ON THE DEVELOPMENT OF "BOOK SMARTS" IN MATHEMATICS: PROSPECTIVE ELEMENTARY TEACHERS' EXPERIENCES WITH INNOVATIVE CURRICULUM MATERIALS

Gwendolyn M. Lloyd Department of Mathematics Virginia Polytechnic Institute and State University Blacksburg VA 24061-0123 <u>lloyd@math.vt.edu</u> Jeffrey A. Frykholm School of Education University of Colorado Boulder CO 80309-249 jeff.frykholm@colorado.edu

Abstract

This paper explores the notion of challenging and re-developing preservice elementary teachers' conceptions of mathematics by engaging them in the use of reform-oriented K-12 curriculum materials during teacher education coursework. Examples of teachers' reflections on course experiences illustrate how innovative curriculum materials can challenge teachers to learn mathematics in reform-oriented ways. As they work in cooperative groups, discuss multiple solution strategies, and approach subject matter through relevant problem situations, teachers are offered invaluable first-hand experiences doing mathematics while bearing in mind questions about teaching.

Introduction

Over the past decade, mathematics education reforms have invited teachers to engage students in active and cooperative exploration and discussion, using a variety of representations and technological tools to solve rich problems and reason mathematically (Mathematical Sciences Education Board [MSEB] & National Research Council [NRC], 1989, 1990; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000). In contrast to traditional classroom activities emphasizing repetition and memorization of skills and rules, reform-oriented views of mathematical activity aim to develop students' conceptions of mathematics as a vibrant, useful subject composed of links among related concepts and procedures, both within mathematics and to the real world (Skemp, 1987; Steen, 1990).

These specific reform recommendations have resulted in, among other things, a flourish of curriculum development activity across grade levels (e.g., TERC Investigations, Connected Mathematics Project, Core-Plus Mathematics Project, Mathematics in Context). The reform-oriented materials of these programs differ from traditional mathematics textbooks in at least three important ways. First, reform-oriented curriculum materials explicitly incorporate reform ideas about mathematics and pedagogy by emphasizing student explorations of real-world mathematical situations and discussions of problem-solving activities. Second, the materials are formatted to support these mathematical and pedagogical differences. Traditional texts are typically divided into chapters outlining self-contained daily lessons for the teacher to present (composed primarily of definitions and examples of the lesson's content) followed by practice exercises for the student. In contrast, reform-oriented curriculum materials are typically published in unit booklets (offering greater flexibility in scope and sequence) that pose large-scale problems, centered on particular mathematical themes and content areas, for students to investigate. A third substantive difference is that reform-oriented materials generally offer more extensive information for teachers. In addition to suggesting acceptable solutions to problems, most of these new materials offer details

about different representations of content, historical information about mathematical and pedagogical ideas, examples of what students might do with or think about particular activities and content, potentially fruitful questions for eliciting discussion, and so on.

This third distinction is important because it highlights the notion that it is teachers, not texts alone, who determine how the innovations envisioned by reformers and curriculum designers become implemented in classrooms. Research supplies consistent evidence that teachers' conceptions strongly impact instructional practice (Brophy, 1991; Fennema & Franke, 1992; Thompson, 1992). Accordingly, teachers' interpretations and implementations of reform recommendations and new curriculum materials are influenced by their conceptions of mathematics teaching and learning (Cohen, 1990; Grant, Peterson, & Shojgreen-Downer, 1996; Lambdin & Preston, 1995; Lloyd, 1999; Lloyd & Wilson, 1998; S. M. Wilson, 1990). Implementation of innovative curricula may also encourage teachers to change their conceptions on the basis of new types of classroom experiences with students and content (Ball & Cohen, 1996).

These issues raise important concerns for those involved with the preparation of future teachers. Many prospective teachers possess weak knowledge and narrow views of mathematics and mathematics pedagogy that include conceptions of mathematics as a closed set of procedures, teaching as telling, and learning as the accumulation of information (Ball, 1990, 1991; Brown, Cooney, & Jones, 1990; Even, 1993; Frykholm, 1996; Thompson, 1992; Wilson, 1994). Such conceptions, many of which are bolstered by years of experience as students in traditional classrooms (Lortie, 1975; Zeichner & Gore, 1990), deeply impact the learning-to-teach process (Eisenhart et al., 1993). If reform themes are to be enacted in the mathematics classrooms of future teachers, conceptions of mathematics and teaching need to be challenged and developed in ways that will support meaningful and lasting change.

