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The Intersection of MET II Content Domains and Mathematical Knowledge for Teaching in Mathematics Content for Elementary Teachers Courses

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Abstract

Mathematics content for elementary teachers (MCfET) courses generally provide prospective elementary teachers (PTs) with opportunities to engage with mathematical content and consider it from a teacher's perspective, yet little is known about the intended curriculum of such courses. Findings from an analysis of MCfET course activities using the Mathematical Knowledge for Teaching framework indicated that activities developing the Mathematical Education of Teachers II recommended content domains of Geometry and Measurement & Data provided fewer opportunities for PTs to develop specialized content knowledge (SCK) than arithmetic- and algebraic-based activities. This led to recommendations of research on developing SCK in MCfET courses.

Introduction

To guide the preparation of future teachers of mathematics, mathematicians and mathematics teacher educators (MTEs) collaborated to create "Essential Ideas" for teacher preparation programs in the *Mathematical Education of Teachers II* (*MET II*; Conference Board of Mathematical Sciences, 2012). Specifically, *MET II* outlined mathematical content domains that prospective elementary teachers (PTs) should engage with during their teacher preparation programs. Here, we illuminate MTEs' attention to these content domains in teacher preparation programs, particularly in coursework focused on mathematical content with some attention to pedagogy.

Mathematical Education of Teachers II

The authors of *MET II* recommended PTs have a "careful study" of identified content domains and their connections to elementary mathematics from a teacher's perspective. The authors also incorporated the content and practices of the *Common Core State Standards of Mathematics* (*CCSSM*; National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010) in their mathematical recommendations for PTs in teacher education programs. In fact, the six proposed content domains for K–5 PTs in *MET II*—Counting & Cardinality, Operations & Algebraic Thinking, Number & Operations in Base Ten, Number & Operations with Fractions, Measurement & Data, and Geometry—align with the content domains of *CCSSM*. Counting & Cardinality is a kindergarten standard, and the first instance of Number & Operations with Fractions is in grade 3. The remaining domains are present throughout grades K–5. The recommendations also suggested integrating content and pedagogy when possible to provide PTs opportunities to "*do* mathematics" with strategic use of technology and "develop mathematical habits of mind." These proposed experiences also included the instructional use of traditional teaching tools (e.g., base-ten blocks, counters).

Here, we considered Mathematics Content for Elementary Teachers (MCfET) courses to be those that provide PTs with opportunities to engage with content through these experiences. Mathematics departments often house these courses which primarily focus on mathematics content and give some attention to pedagogy. We considered any MCfET course instructor to be an MTE, and, therefore, referred to all instructors as such throughout this article.

Mathematical Knowledge for Teaching

One way to conceptualize the mathematical content with which PTs engage is through Mathematical Knowledge for Teaching (MKT; Ball, Thames, & Phelps, 2008). Teachers of mathematics must know about more than mathematics, and this framework categorized the different types of knowledge needed for teaching mathematics. MKT is a practice-based framework developed from studies of what teachers do as they teach mathematics and what they need to know in order to teach mathematics successfully. According to Ball and colleagues (2008), MKT consists of subject matter knowledge and pedagogical content knowledge. Subject matter knowledge contains three domains: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). CCK is “the mathematical knowledge and skill used in settings other than teaching” (p. 399; e.g., carpentry, finance). SCK is knowledge unique to teachers of mathematics in which teachers unpack mathematics in ways atypical to other professions (e.g., sizing up mathematical approaches or errors). HCK is an understanding of connections among mathematical topics across time. One example provided by Ball and colleagues which highlighted these knowledge domains involved inspection of the third-grade multi-digit subtraction problem in Figure 1:

307	307	307
−168	−168	−168
−1	139	2
−60		30
200		107
139		139

Figure 1

Multi-digit subtraction approaches (Ball et al., 2008, p. 397)

In this example, CCK is knowing how to compute and recognize the correct difference. SCK involves assessing the strategies employed in terms of correctness and approach, or being able to recognize what the −1, −60, and 200 represent in the first solution. Knowing the progression of one- and two-digit addition and subtraction in earlier grades and the extension of this concept in later grades would be an example of HCK. In this example, we see CCK embedded in SCK because the knowledge of finding the solution comes before analyzing computational approaches. In addition to CCK, MCfET courses often provide opportunities for PTs to develop SCK.

In the MKT framework, pedagogical content knowledge “bridges content knowledge and the practice of teaching” (Ball et al., 2008, p. 389), particularly focusing on knowledge of content and students, teaching, and curriculum. Although there is likely some overlap, PTs typically engage with pedagogical content knowledge in mathematics methods courses and subject matter knowledge in MCfET courses. In this article, we focus on MCfET courses and, thus, subject matter knowledge. (See Max & Newton, 2015, for connections between the MKT framework and a mathematician's work in teacher preparation.)

