson et al., 1994). Only recently have researchers begun to examine various methodologies to enhance prospective teachers conceptual understandings of the division algorithm. D'Ambrosio and Campos (1992) investigated the effects of engaging prospective teachers in research focused on examining children's understanding of fractions. These examinations elicited inquisitive dispositions, sensitized them to the children's knowledge of fractions, enhanced familiarity with the research literature, induced greater inspection of instructional sequences, and refined reflection on assessment. Tzur and Timmerman (1997) found that a microworld environment aided the prospective teachers' evolving understanding of fraction multiplication and could help to make sense of the division of fractions algorithm. Schifter (1997), drawing from a four-year teacher enhancement project, concluded that teachers need more preparation to confront unexpected and puzzling questions about fractions. In particular, Schifter (1997) asserted that (a) teachers need to develop a richer understanding of the subject matter; (b) teachers need to gain more experience listening to students and sorting out the mathematical issues confronting those students; and (c) teachers need to learn to pose questions in order to gain additional insights into students thinking. In effect, Schifter called for increased attention to the interplay between subject-matter knowledge and certain aspects of pedagogical content knowledge.

Arising from these injunctions to preservice teacher programs, is the question "Can one develop a classroom atmosphere where prospective teachers recognize a personal need to enhance their understandings, identify that questions students ask require preparation, and learn to investigate the students' understandings?" However, Crump (1995) claims that "Students will learn what they want to learn and will have difficulty learning material that does not interest them" (p. 1). In response, the task of reading a research article by Borko et al. (1992)[1] entitled "Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily" was designed to pique the prospective teachers' interests and draw relevancy for the content under discussion. Specifically, this research study sought to investigate the prospective teachers' reactions to reading the Borko article and to answer if the

[1] Henceforth, the Borko et al. (1992) article will be referred to as the Borko article.

prospective teachers, after reading the article, could provide conceptual explanations for why the "invert-and-multiply" (*I&M*) algorithm works for the division of fractions and provide a real-life situation to model the division of fractions for a specific case.

Methodology

This study involved the examination of journal entries where the prospective teachers were entreated to summarize and react to their reading of the Borko article which described a novice teacher's struggles with explaining the division of fractions. In particular, this article described the struggles of a prospective teacher, who due to certain circumstances, becomes an elementary school teacher after dropping out of the secondary mathematics program and having her elementary content courses waived due to her taking advanced mathematics courses. It was intimated that the genesis for her lack of success in explaining elementary mathematics content derived from her lack of preparation and unwillingness to explore to find answers. Combined with this investigation, the study integrated a set of tasks, the first of which was drawn directly from the Borko article, which asked the prospective teachers to explain why the *I&M* algorithm works. The second problem requested, with explanation, a real-life situation corresponding to the following computation: $\frac{15}{2} \Big/ \frac{1}{4}$. Specifically, the following questions were posed:

1. A student stated the following to you:

   > I know that when I'm supposed to divide two fractions, I have to turn one of the numbers upside down and multiply, but I don't know why all of a sudden it gets changed to multiplication, so I forget which one to turn upside down and I get a bunch of the problems wrong.

   How would you respond to the student?

2. Provide a real-life situation which would correspond to the following computation: $\frac{15}{2} \Big/ \frac{1}{4}$.

   Explain why the situation models the computation.

The participants of this study were 29 prospective teachers completing their mathematical content specialization for either an Elementary Education degree (K-8 certification) or a Child and Family Development degree (Pre-kindergarten certification preparing students to work with public or private preschool, day care, or Head Start programs). The discussion will restrict itself to a cohort of 23 of these 29 prospective teachers who provided both a journal entry response and answered the above two questions. All the participants were enrolled in an Advanced Mathematics for Elementary Teachers taught at a regional state university during the Spring of 1997. This senior-level, capstone course consummated the math content specialization where the prospective teachers were expected to have completed classes in number systems, geometry, precalculus, statistics, and calculus although a few concurrently took calculus. The specialization provided these prospective teachers with additional training in a particular content area beyond the two required mathematics content courses. Consequently, the specialization was designed to prepare these prospective teachers to take leadership roles as mathematical specialists in elementary schools.