In light of the differences in the instructional methods prospective teachers will be expected to use in schools and those they likely experienced as students of mathematics, teacher education programs are faced with the task of creating opportunities for prospective teachers to critically consider important mathematical and pedagogical ideas so that more flexible conceptions may develop. As the MSEB and NRC (1989) state:

Teachers themselves need experiences in doing mathematics--in exploring, guessing, testing, estimating, arguing, and proving ... they should learn mathematics in a manner that encourages active engagement with mathematical ideas. (p. 65)

As preservice teachers begin to revisit mathematical content from new perspectives, they may begin to translate the knowledge developed as students of mathematics into pedagogical content knowledge (Shulman, 1987). Facilitating this complicated type of connection, and the resulting development of new conceptions, necessitates extensive reflection and analysis on the part of the prospective teacher.

This paper explores the idea of challenging and re-developing preservice teachers' conceptions of mathematics by engaging them in the use of reform-oriented curriculum materials. Use of curriculum materials in university courses may offer fruitful contexts for preservice teachers to reflect on and analyze very specific mathematical concepts and pedagogical practices. Recognizing that current reforms expect these teachers to teach unfamiliar mathematics using pedagogical methods that they have not experienced personally, we focus on the opportunities afforded by the curriculum materials to engage preservice teachers as *both* learners and teachers of mathematics.

Context for our Discussion

Our project took place between January and May of 1998 in a one semester mathematics course, titled *Geometry for Teachers*, of which the authors were the instructors of two sections. This course is the second of two mathematics courses required of prospective elementary school teachers. Although the primary course goal is to strengthen teachers' mathematical conceptions, the course sequence is also an early opportunity to engage students in thinking about pedagogical issues. Of the approximately 50 students enrolled in the *Geometry* course, most of these students were sophomores (10 juniors), female (3 males), and European-American (1 African-American). 35 of the 50 students took the first course (*Number for Teachers*) during the previous semester. These students, should they maintain this course of study, will take an elementary mathematics methods course and complete their student-teaching field placements during their senior years. In this report, all student names are pseudonyms.

The examples presented in this paper are taken from students' written comments on various course assignments. Students wrote three papers in which they analyzed their experiences as former elementary students, reflected on their studies in the *Geometry* course, and presented visions of themselves as future teachers. Students had opportunities to learn or revisit geometric concepts and mathematical processes, as well as pedagogical issues and practices, as they engaged with three types of materials: (1) student editions of curriculum materials, (2) teachers' guides of curriculum materials, and (3) a reform document for elementary mathematics curriculum.

The bulk of class meeting time in the *Geometry* course involved students working in small groups on activities suggested in reform-oriented middle school materials developed by the Mathematics in Context [MIC] program and the Connected Mathematics Project [CMP]. Interested readers can find examples of activities and general curricular themes at the following internet sites: <u>http://www.ebmic.com</u> and <u>http://www.math.msu.edu/cmp</u>.

Upon students' completion of these activities, usually during the subsequent class period, they shared their work with classmates in the form of short problem presentations. During three weeks of the course, groups of students used teachers' guides from the CMP materials to plan and teach lessons to their classmates. Discussions about pedagogy were enhanced by students' examination of *Geometry and Spatial Sense* (1993), an NCTM reform document. For example, following the students' work on the CMP *Ruins of Montarek* unit about spatial visualization, they were asked to relate their learning experiences to one of the activities described in the NCTM book.

As we read and reflected on students' work in the Geometry course, development of major themes was aided by the use of taxonomic and thematic analytic strategies (Spradley, 1979). Thematic analysis involved identifying and organizing information about opportunities for students to think about the nature of mathematics (geometry in particular), make connections between their own learning and that of their future elementary students, and develop visions of their future classroom practices. These main themes in the students' perceptions and experiences are discussed in the following sections.

