The Effects of Different Undergraduate Mathematics Courses on the Content Knowledge and Attitude towards Mathematics of Preservice Elementary Teachers

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Abstract

Preservice elementary teachers have been shown to generally possess poor mathematical knowledge (e.g. Goulding, Rowland, & Barber, 2002) and also strong negative attitudes toward mathematics (e.g. MacNab & Payne, 2004). Recently, national organizations have proposed interventions to address these issues (Conference Board of the Mathematical Sciences, 2001). This paper analyzes the impact of a content course intervention. When compared to a control group, the experimental group had a significantly more positive attitude toward mathematics. When previous achievement was partially controlled for, the experimental group scored significantly higher than the control group on a measure of content knowledge.

Introduction

Content Knowledge of Teachers

Mounting evidence has revealed serious gaps in preservice and inservice elementary teachers' understanding of elementary school mathematics. For instance, Fuller (1996) found that experienced and novice teachers had only a procedural understanding of fractions, lacking a rich understanding of the concepts associated with fractions. Preservice elementary teachers in a study by Zazkis and Campbell (1996) focused almost exclusively on the procedural aspects when solving problems involving divisibility. Adams (1998) found that fewer than 30% of preservice teacher participants could describe how rationals, integers, naturals, real, and whole numbers were interrelated using diagrams or descriptions. In a study by Stacey, Helme, Steinle, Baturo, Irwin, and Bana (2001), 20% of the preservice elementary teachers did not have a good grasp of concepts related to decimals.

Ma (1999), as part of her seminal work on critical differences of teachers mathematical knowledge between U.S. and Chinese teachers, discovered that the Chinese teachers in her study had a more robust and less rigid understanding of mathematical topics when compared to the U.S. teachers. For instance, both groups of teachers were asked to come up with a situation that they might present to make the following problem meaningful to their students, $1\frac{3}{4} \div \frac{1}{2}$. Of the American teachers, less than half (43%) actually solved the problem correctly (3 ½ being the correct answer) compared to 100% of the Chinese teachers (Ma, p. 58). When creating a situation for the same problem, less than 10% of the American teachers created at least one suitable story problem, while 90% of their Chinese counterparts created at least one suitable story problem (Ma, pp.64-72). Stoddart, Connell, Stofflett, and Peck (1993) found that the preservice teachers in their study solved procedural questions involving rational numbers with accuracy between 37% and 98% depending on the problem. However, on problems involving similar content but modified to be more conceptual, the same teachers' accuracy ranged between 5% and 10% correct. For instance, when asked to multiply two decimal numbers or to compare two fractions, approximately 80% of these preservice teachers correctly solved the problem. However, when these preservice teachers were asked to arrange a list of decimals and fractions from smallest to largest, only 10% of these teachers responded correctly.

Intuition suggests and some research supports the theory that the mathematical content knowledge (MCK) of elementary teachers is related to their teaching ability and eventual student's achievement. For example, Goulding, Rowland, and Barber (2002) reported that among teachers in the United Kingdom, teaching performance has a significant correlational relationship with MCK. Rowan, Chiang, and Miller (1997) found that students whose teachers had better mathematical preparation and knowledge outperformed the students of other teachers. The effect size was quite small, in the range of .05 to .15 standard deviations. Upon analyzing interaction effects, these researchers discovered that for students of low ability the effect of the teacher mathematical preparation and knowledge was greater. Hill, Rowan, and Ball (2005) reported that teachers with better mathematical knowledge had students with better annual gains in mathematical knowledge.

Teachers' Attitudes toward Mathematics

Regarding attitudes toward mathematics, much of the recent research is also associated with research on beliefs, anxiety, and efficacy towards mathematics (Beswick, 2006; Beswick & Dole, 2001; McGinnis et al., 2002; Swars, 2004). Definitions of these terms are complex and often dependent on each other. Bandalos, Yates, & Thorndike-Christ (1995) define anxiety partially in terms of attitude. Efficacy has also been shown to be related to attitude toward mathematics (Randhawa, Beamer, & Lundberg, 1993). Furthermore, attitude toward mathematics has been shown to be negatively correlated with anxiety toward mathematics (Brady & Bowd, 2005).

Research in the attitude of preservice elementary teachers has generally found that they have negative attitudes towards mathematics. MacNab and Payne (2003) reported that among preservice elementary teachers in their first undergraduate year, 46% listed the word "worried" when discussing working on mathematical tasks. Kolstad and Hughes (1994) found that 34% of the K-4 teachers in their study had strong negative attitudes toward mathematics, a significantly higher percentage than other educators.