Analysis of the participants' responses to these assessment questions and reactions described in journal entries focused on the qualitative aspects identified by Silver and Cai (1993). In particular, analysis centered on classifying the various response types and quantifying the number of respondents displaying similar response characteristics concerning the answer, the explanation type, the usage of various representations, and other salient characteristics. In order to accomplish this, the data collected were double-coded by raters and examined for consistency between codings. For any responses evidencing a discrepancy between the two codings, the response was reviewed and a consensus was reached concerning the final coding of the response.

Results

As mentioned previously, each of the participating prospective teachers were required to discuss their reactions to the Borko article in a journal entry. These reactions included emotional phrases such as "very negative feelings", "scared and nervous", and "very upsetting" as well as non-emotional reactions. In addition, these journal responses provided evidence that some of the students took the lessons drawn from the article and applied those lessons on a personal level. For instance, lines in the journals included descriptions such as "The one thing I would hope I would never do, is promise an explanation the next day and then never follow through", and "I want to be able to answer questions like this or at least be able to humble myself enough to say, I don't know, why don't we find out together".

Table 1 looks across all the submitted journal responses in reaction to the Borko article.

Table 1. Prospective teacher's reactions to reading the Borko et al. (1992) article

	Emotional	No indication of emotional reaction
Personal application	11	2
No indication of personal application	3	7

Examination of Table 1 reveals that emotional responses were not always linked with personal identifications with the article's lessons. However, this type of response and a non-emotional/non-personal identification type of response were the typical norms. In order to get a better feeling for these types, the following examples help illustrate the differences.

Alice's emotional response containing indications of personal application.

It made me realize that mathematics is more than knowing the concepts and definitions. You have to understand it before trying to have the students understand it. Also, you need to become prepared for classes. Making lesson plans involve problems, lecturing, and understanding. The teacher has homework, as well as the students. I could somewhat understand when the teacher had a hard time explaining fractions to the class, but not investigate into it so she could tell the class the next day was quite angering to me. I would expect that teacher to look into it and find an answer. It helped me realize that the students are going to have lots and lots of questions on everything. You, the teacher, need to come prepared for that in some way.

Darla's emotional response lacking indication of personal identification.

The article about Ms. Daniels is discouraging not only as a future educator, but as a future parent as well. Children need teachers who are confident as well as competent in all subject areas. I feel that her education at her selected college may not have been up to par. I do not think that she should have been allowed to test [out] of her elementary math classes. I also feel that her uneasiness with teaching the division of fractions should have been dealt with.

Kayla's non-emotional response containing indications of personal application.

> As for the paper I found it very interesting. The student teacher was asked to review division of fractions and when a student asked a question she learned that she really didn't understand them herself. I can see how this could easily happen to a teacher because there are many things we do simply because that is the way we learned it. As teachers we really need to understand concepts and be armed with examples if we are going to be effective and credible.

<u>Tom's non-emotional response lacking indication of personal application.</u>

> I also read the handout we received in class. It was mainly about a student teacher who they called Ms. Daniels. She is an elementary education major who specializes in mathematics. It shows the changes in her attitudes and beliefs she makes from before she student teaches until after she teaches. A student asked her why her multiplied by the reciprocal when she was dividing fractions. She really couldn't come up with a good answer. It showed her problems that she had with relating what she was teaching to life. I think that this may be a typical experience for some people when they start student teaching.

In the first example, Alice's response reveals a prospective teacher emotionally involved and evaluative of her understandings and her teaching duties. In contrast, Tom's journal response contained neither an emotional reaction nor a personal application. Essentially, Tom summarized the facts without embellishing with his own feelings concerning Ms. Daniel's actions in the classroom. As a result, it is evident that reading the Borko article incited various reactions amongst the prospective teachers. The article caused some to reevaluate their understandings and others to reflect on their role as teachers. Consequently, the reading of the Borko article encouraged, in some participants, a growing personal awareness of their own inadequate understandings and their need to search for answers as to why things work.