Purpose of this Report

While *MET II* was clear on the content domains with which PTs should engage during teacher preparation, investigation of the presence of these content domains in MCfET courses has yet to be done. This report was an effort to help MTEs understand the ways in which MCfET instructors integrate content into their courses. Therefore, we analyzed content activity descriptions provided by instructors of MCfET courses by *MET II* content and MKT subject matter domains. We sought to give perspective on the ways MTEs intend to support PTs as it relates to content. We have particular interest in SCK because of its unique application to teaching mathematics. MTEs benefit from knowing how instructors address MKT domains through content because, as recommended in *MET II*, attention to both mathematical content and pedagogy is a necessary part of teacher preparation programs.

We drew from a larger study on the mathematical preparation of elementary teachers in which a survey of MTEs asked respondents to provide information about teacher education program requirements, MCfET course instructor backgrounds, and details about MCfET courses that the respondents had experience teaching. We focus our findings described here on responses to two of the 40 survey questions that requested information about the attention to *MET II* content domains (see Figure 2). Respondents indicated the content domains in Q4.16 for 81 courses and we analyzed the 68 content activities they described or uploaded for Q4.17 (see Figure 2 for questions).

Analysis was completed separately for content domain and MKT domain. First, the two authors each individually coded every content activity according to the *MET II* content domains, including multiple domains where appropriate. Next, the 68 activities were assigned to the MKT subject matter domains of CCK, SCK, and HCK, with one domain identified as the core domain for each activity. We utilized this coding scheme because the authors recognized CCK in all activity descriptions. Therefore, activities without SCK were generally coded as CCK. The goal in this round of coding was to distinguish between the activities that primarily focused on developing CCK, SCK, and HCK.

Q4.16 The Mathematical Education of Teachers II (CBMS, 2012) recommends the following areas be addressed in mathematics education programs. Does this course provide opportunities to learn in the following areas? Please check all that apply:

- ❑ Counting and Cardinality (1)
- ❑ Operations and Algebraic Thinking (2)
- ❑ Number and Operations in Base Ten (3)
- ❑ Number and Operations - Fractions (4)
- ❑ Measurement and Data (5)
- ❑ Geometry (6)

Q4.17 For the area selected above that gets the most attention in the course, please upload or describe one activity, assignment, or reading used in the context of this content.

- ❍ Describe the activity: (1) ____________________
- ❍ Upload file (2)

Figure 2

Content domain questions presented to instructors of MCfET course

Who were the respondents?

Thirty-four instructors provided 68 content activities or descriptions that highlighted the most-addressed *MET II* content domain. They were from 33 unique institutions across 19 states. The instructors had an average of 10 years of experience at the postsecondary level. Twenty-nine (85%) of the instructors had an affiliation with mathematics departments, seven (21%) of whom had an affiliation with mathematics and education departments, and five (15%) of whom were solely affiliated with education departments. The 30 respondents who indicated requirements of MCfET courses reported an average of 2.2 MCfET courses worth 6.8 semester credit hours. These courses were offered in the mathematics department 88% of the time, with the remaining courses offered most often in education.

Which content domains were present in MCfET courses and content activities?

We report the respondents' attention to each of the *MET II* content domains in the MCfET course in Table 1 along with the results of our analysis of the content activities through the lens of *MET II* content domains. We were unable to classify three content activities because of brevity or difficulty in discerning the objective of the task, bringing our count to 65.

Table 1
Attention to *MET II* Content Domains in MCfET Courses

MET II Content Domain	Present in the Course (n = 81)		Present in Content Activity (n = 65)	
	Freque	%	Freque	%
Counting & Cardinality	38	4	1	2
Operations & Algebraic Thinking	53	6	29	45
Number & Operations in Base	48	5	21	32
Number & Operations with	50	6	18	28
Measurement & Data	46	5	17	26
Geometry	42	5	21	32

Respondents indicated Operations & Algebraic Thinking as addressed in the highest number of MCfET courses, and it was the most frequently present in content activities. The spread of content in the activities is relatively consistent for the other content domains except for Counting & Cardinality; this exception may be because this domain is included for kindergarten only.

Which domains of Mathematical Knowledge for Teaching were present in the content activities?

We identified the MKT domain addressed by the activities and then inspected the intersection of content and MKT domains. Within the 65 activities coded, the primary MKT domain was SCK for 62% of the activities and CCK for 38% of the activities; none of the activities addressed HCK. Table 2 disaggregates the content domains by SCK and CCK. In the table, we also provide an example of content activities that primarily addressed SCK in each content domain. We chose to highlight SCK because of its unique application to teaching and as mentioned earlier, all activities embodied CCK in some form.