Thinking about Mathematics

At the beginning of the course, many of the prospective teachers expressed apprehension about enrolling in a mathematics course devoted to the study of geometry. They described their previous experiences as learners of mathematics, and geometry in particular, in negative terms. "When I realized that I would be taking geometry in college," said Anne, "I immediately became scared." Despite their concerns, nearly all students suggested that they were looking forward to experiences that would "fill the gaps" in their knowledge of geometry. As Emily summarized, "I would like to relearn the stuff that I have forgotten from past years and better understand the stuff I already know."

As the preservice teachers began their work with the student editions of the curriculum materials, the most common remark was that the activities posed significant mathematical difficulties to them-- even as college students. For example, Holly said, "It is hard to believe that this is curriculum developed for a middle school age group, as some of the activities are difficult *for me* to understand." This sentiment was communicated by almost every student in the class, often with a tone of surprise: "This middle school work is harder than it seemed like it was going to be."

The types of mathematical activities found in the CMP and MIC units were novel to all of the students. They communicated being unfamiliar with "group work," "hand-on activities," "open questions," and "experiments in math." Sometimes the differences between these activities and their past experiences created frustration for students. Leah confessed,

The lack of structure confuses me. I like that we use manipulatives. I've never done that in math class before. However, I must admit I found junior high math easier to understand the first time around.

Although Leah and some of her classmates began to wish they were not required to revisit middle school mathematics and develop new understandings, other students seemed to thrive on the novelty of the reform-oriented activities and their underlying philosophies. Stella articulated her view of the mathematical novelties:

I am learning about how to look for reasons and explanations as opposed to simply believing "the rules" that some really ancient dead guy came up with. I prefer being able to use my own

mind in solving problems. This class seems to use more common sense instead of book smarts. It is noteworthy that Stella's comment refers to "book smarts": in contrast to other experiences where the textbook is a primary source of mathematical authority, her work with these materials may have allowed her to recognize herself as a sense-making authority for the first time. Understanding mathematics demands much more than reproducing rules found in textbooks, and as this student indicated, curriculum materials can provide the basis for conceptual exploration and knowledge.

The students were also challenged to think about and learn mathematics as they prepared lessons and taught their classmates. Teaching the class helped them to understand the mathematical ideas more deeply. For example, Vicky contrasted preparing to teach with working on problems in the student textbook:

Even though these exercises might seem easy to write down on paper, they become much more difficult to say them aloud and teach the concepts. I had to make sure I knew the material backwards and forwards. The Teachers' Notes help if we miss something in the book or are not quite sure how to explain it.

Vicky was not the only student who felt that a teachers' edition was helpful for enhancing or supporting her mathematics knowledge-- almost every student identified this role of the text. As Anne explained, the teachers' edition helped her to "understand things more clearly so [she would] know what [she's] teaching." The importance of understanding for teaching became extremely clear to students as they taught the class. Michelle expressed the following: "You must be extremely prepared. Even as I spoke and taught, I wished I had thought through some other

points." Because the open-ended problems in the materials challenged their own mathematical understandings, the teachers were beginning to recognize that, among other things, teaching demands extensive subject matter knowledge.

Thinking about Learning Mathematics

Students' interactions with the curriculum materials appeared to help them to recognize explicit connections between their understandings of specific geometry content, and their ability to teach it effectively in the classroom. As Maxine reported,

Geometry is not an easy subject and sometimes can confuse students terribly. I want to have enough knowledge about geometry in order to help sort out the difficulties that students will have....I think these geometry materials will teach me a lot.

Similarly, Emily suggested

I would like to relearn those [geometry principles] so that they stick this time. From the curriculum materials we have I think we will attain a better grasp of geometry because we are learning it the way the students are expected to learn it. Teachers often think these little workbook exercises are easy.... There is actually a whole lot more to it.

This student recognized one of the underlying themes and goals of the course -- that beginning teachers need to experience geometry content and pedagogy as learners through activities that they might facilitate one day with their own students.