<u>Explaining why the *I&M* algorithm works</u>

The prospective teachers' responses to the question of why the *I&M* algorithm works as it does with turning fractions "upside down" and changing to multiplication provided insights into the knowledge and resources applied to the situation. In response, The prospective teachers provided three different types of explanations: *Conceptually-based* (10 participants), *Procedurally-based* (12 participants), and *Idealic* (1 participant).

Conceptually-based, in this instance, means the response contained elements providing answers to the question of why the procedure yielded it's intended result. For example, Kim provided the following conceptual response:

> I would try to explain to the students that you can look at the problem in two different ways. Take the problem $\frac{\frac{6}{2}}{\frac{1}{4}}$ for example. It may be easier to write it $\frac{6}{2} \div \frac{1}{4}$. From there you can ask the student what is $\frac{6}{2}$ and then take that number and multiply it by four. Or the student can ask themselves what is the value of 3 divided into $\frac{1}{4}$ sections.

$$\frac{\otimes}{4} \quad \frac{\otimes}{4} \quad \frac{\otimes}{4} = 12$$

Of those 10 prospective teachers who did respond with at least one conceptually-based statement to explain the reasons behind the *I&M* procedure, they drew upon a variety of explanations isomorphic to: (a) "Division is the same as multiplication by reciprocal" (4 participants); (b) "How many times will y go into x?" (3 participants); (c) "Division is the inverse operation of multiplication" (2 participants); (d)

"Multiplication is the inverse (reverse) operation of division" (3 participants); and (e) "What value times y equals x?" (4 participants). Evidently, a few of the prospective teachers utilized more than one of these explanations as part of their response.

A response focusing solely on the *procedural* aspects of solving a division of fractions does not address the reasons underlying the process but rather concentrates on the process itself. An example of this type of response is Kerry's explanation shown below:

> The number you always want to flip ("turn upside down") is the bottom fraction. For example you want to divide $\frac{\frac{1}{2}}{\frac{1}{4}}$ you are going to take the bottom fraction $\frac{1}{4}$ and flip it ("turn it upside down") so then it will be $\frac{1}{2}\times 4=2$. The most important thing to remember is always take the <u>bottom</u> <u>fraction</u> and flip it ("turn it upside down").

As identified previously, a greater number of the prospective teachers responded with procedurally-based explanations compared to conceptual or idealic. This attraction to rule-based explanations was identified by Ball (1990) when she stated ". . . the prospective teachers, both mathematics majors and the elementary candidates, tended to search for the particular rules . . . rather than focusing on underlying meanings. They seemed to assume that stating a rule was tantamount to settling a mathematical question" (p. 141). Unfortunately, this means that one of the goals of having the prospective teachers read the Borko article to illustrate the insufficiency of relying upon procedural explanations was not entirely achieved.

A third type of response was given by one prospective teacher in reaction to this first question. This response type, classified as *idealic,* corresponded to a discussion of the pedagogical techniques the prospective teachers would employ. Jamie's response exemplified an idealic characterization of what she would do in response to the student's question:

> You could always show the student the algebraic explanation of the problems, but that does not work for me. What I would do is take an example of division of fractions problem, one that reflected a real life experience, & explain the problem using manipulatives. I think that visuals & real life examples help students to understand better. I would solve the problem w/ only manipulatives first, the relate it to the flip and multiply routine. Explain how we get the same answer to the problem by flipping and multiplying.

Neither descriptive of the procedure surrounding *I&M* nor the conceptual basis for the procedure, this response merely identified the various pedagogical strategies the prospective teacher would use to address the student's question.

<u>Modeling the division of fractions</u>.

The prospective teachers' responses to providing a real life situation modeling the division of $\frac{15}{2}$ by $\frac{1}{4}$ and explain why the situation models the computation indicated that most could supply reasonable real-world situations modeling the division of fractions. In Table 2, the first three categories correspond to the ability to model the division of fractions and the next four categories indicate if a prospective teacher's provided situation modeled an operation other than the division of $\frac{15}{2}$ by $\frac{1}{4}$ or contained an element that is impossible.