Similar to content knowledge, anxiety, attitudes, and beliefs towards mathematics have been associate with teaching ability. For example, "poor" beliefs have been associated with rote teaching (Richardson, 1996, Beswick & Dole, 2001). Teachers identified as having high efficacy practice teaching techniques were associated with higher student achievement (Gibson & Dembo, 1984).

Efforts to Change Teachers' Content Knowledge and Attitude

Previous studies have documented efforts to improve teachers' MCK and attitude toward mathematics. Some of these studies have documented the impact of methods courses. Both Quinn (1997) and Stoddart, Connell, Stofflett, and Peck (1993), analyzed the effect of a methods course that emphasized understanding both content knowledge and related pedagogy. Quinn found significant increases in MCK and attitude toward mathematics. Stoddart et al. reported descriptive statistics that indicated MCK increases. Neither study compared results to a control group. Both of these studies had a testing effect limitation because the same instrument was used for the pre and posttest data. Knight (1993) compared a similar methods course with a more traditional methods course. Using qualitative methods, Knight found that participants of the MCK-focused methods course had higher MCK and lower mathematical anxiety than participants of the more traditional course.

No previous study directly measured the effect of a mathematics content course (i.e. a course taught in a mathematics department). Mathematics content courses differ from methods courses in their scope and the potential to deeply investigate mathematical concepts. Much of the time used of a methods course is spent focusing on the procedures of teaching, (i.e. effective evaluation tools, classroom management, etc.), rather than on the mathematics. Historically, preservice teachers have taken general mathematics courses, such as statistics or college algebra, as part of their preservice preparation. Earlier this decade, national organizations concerned with teacher education have suggested that preservice elementary teachers take specialized mathematics courses instead (Conference Board of the Mathematical Sciences, 2001; Kilpatrick, Swafford, & Findell, 2001). These organizations propose that these specialized courses address the mathematics typically taught in elementary school from an advanced prospective.

Despite these guidelines, the mathematical content preparation of preservice elementary teachers still varies widely across the nation. The authors randomly chose 59 out of 1,297 higher education institutions classified at The Chronicle of Higher Education website, http://chronicle.com/, in April, 2007. The authors retrieved information about elementary education degree requirements for this sample. Eleven did not offer a degree in elementary education. Twenty-nine offered a degree and had at least one mathematics course specifically designed for preservice elementary teachers. Fourteen offered a degree but did not have this type of course, requiring a general mathematics course instead. Five were unable to be classified into the previous categories for a variety of reasons, such as insufficient course descriptions. Thus in some programs, preservice elementary teachers take only general mathematics courses, such as statistics or college algebra; in other programs, they take specialized courses designed to specifically address elementary mathematics from an advanced perspective.

Some research has focused on specialized mathematics content courses. Leapard (2000) found significant decreases in mathematical anxiety but no increases in MCK. Leonard and Joergensen (2002) used a continuous diagnostic tool, post-test data, interviews, and journals to measure the MCK increase. The results did indicate significant growth in MCK throughout the semester, with some subjects like area and perimeter problems still relatively misunderstood. Lubinski and Otto (2004) reported that preservice elementary teachers' attitudes and beliefs changed positively via the impact of a standards-based content course. Beswick (2006) found that preservice elementary teachers' attitudes and beliefs significantly changed in the expected direction via a combination of content and methods courses.

These previous studies allow some insight into improving MCK and attitudes for preservice elementary teachers. However, no previous studies exist that look at the effects of a specialized mathematics content course with a control group. This study contrasts the effects of a specialized mathematics course titled Logic of Arithmetic (LOA) as compared to the effects of more general mathematics courses on preservice elementary teachers. The specific research questions were as follows.

1. Do the LOA course and general mathematics courses differ on their impact of preservice elementary teachers' content knowledge (MCK)?

2. Do the LOA course and general mathematics courses differ on their impact on preservice elementary teachers' attitude toward mathematics?
3. What general areas of MCK does this study reveal as problematic or successful for preservice elementary teachers?

Methodology

Participants

The teacher, who is not one of the authors, of the fall 2005 methods course for elementary mathematics at a large Midwestern public university recruited the participants for this study. The preservice elementary teachers in the experimental group (n = 29) had taken the LOA course. The preservice elementary teachers in the control group (n= 19) had taken a general mathematics course instead. Prior to the fall 2004 semester, the LOA course was unavailable. After the fall 2004 semester, preservice elementary teachers were advised to take LOA. Thus, these intact groups were not randomly selected, but were also not self-selected.