This theme was prominent during students' work with the materials. Some of the students who reacted most positively to the middle school work, CMP's *Ruins of Montarek* in particular, were those who struggled the most with it. These students recognized the future benefits of their struggles. For example, Cathy suggested

Working in this unit showed me my own weaknesses in visual perception and paper-pencil perception. I had a lot trouble translating 3-dimensional ideas onto paper. This will be helpful to me later on when I will have students with areas of weakness which differ from mine

to me later on when I will have students with areas of weakness which differ from mine. As this comment suggests, during their work on the unit, students were required to draw building plans to match a given building – thus creating two-dimensional representations of threedimensional objects. For instance, one of the introductory problems states, "Make a stack using three cubes. Hold the stack in the air and turn it, observing it from many views. Your challenge is to find every way this stack of three cubes can be pictured on isometric dot paper." Cathy's insightful connection between her own learning and the future learning of her students suggests that she had begun to transform some of her mathematical knowledge into pedagogical knowledge. As the above excerpt exemplifies, one of their envisioned future students. They were able to better appreciate the complexity of mathematical concepts and processes: "We have been working through math concepts and problems which seem difficult but are actually for even younger children."

Although the preservice teachers who encountered difficulties and persevered through the activities were most likely to articulate explicit connections to teaching children, a few other teachers (who struggled less) also connected their experiences to children's learning. These teachers seemed to have developed an appreciation for the value of certain mathematical topics in a child's development. For example, describing what she learned from CMP's *Ruins of Montarek* unit, Michelle stated

As an adult, I can easily visualize these towers. I have been to cities and have worked a lot with shapes. As a child, they are still discovering the world of shapes and often haven't seen a city. Working with three dimensional objects and visualizing them on paper is very different. Many children may not be able to visualize these things but will learn in time and from experience.

This type of reflection indicates that working with the curriculum materials seems to encourage preservice teachers to think about the children who they will be teaching and the types of mathematics that are most relevant to those children's lives.

Thinking about *Teaching* Mathematics

There was also nearly uniform agreement among the students that engagement with the reform curriculum materials would provide valuable insights and concrete suggestions for classroom activities and lesson ideas that they could later apply in elementary classrooms. Central to their insights was the recognition that these innovative materials and activities necessarily would lead to different teaching strategies than they had seen previously. Emily, for example, noted that, "After looking over the materials we will use in the class I found that the ways in which geometry is being taught are much different from when I learned." Most students embraced these differences, and looked forward to the teaching process. At the beginning of the course, Anne made the following statement in this regard: "The books and manipulatives that we will learn to use in this class will give me new ideas about teaching and tactics that I will one day be able to use in the classroom."

After discussing and working cooperatively on problems from the student editions of the CMP and MIC materials, the students began to communicate even more strongly their awareness of important lessons about teaching. For example, Vicky said "The use of manipulatives helps to create a more student-centered activity. Real life applications show students how important mathematical concepts are." One student was "thrilled" with the explicit focus in the materials on "comparing answers to classmates'." Students believed this discussion was essential for creating a supportive, student-centered environment in which multiple perspectives could be explored. This notion is consistent with Tami's claim that she learned

to be aware that children probably see these things differently and also approach solving them differently than I do. I need to be open-minded and see everyone's point of view and method of solving the given problem.

The real-world contexts of the activities were also recognized and appreciated by almost every student. Meg explained

I felt the problems allowed the children to see how they could relate their work to real life which oftentimes makes children realize the importance of the concept. I know a lot of times I never understood why we had to learn something. I would always say "When will I ever use this again?" *Ruins of Montarek* allows the concept to be easily understood and grasped by a student, while at the same time giving them an explanation of a future use in life.

Maxine suggested, more specifically, that the fictitious female archeologist Emily Hawkins, whose explorations tie together the activities of CMP's *Ruins of Montarek*, played a critical teaching role:

The book brings it closer to a personal relationship with the "author" rather than an abstract view on the facts. Having the little tidbits about the different places and the different buildings

of ancient times was a nice touch to say, "Hey it is not just mathematicians who use this stuff that you are learning."