Table 2. Models used by the prospective teachers

	Number of prospective teachers
Reasonable model	
Models division of fractions correctly using $7\frac{1}{2}$	3
Models division of fractions correctly using $\frac{15}{2}$ only	8
Models division of fractions although problem situation contains an unfocused question	5
Unreasonable model	
Models multiplication of $\frac{15}{2}$ and 4 rather than division of fractions	3
Models multiplication of $\frac{15}{2}$ and $\frac{1}{4}$ rather than division of fractions	2
Unreasonable problem - partition of element which cannot be divided, i.e., a student	1
No problem situation provided	1

A problem situation was judged as correct if the context correctly modeled the division of $\frac{15}{2}$ by $\frac{1}{4}$ or $7\frac{1}{2}$ by $\frac{1}{4}$. Several of the prospective teachers translated $\frac{15}{2}$ into $7\frac{1}{2}$ to make the problem situation more realistic. For instance, Hermina identified this transformation in her situation.

> I would turn the $\frac{15}{2}$ into $7\frac{1}{2}$ and take that many pizzas. So I would have $7\frac{1}{2}$ large pizzas and each one would be divided into four pieces or fourths and we would end up with 30 pieces.

Other students chose to maintain the $\frac{15}{2}$ without transforming it. The lack of transformation caused some awkwardness in reading in some of the problem situations while other students elegantly used the $\frac{15}{2}$ in ways similar to the following example from Darla:

> You have 15 candy bars for 2 groups of kids. The candy bars are big enough to be cut into fourths. How many kids could be in each group and still receive a piece of candy bar?

Lastly, some situations modeled the division of $\frac{15}{2}$ by $\frac{1}{4}$ but included questions not completely linked with the computation. For instance, Mandy's provided problem exemplified the difficulty some of the prospective teachers had in constructing a reasonable question as part of a problem situation.

> If you have a piece of yarn that is $\frac{15}{2}$ inches long, and you have to cut it into pieces that are $\frac{1}{4}$ inch long for each of your students, how many students must you have in your class?

The answer to Mandy's question could be anything but could be answered from the division of $\frac{15}{2}$ by $\frac{1}{4}$ assuming that the number of students in the class equated with the number of $\frac{1}{4}$ inch long pieces of yarn.

Situations which were considered incompatible either modeled the multiplication of $\frac{15}{2}$ and either 4 or $\frac{1}{4}$. For instance, Jane's provided situation modeled the multiplication of $\frac{15}{2}$ and $\frac{1}{4}$:

> You have 15 candy bars and you are asked to give $\frac{1}{2}$ of them to the teacher ($\frac{15}{2}$). The teacher then tells you to give $\frac{1}{4}$ of the remaining candy to the student sitting next to you $\left(\frac{15/2}{1/4}\right)$. How much do you have to give you classmate?

One prospective teacher, Becky, provided a situation which modeled an impossible partition:

> I have 15 students in my class. I want two groups to play a game. Each group needs the same number of the students to read the problem and keep score for the rest of the group.

These faulty situations revealed that some of the prospective teachers had difficulty blending division into a realistic problem situation that modeled the division of $\frac{15}{2}$ by $\frac{1}{4}$.

These data shown in Table 2 contrast the conclusions drawn by Ball (1990), Simon (1993), and Ma (1999). However, in each of those studies, the population under examination differed from that selected for this study. Both Ball and Simon looked at prospective teachers early in their programs and each found that about 70% could not provide an appropriate representation for a division of fractions problem such as $1\frac{3}{4} \div \frac{1}{2}$ or $\frac{3}{4} \div \frac{1}{4}$. Ma (1999), in comparing U.S. and Chinese inservice teachers, found that only one of the 23 U.S. teachers in that study could provide an acceptable story problem for the division of fractions whereas nearly all the Chinese teachers constructed a correct story problem. As a result, this study contrasts with these studies both with respect to the demographics of the participants and the abilities of the prospective specialists in elementary mathematics' to provide acceptable situations modeling the division of fractions.