Of the participants, only one was male (control group), two were non-Caucasian (both in the experimental group), and one over the age of 23 (control group). For methods classes in elementary mathematics at this particular institution, these demographics of participants were representative of the population. The control group and experimental group did not differ significantly on any major demographic variable, with the exception of sex and ethnicity because the one male and the two non-Caucasians.

Description of Intervention

The mathematics and mathematics education faculty, including the second author, at the institution where the study was conducted developed the LOA course collaboratively. The course content included an in-depth study of the number system and place value, and a variety of algorithms for operations in the natural numbers, including extending algorithms in bases besides base 10. The course developers believed that the preservice teachers' work on understanding these extensions into other bases simulated the learning processes that these preservice teachers' future students (i.e. grade school children) may undergo when learning with the natural numbers. Moreover, the study of various algorithms may help teachers aim for a conceptual base instead of just rote learning (CBMS, 2001; Graeber, 1999; NCTM, 2000). Throughout the course, student errors arising from the use of different algorithms instigated discussion and explorations (Mapolelo, 2003). Additionally, the course extended the arithmetic operations and algorithms to the positive rational numbers and integers, providing rich sources for investigations. Finally, the course covered problem solving techniques such as those suggested by G. Pâolya (1957), pattern recognition, models of natural numbers, integers, prime numbers, least common multiples, greatest common divisors, and irrationals. A lecture session was held twice a week. Smaller discussion sessions were also held twice a week enabling in-depth discussions, hands on activities, explorations, and some group work.

Instruments

The Mathematical Content Knowledge for Elementary Teachers test was used to measure MCK of the participants (see Appendix A for exact items). This instrument was designed by a panel of experts, including both authors, and covered elementary-school mathematical topics. The range of possible scores is 0 to 20. Half of the test questions came from previously established tests on teachers' MCK (Quinn, 1997; Stoddart, Connell, Stofflett, & Peck, 1993;

White, 1986). Additional items were created to expand the content coverage of the test. The test was evaluated by a different panel of expert mathematics educators and deemed externally valid. To judge the reliability of the instrument with this population, the Cronbach alpha was calculated at $\alpha= 0.80$. Attitude towards mathematics was measured using the Aiken's Revised Mathematics Attitude Scale (Aiken, 1963). This instrument was chosen for its ability to measure attitude toward mathematics in college women. Aiken and Dreger (1961) report the test-retest reliability for their sample to be 0.94. Typical items on this instrument included questions asking whether mathematics was an enjoyable or dreaded subject. The specific hypotheses of this study are that there will be no differences between the experimental group and the control group on the MCK test scores and the attitude scores.

Procedure

Each participant's descriptive statistics were acquired from the school's database. Furthermore, for each participant, the grade, semester taken, and credits earned of all mathematics courses taken at the university where the study took place were also obtained. The attitude survey and MCK test were both administered during the first week of the methods course, so as to minimize the impact of the method's course instruction on the results.

Data Analysis

Independent sample *t*-tests were conducted to compare the two groups along the MCK and attitude variables. For the MCK variable, in order to partially control for ability differences between the experimental and control groups, a linear regression analysis was also conducted. In this analysis, the ACT comprehensive score and cumulative university g.p.a. were entered into the model first, and then the group variable (control or experimental) was added to determine if there were significant differences between groups on the MCK variable once group differences in previous achievement had been controlled. Interaction terms and quadratic relationships were also considered for inclusion into the model and reported if included. For the attitude survey, the results presented here include all participants. However, the data was also run excluding the one male participant without any major differences in the results, to address possible test bias.

Results

Table 1 contains descriptive summaries of the scores on each instrument.

Table 1

Means and Standard Deviations of Scores by Group

	Control Group			Experimental Group		
	N	M	St.D	N	M	St.D
CKM Score	19	8.58	2.27	29	9.66	2.33
Attitude Score	19	-9.32	9.74	29	-0.15	15.8

Content Knowledge Comparison Results

For the content test analysis, a test for equality of variance was conducted and retained (F = 0.10, p>.10). The *t*-test results were not significant (t = -1.53, df = 46, p>.10). In the linear regression analysis, ACT comprehensive and cumulative university g.p.a. did significantly predict MCK score (R^2 = .232, p <.01). Adding group effect, i.e. adding the effect of taking the

LOA course, also significantly contributed to the model (R^2 change = .072, p <.05). Table 2 contains the associated beta weights and p-values for this regression analysis.

Table 2
Linear Regression Results on the CKM Measure

		Unstandardized Coefficients B	Standardized Coefficients b	t	p
Model 1	(Constant)	-1.345			
	Cumulative GPA	0.4	0.042	0.329	0.744
	ACT score	0.399	0.508	3.946	<0.000
Model 2	(Constant)	-1.55			
	Cumulative GPA	0.161	0.017	0.137	0.892
	ACT score	0.411	0.522	4.195	0.000
	Group Variable	1.219	0.257	2.061	0.045

Therefore, once previous achievement was controlled, there was a significant difference between students in the control and experimental groups in average MCK.