These quotes suggest that students were able to extract rich pedagogical notions and examples from their work with the student editions of the middle school mathematics curriculum materials.

Using teachers' editions also provided relevant contexts for thinking about teaching. Kelly described how using the teachers' edition made her "feel like a *real* teacher.... It helped show other angles to take when explaining certain methods." The teachers' edition was viewed as a prized possession by many- Missy commented excitedly that "this was the first time [she] was able to see one or use one." After teaching, she elaborated:

I thought that using the Teachers' Guide was helpful. Although I didn't follow it exactly, I used the best information and integrated my own thoughts and activities to allow the students a good lesson which touched on all important details without repeating stuff which makes the class bored.

Students also learned various lessons, positive and negative, about teaching from being taught by classmates. For instance, Tracie pointed out that she learned not to "automatically think everyone is going to know what you are teaching." In other words, connections that are clear to the teacher may not be so clear to students. This important observation suggests that participating in peer-taught lessons may offer viable opportunities for students to see themselves in teaching roles and figure out "what [they] might have done" in similar situations.

Because students' reading of the NCTM document came immediately after working on the CMP *Ruins of Montarek* unit, the bulk of students' comments integrated ideas from both books. Students' comments indicate that reading this *Standards* document helped many of them to make better sense of what they had done during the *Ruins* unit. After reading about specific activities for young children in the NCTM book, many students *then* connected their experiences with *Ruins* to children's learning. For example, Vicky wrote,

Exploring *Ruins of Montarek* as an adult is more difficult than one might think. It was challenging and probably just as challenging as "Making Twins" [an activity described in NCTM book] would be for a kindergartner. The objects made in "Making Twins" might seem simple to an adult, however, *Ruins of Montarek* was basically the same to an adult as "Making Twins" is to the child. They are both difficult, just to different ages. They are basically the same concepts, just at a different level.

This example suggests how important it was for Vicky to have experienced a challenging activity herself. When she read in the NCTM book about an elementary school activity involving similar concepts, she was prepared to make a connection to her own experience.

Many other students indicated that the NCTM book clarified for them the value of the *Ruins* of *Montarek* activities. As Emily wrote, "After reading the NCTM book, I was able to realize the significance of the assignments." Whereas *Ruins* helped students in understanding or learning mathematics, it did not help them, as much as they would have liked, to translate that knowledge into pedagogical content knowledge: "We had to *infer* what concepts were supposed to be stressed to a middle school student." Students' comments seem to indicate that they *expected* to learn pedagogical skills and ideas from the curriculum units, as suggested by Sally:

The NCTM book explained to me why I was doing what I was doing - the concepts I would be teaching. The *Ruins* book was not very straightforward as to what was being accomplished by each activity. This interest in identifying explicit teaching goals for the curriculum units was echoed by many students who lamented the fact that the workbooks "simply present problems."

Most students' comments suggest that reading the NCTM book allowed them to make sense of and extend the experiences gained from working solely with the curriculum materials. Whereas the NCTM book did not seem to offer many of these students opportunities to learn new mathematics, it did allow them to recognize the relevance of their learning with the curriculum materials and to relate that learning to their future elementary school students.

Issues and Further Questions

The course in which these students enrolled, *Geometry for Teachers*, is intended to prepare them to deal successfully with mathematical subject matter as teachers of elementary students. Our rationale for using middle school mathematics curriculum materials was to offer preservice teachers opportunities to learn reform-oriented mathematics in reform-oriented ways. It is critically important that we document students' experiences with mathematics in classes where this mysterious type of "reformed learning" occurs.

One of the most important implications of our project is that middle school curriculum materials allowed our preservice elementary teachers to extend their understandings of and beliefs about mathematics. Some teacher educators and mathematicians may argue that middle school texts are not "advanced" enough for use in a college-level mathematics course. Although they were using middle school materials, the preservice teachers in our course were *not* middle school students. They brought to their work with the materials a complex perspective based on their past experiences as students, their beliefs about mathematics and mathematical activity, and their emerging projections of themselves as future classroom teachers.