Discussion

For many of the prospective teachers, the reading of the Borko article served the purpose of causing them to be introspective of their ability to provide explanations and examples in similar situations. For instance, some students, after reading the article, stated some the following in their journal: "The article we were assigned to read really made me think. I never really thought about how much goes into every part of every subject taught." and "The research article given to us was both interesting and eye-opening. It really helped me realize that teaching does not only deal with learning the basic concepts. Children have very inquisitive and wondering minds." This introspection, typically did not have a personal negative side. However, for two of the prospective teachers, Connie and Jamie, the introspections resulting from reading the article increased their fears of instructional sequences which extend beyond the scope of their knowledge. In the case of Connie, she reacted to the Borko article in the following manner:

> This article is scary because as I was reading I realized that I am not sure I could devision [sic] of fractions to a 6th grade class either. It made me thind [sic] that I too might have difficulty answering unpredicted [sic] questions my students may have. Students are always thinking of things that teachers never thought of and I don't want to be caught of guard and unable to explain to my students, but I am sure it will happen to me at some point. . . . it's scary to think that her own methods teacher probably couldn't have taught it to a 6th grade class. . . . his explaination [sic] probably would be too difficult for a 6th grade class. I think he should of taught it like the students could teach it to their own classes.

The above quotation exposed a prospective teacher who is virtually certain that she will be unable to explain a concept to her students. However, her statements to a writing assignment assigned after the instructional sequence revealed that her fears and lack of understanding go deeper.

> If I walked into a classroom and was asked to teach fractions. I would become a nervous wreck. This would probably [sic] the worst math topic for me to teach. I think it would be hard because I am not sure that I understand everything about fractions myself. . . . When I do fractions I don't know why I have to get a common denominator when I add and subtract; why I don't have to get a common denominator when I multiply; why you have to flip the second fraction and multiply when you are supposed to be dividing; etc. I just simply know that those are the rules and that is the only way to get the correct answer. That is how I was taught. . . . I am sure, though, that I won't research this until I have to. If I were to just walk into a class one day and was asked to teach fractions, I probably still wouldn't know the reasons behind the rules. This would make it very hard to teach it. . . . Fractions are the only subject I can remember having difficulty with in my whole education career. I was in the fourth grade when I learned them, and I will never forget thinking this was too hard of a subject for us to deal with. Now that I look back at the time, I had a new teacher and I don't think that she was very comfortable with fractions. This made the topic even more difficult for us, and it didn't give us a very strong background on them. I feel comfortable working with fractions now, but I think the first few time [sic] of teaching it will be difficult.

In essence, Connie potentially resigned herself to repeat the mistakes of her elementary school teacher. In fact, her responses to the questions about explaining the *I&M* algorithm and then providing a real-life situation revealed a procedural conception and an unreasonable model. In particular, her response explaining the *I&M* algorithm was:

> $\dfrac{\frac{15}{2}}{\frac{1}{4}}$ Since this looks really complex manipulate it so it doesn't seem so bad. My suggestion is this:
>
> You have $\dfrac{\frac{15}{2}}{\frac{1}{4}}$ multiply this fraction by one in such a way that you can cancel out the bottom fraction. $\dfrac{\frac{15}{2}\cdot\frac{4}{1}}{\frac{1}{4}\cdot\frac{4}{1}}$. Does this cancel out the bottom fraction? You are left with $\frac{15}{2}\cdot\frac{4}{1}=\frac{60}{2}=30$. Doing this will help you to determine which fraction should be flipped and multiplied.

And in response to the second question, Connie wrote:

> Susie has $\frac{15}{2}$ tbsps of oil for her cookies. She put in one tbsp. of oil for every $\frac{1}{4}$ tbsp of flower [sic].
>
> How many tbsp. of flower [sic] does she have in all? $\frac{15}{2}=7.5\times 4=30$
> $\frac{1}{4}x=\frac{15}{2}\quad x=30$.

Although Connie used proportional reasoning to form her real-life situation modeling the division of fractions, she failed to recognize the flaws in her situation. In particular, her situation modeled:

$$\frac{1\,tbsp\ oil}{\frac{1}{4}\ tbsp\ \ flour}=\frac{\frac{15}{2}\ tbsp\,oil}{\frac{15}{2}\cdot\frac{1}{4}\ tbsp\ \ flour}$$

Even though she recognized what the appropriate result should be, her provided situation did not correlate with those computations. Consequently, the reading of the Borko article clearly did not have sufficient power to overcome such ingrained fear and lack of understanding toward fractions and possibly appeared to exasperate the situation.