These results only partially support the rejection of the first hypothesis because the *t*-test results were not significant. However, once previous achievement was controlled for the groups were significantly different on the MCK measure. Thus, the LOA course may have a larger impact on preservice teachers' MCK than a general mathematics course.

Attitude Comparison Results

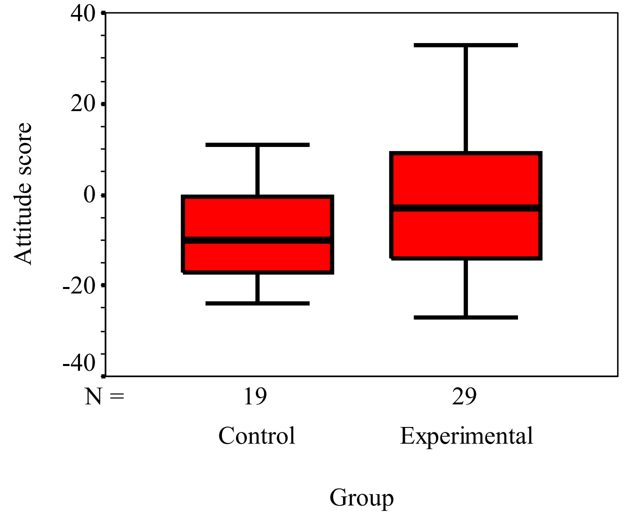
For the attitude analysis, a test for equality of variance was conducted and rejected (F = 4.25, p<.05). The non-equal-variances t-test results were significant (t = -2.49, df = 45.89, p<.05). The effect size for this result, -0.74, is considered a moderate to large (Sprinthall, 2000). At the standard alpha level of .05, these results are similar to previous studies that have demonstrated significant increases in attitude toward mathematics with MCK interventions.

Discussion

Interestingly, the variances of the two groups were significantly different. Figure 1 shows a box plot of the two groups on the attitude score. This data may imply that the LOA course had the effect of making differences in attitude toward mathematics more pronounced as compared to the control group of courses. Perhaps the exposure to elementary mathematical topics at a rigorous level has a polarizing effect that makes those who succeed at understanding like mathematics even more; while the exposure makes those who struggle with the material feel even more dislike for mathematics.

Figure 1

Box and Whisker Plot of attitude scores by group



Item Differences between Groups

An item by item comparison between the experimental and the control group revealed some interesting discrepancies in the average percent correct for each item. See Table 3 below.

Table 3

Percent correct per item for the experimental and control groups

Question Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Control Group	95	79	95	58	74	63	32	32	21	47	21	0	74	42	26	58	16	5	5	16
Experimental Group	83	55	100	79	62	76	31	45	17	55	38	0	72	41	66	45	28	41	17	14

Caution should be taken when drawing any conclusions about the significance of these comparisons. Statistical tests were not performed and were unlikely to produce any significant results, due to the necessary Bonferroni correction of 1/20 on the α-level and low sample size. However, the control group did considerably better than the experimental group on question 2, and somewhat better on questions 1, 5, and 16. Questions 1 and 2 dealt with the concept of division. The experimental group did considerably better than the control group on questions 4, 15, and 18 and somewhat better on questions 6, 8, 11, 17, and 19. Questions 15 and 19 dealt with understanding the standard U.S. algorithms for multiplication and division.

General Problematic Areas in Content Knowledge

As mentioned previously, the content test was created using some items from previous studies. Discussing which items are causing difficulties for teachers in two different studies (and thus different contexts) may be useful. Table 4 indicates the percentage correct for all participants in the study on the content test, with associated percentage correct reported in other studies.

Table 4
Percent correct per item from CKM test and previous studies using the item

Question Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Current Study	87	65	98	71	67	71	31	40	19	52	31	0	73	41	50	50	23	27	13	15
Previous Study	89	52	96	44	67	70	41				37	5					58			

The percent correct attained by participants in this study was similar to the percent correct from the prior studies (with noticeable exceptions on items 4 and 17). Preservice elementary teachers in multiple studies struggled with problems that deal with averages (like #7 or #12). This research supports previous findings of preservice elementary teachers' poor conceptual understandings of measures of central tendency (Groth & Bergner, 2006). Seemingly accessible, a correct solution to item # 12 eluded the preservice elementary teachers in multiple studies. The typical response (to add the fractions) is incorrect. The authors have observed that simple substitution of percentages makes the incorrect solution seem unreasonable and leads to a correct solution (averaging). However, using this problem to talk about reference units (i.e. "What is the whole?") leads to rich discussion. Analysis of the results on items created for this study also show some points of interest. Preservice elementary teachers in this study did poorly on problems dealing with conceptual understanding of algorithms used to solve whole number computation problems (Items 15, 19, and 20). Since the ability to understand and analyze different algorithms is necessary for excellent teaching, these results are problematic (Jacobs & Phillips, 2004).