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Table 2
Intersection of *MET II* Content and MKT Domains in Activities

Content Domain	CCK	SCK	SCK Sample Activity
Counting & Cardinality (n = 1)	0	1	For patterns, we ask students to write number sentences of how they counted each of the figures building on their same method of counting. For example, if they are counting the number of blocks in each row, then continue making number sentences based on the number of blocks in each row. If they are counting a large growing square first, and then adding on a linear pattern somewhere else, they should describe that, make a number sentence based on that pattern, and then continue counting and making number sentences in that way.
Number & Operations in Base 10 (n = 20)	1	19	Understanding algorithms for operations and the connection to our base ten number system…comparing the standard algorithm to the partial products algorithm and area models for connecting to the distributive property.
Number & Operations with Fractions (n = 18)	3	15	Using context problems and models to make sense of fraction operations. For example, solving fraction context problems that suggest either area or number line model and examining both models, how they work, what they mean, how they represent the operation.
Measurement & Data (n = 17)	13	4	In groups, design and make 3 types of Longorian measuring instruments. Demonstrate your instrument to the class and explain how to use your instruments to measure items in the class[room].
Geometry (n = 21)	15	6	To PTs: A 6th grade student says that if the perimeters of two rectangles are the same, the areas of these figures must also be the same. How might you respond to this?

The examples provided offer a range of opportunities for PTs deepen their own understanding and make sense of mathematics, as evident in the three “Operations” strands where PTs are using models to represent and make sense of the mathematics. There were several differences in the MKT domains addressed in the activities. As seen in Table 2, the respondents provided activities that mapped overwhelmingly to SCK on the three algebraic- and arithmetic-focused domains of (1) Operations & Algebraic Thinking, (2) Number & Operations in Base Ten, and (3) Number & Operations with Fractions. Attending to SCK typically involved PTs describing multiple strategies (e.g., takeaway and difference in subtraction), multiple representations (e.g., number line jumps, alternative and standard algorithms), or other base systems (e.g., base two). The two content domains of (1) Geometry and (2) Measurement & Data generally involved CCK only. More than half of the CCK-mapped activities in these two domains involved classifying or finding area and perimeter of two-dimensional figures and volume of three-dimensional solids.

What does this mean moving forward?

The activities provided by these MTEs afforded PTs learning opportunities related to the *MET II* content domains and MKT domains and can help them as they prepare to be successful teachers of mathematics. Opportunities for PTs to develop SCK in all content areas supports their well-rounded mathematical preparation. We found that the majority of the content activities in the domains of Operations & Algebraic Thinking and Number & Operations in Base Ten and with Fractions addressed SCK. Therefore, we do not feel obliged to discuss those further. Because we see SCK as the domain that separates general mathematics and MCfET courses, we chose to focus on Geometry and Measurement & Data, where the content activities mapped more often to CCK than SCK.

MCfET course instructors consider necessary mathematical content for PTs. If they attend to the recommendations of *MET II* (CBMS, 2012), pedagogy is also considered to a lesser degree. We see activities that develop SCK as opportunities for instructors to attend to both content and pedagogy in MCfET courses. However, opportunities to develop SCK were available less often in the Geometry and Measurement & Data activities submitted. One potential reason for the CCK focus in Geometry and Measurement & Data could be a result of PTs’ struggles with geometric conceptual understanding, which Chamberlin and Candelaria (2014) investigated in their study with 10 PTs in a MCfET course focused on Geometry and Measurement. Their study suggested that group activities in MCfET courses with what I determined to have an SCK focus improved PTs’ understanding of area (e.g., comparing their results and approaches on an activity to another group’s results and approaches), volume, (e.g., comparing use of base-ten blocks with other nontraditional tools), and measurement (e.g., determining what part of the measurement learning progression an activity addresses). We assert that connecting the content of Geometry and Measurement & Data in ways that develop SCK may provide motivation for PTs to understand the content in various ways. Because PTs have likely been exposed to this content in their K–12 experiences, approaching this content through activities that develop SCK has potential to inform a perspective connected with a purpose for the PTs–the approaches of their future students. For example, MTEs could modify an area and perimeter activity like that of the multi-digit subtraction problem in Figure 1 by presenting PTs with multiple approaches that students (or their classmates) used to determine the area and perimeter of a given figure. PTs could then analyze the approaches, affording them opportunities to develop SCK in Geometry and Measurement & Data. These opportunities allow PTs to unpack mathematics in ways they have potentially not considered as K–12 students.

Conclusion

The MTEs who provided these activities are charged with the challenging task of preparing elementary teachers to teach mathematics. Giving sufficient attention to the variety of content domains is part of that challenge. Additionally, MTEs must also consider ways they can develop SCK within the content domains, because SCK is knowledge unique to teachers of mathematics

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and thus may not be a part of traditional mathematics coursework. Therefore, we encourage MTEs to consider ways in which their Geometry and Measurement & Data activities could address SCK. While CCK also needs developed, finding ways to develop both CCK and SCK in these content domains can strengthen the overall MKT of PTs. MTEs from both mathematics and education may need to work together to develop ways that SCK can be addressed in MCfET courses, especially in the areas of Geometry and Measurement & Data. This will support PTs—and, eventually, their students—as they prepare the next generation of mathematicians and MTEs.

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