Jamie's reaction to the Borko article was quite similar to Connie's. She assessed her repertoire of understandings which provide the reasons for algorithms and concluded that she was lacking many of the reasons.

> I was reading the example of the student teacher teaching division of fractions and I realized that I do not know a lot of the background reasons for algorithms. As students, you are just expected to take the "formulated way" & apply it. My past teachers have not challenged me to think beyond that point. So, I now fear this example of teaching mathem [sic] happening to me. I don't want to get stuck & not know how to explain the "why?" of a process! I want to be able to answer questions like this or at least be able to humble myself enough to say, I don't know, why don't we find out together. Lately, as I have been thinking of teaching, some aspects have been scaring me. . . .

Jamie asserted that her teachers did not challenge her but she did show a desire to be able to address the students' questions. However, when Jamie addressed the student's question about why the *I&M* algorithm works, she supplied the only idealic response. It was evident in her response presented previously that Jamie provided a reasonable sequence of actions which a teacher could take to help a student make sense of the *I&M* procedure without providing evidence that she could supply the student with an explanation of the reasons why.

Now, when Jamie was asked to provide a real-life situation modeling the division of $\frac{15}{2}$ by $\frac{1}{4}$, she provided a situation which modeled the multiplication of $\frac{15}{2}$ by 4. In particular, she stated:

> You are on the decoration committee for the school dance, right? Well, it's your responsibility to divide the crepe paper to that it is evenly divided and covers the gym. If you have four walls that are all $\frac{15}{2}$ yards long and you need to find enough crepe paper for all four sides how many yards will you need?

This real-life situation, although computationally yielding the same result as $\frac{15}{2} \div \frac{1}{4}$, does not give direct meaning to the $\frac{1}{4}$. The $\frac{1}{4}$ can be connected with the one-fourth of the walls of the gym; however, Jamie did not clearly identify this connection in the real-life situation.

In fact, story problems like those required for a model of the division of fractions caused fear in Jamie. This fear can be seen in Jamie's response to a writing assignment asking her to discuss what topic or concept she would not want to teach unprepared:

> I would not want to teach story problems if I were not prepared for the lesson. . . . The reason I would not want to teach them is because I still am leery on dealing with them today. . . . When I see a story problem I tend to cringe or moan, for I know that the words within the problem always tend to confuse me. You think that if a problem explained a real life incident, that I would understand it better and would be able to compute it easier because it would be applicable. Yet, there is something about story problems that create anxiety and because of the anxiety I would not feel confident enough to teach the lesson unprepared. I would need the teacher's manual, answers, formulas, and examples to help me understand it first.

Both Connie and Jamie engaged in introspection and uncovered that they themselves did not know the underlying conceptual reasons why many mathematical algorithms work. Their responses also revealed that these two prospective teachers lacked confidence in their own abilities to the point that they both expressed fear of such a situation as well as demonstrated a focus on the external problem rather than internalizing a solution. The fusion of a decreased confidence and a focus on the inability of others to provide reasonable explanations acted as validation of their shortcomings. According to McLeod (1992), confidence correlates positively with achievement in mathematics and relates to patterns of classroom interaction between teachers and students. Even though disquieting articles can excite personal introspection and empower both the development of understanding and the application of the resolution phase's lessons, introspection that reveals gaps in knowledge and culminates in fear can leave the learner feeling helpless to fill in those gaps. As a result, instruction needs to be designed to both challenge learners' knowledge structures while ensuring that the turmoil does not result in divestment of the learners.

Conclusion

This study extends the information available on teachers' knowledge of fractions. Previously, studies painted gloomy pictures of precollege education and prospective teacher training with respect to understanding the division of fractions. However, this study offers some hope and evidence that the acquisition of a specialization in mathematics beyond the typical one or two mathematics courses of the general elementary education major significantly improves performance in comparison to the results reported by Ball (1990), Simon (1993) and Ma (1999). In particular, nearly half the mathematics specialists provided conceptually-based explanations and about 70\% supplied a reasonable situation modeling the division of fractions. It is likely that the additional training may have contributed to the prospective teachers' enhanced understandings of the links within mathematics. Even though these levels do not correspond with the ideal, they show a marked improvement over the U.S. contingents described by Ball, Simon and Ma.