Many of these preservice teachers did not recall any explicit learning of geometry when they were in the elementary grades themselves. Their primary experiences with geometry occurred in high school classes and in non-school settings. However, they seemed genuinely motivated to deeply understand geometry so that they would be able to develop ways of engaging students in its exploration. The middle school materials enabled the students to engage with challenging mathematical material, but also feel "safe" while they were doing so. Their work in the course indicates that the problems and activities outlined in the materials were substantial and open-ended enough to offer all of the students challenges to their mathematical conceptions. The educative value of middle school materials is "advanced" to a more sophisticated and powerful level when preservice teachers explore them as adults possessing unique professional needs and perspectives.

While appreciating the powerful role that curriculum materials can play in extending preservice teachers' mathematical conceptions, we must also bear in mind the following question: How does preservice teachers' mathematical learning play out when they are in the "real world" of the elementary school classroom? This issue is important because, after all, the students enrolled in our mathematics courses are future *teachers* who will need more than extensive subject matter backgrounds. These students' appreciation for the use of "real" curriculum materials is a reminder that future teachers also need and want to develop their pedagogical conceptions and skills.

It was not uncommon for the students in our course to attempt to develop a teaching perspective by reflecting on the mathematical activities in which they were participating. In fact, one of the most fascinating aspects of students' reactions was the way they made connections between their own learning and their ideas about their future students' learning. For most students, this connection was expressed as being both "student" and "teacher" in the course activities, or as one student expressed "being on both sides of the spectrum at the same time."

This theme suggests the importance of reflection in the development of pedagogical conceptions: by reflecting on their own learning with the curriculum materials, the teachers were able to begin to consider appropriate ways of teaching to bring about meaningful mathematical understandings in their own students. An implication for mathematics teacher educators is that integrating mathematics and pedagogy, as many of the activities in our course did, may encourage teachers to make critical connections between different aspects of their teaching and learning experiences. Further analysis of the nature of such connections, and the contexts in which they occur, will bring us a richer understanding of what it means to learn in reform-oriented ways and how such learning might benefit teachers as they interact with children in elementary classrooms.

Perhaps the most important reason to encourage preservice teachers to forge such connections is that doing so provides a first step toward the development of rich pedagogical conceptions. The curriculum materials offered detailed images of what reformed mathematics teaching can look like. Images of classroom discourse, for example, are highly valued by teachers at early stages in their learning about reformed instruction (Davenport and Sassi, 1995). Preservice teachers, whose process of learning to teach in reformed ways is compounded by the pressures of learning to teach for the first time, may greatly benefit from explicit attention to the development of models of practice during teacher education experiences.

We are not suggesting that access to visions or models of practice will result in swift changes in teachers' conceptions of appropriate classroom activity. In fact, most of the preservice teachers in our course communicated (in both their comments and their practices leading the class), that teaching centers on *explaining correct solutions and answers*, despite the fact that the MIC and CMP curriculum materials emphasize cooperation among students and discussion of multiple perspectives and solutions. Although this is a pedagogical conception that we would eventually like to see changed or extended, there is at least strong evidence to suggest that our students had *begun* to think about teaching. It is perhaps not so surprising that students' initial thoughts about and efforts at teaching focus almost exclusively on teacher explanation- for this has been the most prominent form of instruction in their experiences as students. Our goal as teacher educators who work with these preservice teachers throughout their programs is to help them extend their views about teaching to include additional pedagogical methods and insights. However, the strength of these conceptions necessitates that teacher educators begin to engage students in thinking about mathematics *for teaching* early in their programs.

Given the extremely brief time period in which teacher educators interact directly with preservice teachers, it is important to consider how to effectively prepare our students for ongoing learning in their careers. A motivating factor in our decision to use reform-oriented curriculum materials for this course was that we wanted to expose our students to the type of school mathematics materials that will likely continue to emerge in the coming years. Given the prominent role of textbooks in guiding practice in American classrooms (Tyson-Bernstein & Woodward, 1991), we also feel the need to help students develop ways of learning from other materials in the future. Beginning teachers need guidance in learning to make reasoned pedagogical decisions about how to incorporate the recommendations of curriculum materials into their own instruction. Such learning must extend beyond making choices among particular practices or activities to the broader development of personal theories of teaching and learning.