This study also indicated that the reading of the Borko article set the stage for investigation and encouraged introspection. In some cases, the article elicited strong reactions from the prospective teachers and corresponded to better explanations and ways of presenting the concept of the division of fractions. For others, the introspection resulted in heightened degrees of fear which in turn may have contributed as a debilitating factor to their inability to discuss the reasons behind the *I&M* procedure or to provide a reasonable situation modeling the division of fractions. The potentiality of engendering debilitating fear brings to question the overall effectiveness of the described instructional intervention. The goal of invoking personal introspection in hopes of propelling the prospective teachers to higher levels of understanding which in turn would effect their ability to explain the division of fractions to their eventual students appears reasonable. However, the total instructional intervention and discussion needs to incorporate elements which alleviate the agitated level of fear brought to the foreground.

After creating emotional upheaval through the reading the Borko article, one must address the subject-matter and pedagogical content knowledge along with the emotional state of each student. Some educators have focused their attention on the necessary subject-matter and pedagogical content knowledge while failing to address these latter components of emotional state created. Such a focus can leave the prospective teachers with an introduction to the concepts and techniques for teaching them without addressing their desires or abilities to express them to their future students. As a result, additional attention needs to be spent on these components when designing instructional interventions which potentially could cause volatile emotional reactions.

Finally, this noticeable lack of treatment of emotional issues brings to rise several questions. 1) What additional instructional activities can be added to alleviate the potential emergence of fear and further illustrate the need for conceptual explanation? 2) Would the discussions have been more effective during a methods course rather than a content course, during observational field experiences, or during student teaching? 3) Would augmenting the reading assignment with an article descriptive of a teacher who successfully dealt with the division of fractions such as "How children think about division with fractions" by Warrington (1997) alleviate some of the debilitating fears? and 4) Are there other, more effective ways of encouraging prospective teachers to reevaluate their knowledge of both the subject matter and pedagogical techniques associated with the division of fractions? Answers to these questions will need to be examined in future studies focused on the interplay of cognitive and affective issues.

References

Azim, D.S. (1995). *Preservice elementary teachers' understanding of multiplication involving fractions*. Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education - PME-NA 17, Columbus, Ohio. (ERIC Reproduction Number ED 389 612)

Ball, D.L. (1989). Research on teaching mathematics: Making subject matter knowledge part of one equation. In J. Brophy (Ed.), *Advances in Research on Teaching: Vol. 2. Teacher's Subject Matter Knowledge and Classroom Instruction* (pp. 112-136). Greenwich, CT: JAI Press.

Ball, D.L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21*(2), 132-144.

Ball, D.L. (1993). Halves, pieces, and twoths: Constructing representational contexts in teaching fractions. In T.P. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157-196). Hillsdale, NJ: Erlbaum.

Ball, D.L. (1996). Connecting to mathematics as part of learning to teach. In D. Schifter (Ed.), *What's Happening in Math Class, Volume 2: Reconstructing Professional Identities* (pp. 36-45). New York: Teachers College Press.

Behr, M.J., Harel, G., Post, T. & Lesh, R. (1991). Rational number, ratio, and proportion. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 296-333). New York, NY: Macmillan.

Behr, M.J., Lesh, R., Post, T., & Silver, E.A. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematical concepts and processes* (pp. 91-126). New York, NY: Academic.

Behr, M.J., Wachsmuth, I., Post, T., & Lesh, R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for Research in Mathematics Education, 15*, 323-341.

Behr, M.J., Wachsmuth, I., & Post, T. (1985). Constructing a sum: A measure of children's understanding of fraction size. *Journal for Research in Mathematics Education, 16*, 120-131.

Borko, H., Eisenhart, M., Brown, C.A., Underhill, R.G., Jones, D., & Agard, P.C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*(3), 194-222.

Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C.P. & Loef, M. (1989). Using knowledge of childrens' mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-532.

Crump, C.A. (1995, April). *Motivating students: A teacher's challenge* Paper presented at the 6th Annual Sooner Communication Conference, Norman, OK. (ERIC Reproduction No. ED 378 840)

D'Ambrosio, B.S. & Campos, T.M.M. (1992). Pre-service teachers' representations of children's understanding of mathematical concepts: Conflicts and conflict resolution. *Educational Studies in Mathematics*, *23*, 213-230.

Even, R & Lappan, G. (1994). Constructing meaningful understanding of mathematical content. In D. Aichele & A. Coxford (Eds.), *Professional Development for Teachers of Mathematics: 1994 Yearbook* (pp. 128-143). Reston, VA: NCTM.

Fennema, E. & Franke, M.L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 147-164). New York, NY: Macmillan.

Hunting, R. (1983). Alan: A case study of knowledge of units and performance with fraction. *Journal for Research in Mathematics Education, 14*, 182-197.

Hunting, R. (1986). Rachel's schemes for constructing fraction knowledge. *Educational Studies in Mathematics*, *17*, 49-66.

Johnson, N.R. (1999). A descriptive study of number sense and related misconceptions about selected rational number concepts exhibited by prospective elementary teachers. *Dissertation Abstracts International*, *59*(11), 4088.

Katzman, P.A. (1997). Elementary inservice teachers' fraction content knowledge: An exploration of content knowledge in instruction. *Dissertation Abstracts International*, *57*(8), 3434.

Khoury, H.A. & Zazkis, R. (1994). On fractions and non-standard representations: Pre-service teachers' concepts. *Educational Studies in Mathematics*, *27*, 191-204.

Kieren, T. (1988). Personal knowledge of rational numbers: It's intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades* (pp. 162-181). Reston, VA: NCTM.

Lehrer, R. & Franke, M.L. (1992). Applying personal construct psychology to the study of teachers' knowledge of fractions. *Journal for Research in Mathematics Education*, *23*(3), 233-241.

Leinhardt, G. & Smith, D.A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, *77*(3), 247-271.

Lester, F.K. (1984). Teacher education: Preparing teachers to teach rational numbers. *Arithmetic Teacher, 31*(6), 54-56.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Hillsdale, NJ: Erlbaum.

Mack, N. (1990). Learning fractions with understanding: Building on informal knowledge. *Journal for Research in Mathematics Education*, *21*(1), 16-32.

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education*, *41*(3), 3-11.

McLeod, D.B. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). New York, NY: Macmillan.

Piel, J.A. & Green, M. (1994). De-mystifying division of fractions: The convergence of quantitative and referential meaning. *Focus on Learning Problems in Mathematics*, *16*(1), 44-50.

Schifter, D. (1997). *Learning mathematics for learning: Lessons in/from the domain of fractions*. Newton, MA: Education Development Center. (ERIC Reproduction No. ED 412 122)

Silver, E.A. & Cai, J. (1993, June). *Schemes for analyzing student responses to QUASAR's performance assessments: Blending cognitive and psychometric considerations*. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.

Simon, M.A. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education*, *24*(3), 233-254.

Thipkong, S. & Davis, E.J. (1991). Preservice elementary teachers' misconceptions in interpreting and applying decimals. *School Science and Mathematics*, *91*(3), 93-99.

Thompson, A. (1985). Teachers conceptions of mathematics and the teaching of problem-solving. In E.A. Silver (Ed.), *Teaching and Learning Mathematical Problem-Solving: Multiple Research Perspectives* (pp. 281-294). Hillsdale, NJ: Erlbaum.

Thompson, A., Phillipp, R., Thompson, P., & Boyd, B. (1994). Calculational and conceptual orientations in teaching mathematics. In D. Aichele & A. Coxford (Eds.), *Professional Development for Teachers of Mathematics: 1994 Yearbook* (pp. 79-92). Reston, VA: NCTM.

Tzur, R. & Timmerman, M. (1997). Why do we invert and multiply? Elementary teachers' struggle to conceptualize division of fractions. In J.A. Dossey, J.O. Swafford, M. Parmantie & A.E. Dossey (Eds.) *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 553-559). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Warrington, M.A. (1997). How Children Think about Division with Fractions. *Mathematics Teaching in the Middle School*, *2*(6), 390-94.