Similarly, teacher educators can use curriculum materials as a context for engaging future teachers in learning new mathematics, and for helping them to recognize how such learning occurred. Better awareness of the learning process may support these teachers in using textbooks and other resource materials to teach themselves about new mathematical and pedagogical concepts in the future. We must help our future teachers learn to employ curriculum materials with the type of "book smarts" that will enable them to not only engage their students in meaningful classroom mathematics, but also appreciate the potential of curricula to support their own continued learning.

References

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, *21*, 132-144.

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter part of the equation. In J. Brophy (Ed.), *Advances in research on teaching*. Greenwich, CT: JAI.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is - or might be - the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher*, 25(9), 6-8, 14.

Brophy, J. (Ed.). (1991). Advances in research on teaching: Teachers' knowledge of subject matter as it relates to their teaching practice (Vol. 2). Greenwich, CT: JAI.

Brown, S., Cooney, T., & Jones, D. (1990). Mathematics teacher education. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 639-656). New York: Macmillan.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis*, *12*, 327-345.

Davenport, L. R., & Sassi, A. (1995). Transforming mathematics teaching in Grades K-8: How narrative structures in resource materials help support teacher change. In B. S. Nelson (Ed.), *Inquiry and the development of teaching: Issues in the transformation of mathematics teaching* (pp. 37-46). Newton, MA: Education Development Center.

Eisenhart, M., Borko, H., Underhill, R, Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education*, *24*, 8-40.

Even, R. D. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*, 94-116.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Frykholm, J. A. (1996). Preservice teachers in mathematics: Struggling with the Standards. *Teaching and Teacher Education*, *12*(4), 665-681.

Grant, S. G., Peterson, P. L., & Shojgreen-Downer, A. (1996). Learning to teach mathematics in the context of systemic reform. *American Educational Research Journal*, *33*, 509-541.

Lambdin, D. V., & Preston, R. V. (1995). Caricatures in innovation: Teacher adaptation to an investigation-oriented middle school mathematics curriculum. *Journal of Teacher Education*, *46*, 130-140.

Lloyd, G. M. (1999). Two teachers' conceptions of a reform curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education*, 2, 227-252.

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education*, *29*, 248-274.

Lortie, D. (1975). Schoolteacher. Chicago: University of Chicago Press.

Mathematical Sciences Education Board and National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington DC: National Academy Press.

Mathematical Sciences Education Board and National Research Council. (1990). *Reshaping school mathematics: A framework and philosophy for curriculum*. Washington DC: National Academy Press.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1993). *Geometry and spatial sense: Addenda series, grades K-6*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-22.

Skemp, R. R. (1987). The psychology of learning mathematics. Hillsdale, NJ: Lawrence Erlbaum Associates.

Spradley, J. P. (1979). *The ethnographic interview*. New York: Holt, Rinehart, & Winston. Steen, L. A. (Ed.) (1990). *On the shoulders of giants: New approaches to numeracy*.

Washington, DC: National Academy Press.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan Publishing Company.

Tyson-Bernstein, H., & Woodward, A. (1991). Nineteenth century policies for twenty-first century practice: The textbook reform dilemma. In P. G. Altbach, G. P. Kelly, H. G. Petrie, & L. Weis (Eds.), *Textbooks in American society* (pp. 91-104). Albany: SUNY Press.

Wilson, M. R. (1994). One preservice secondary teacher's understanding of function: The impact of a course integrating mathematical content and pedagogy. *Journal for Research in Mathematics Education*, 25, 346-370.

Wilson, S. M. (1990). A conflict of interests: The case of Mark Black. *Educational Evaluation and Policy Analysis*, *12*, 309-326.

Zeichner, K. M., & Gore, J. N. (1990). Teacher socialization. In W. R. Houston (Ed.), *Handbook of Research on Teacher Education* (pp. 329-348). New York: Macmillan.