Successful Areas in Content Knowledge

Somewhat contradictory to previous results (Ball, 1990; Ma, 1999) the participants in this study and others had moderate success (65% correct in this study) on a problem dealing with a word problem for division of fractions. Table 4 also shows that preservice elementary teachers have done reasonably well on problems dealing with understanding of division in a variety of contexts (items 1, 2, and 5) reasoning with measurement combinations (item 6), and estimation (item 3). Participants in this study also did relatively well on ordering decimals (item 13, created for this study only).

Limitations

This study does have some limitations. The groups were not randomly assigned, but were rather intact groups. However, differences in achievement between the groups were partially controlled for by using regression analysis. Moreover, there was what might be termed a delay effect that was confounded with the grouping effect. The time between when the participants took their mathematics content course and the methods course was slightly longer (on average, four to six months longer) for the control group than for the experimental group. In addition, widely accepted measures of MCK were not known to the authors when the research was planned in early 2004. The instrument to measure MCK was created for this study and has not been extensively tested.

Conclusion

Further research is needed in the field of preservice elementary teachers' preparation in mathematics. New measures of teacher content knowledge are available and possibly more efficient than the test used in this study (Center for Research in Mathematics and Science Teacher Development, 2006; Hill, Schilling, & Ball, 2004). Replicating this study with these new instruments would be worthwhile. Further research could look more closely at how these specialized mathematics courses affect preservice teachers' learning of the pedagogical content knowledge taught during their methods course. Furthermore, are the differences found in this study robust? Do these differences still exist at the end of the methods course or after a few years of teaching? Finally, the overall percentages correct in certain areas were troubling. For example, both the experimental and control group did poorly (<25% correct) on procedural problems dealing with fractions. Other areas that both groups did poorly on dealt with conceptual problems such as the concept of average (<35% correct), models of integers (<50% correct), and explaining portions of the standard long division algorithm (<60% correct). Each of these areas was specifically addressed during the LOA course and future research in improving these areas would be useful.

As institutions consider their own preparation of elementary teachers, we believe that the results presented in this paper support the recommendations given by the Conference Board of Mathematical Sciences and other national organizations. Teacher preparation institutions may resist requiring a content course for teachers because of logistical, financial, and philosophical reasons. However, this research does indicate that including content courses as a requirement for preservice teachers should be given careful consideration by teacher preparation programs.

References

Adams, T. L. (1998). Prospective elementary teachers' mathematics subject matter knowledge: The real number system. *Action in Teacher Education, 20*(2), 35-48.

Aiken, L. R. (1963). Personality correlates of attitudes toward mathematics. *The Journal of Educational Research*, *56*, 476-480.

Aiken, L. R., & Dreger, R. M. (1961). The effects of attitudes on performance in mathematics. *Journal of Educational Psychology*, *52*(1), 19-24.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *Elementary School Journal*, *90*, 449-466.

Bandalos, D. L., Yates, K., & Thorndike-Christ, T. (1995). Effects of math self-concept, perceived self-efficacy, and attributions for failure and success on test anxiety. *Journal of Educational Psychology*, *87*(4), 611-623.

Beswick, K. (2006). Changes in preservice teachers' attitudes and beliefs: The net impact of two mathematics education units and intervening experiences. *School Science and Mathematics, 106*(1), 36-47.

Beswick, K. & Dole, S. (2001). Dispelling the myths: Influencing the beliefs of preservice primary teachers. In J. Bobbis, B. Perry, & M. Mitchelmore (Eds.), *Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australia: Numeracy and beyond* (pp. 90-97). Sydney, NSW: Mathematics Education Research Group of Australia.

Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and Teaching: Theory and Practice*, *11*(1), 37-46.

Center for Research in Mathematics and Science Teacher Development. Retrieved May 29th, 2006 from http://www.louisville.edu/edu/mathscience/index.html.

Conference Board of the Mathematical Sciences. (2001). The mathematical education of teachers, part I. Retrieved September 8th, 2003, from www.cbmsweb.org/MET_Document/index.htm.

Fuller, R. A. (1996). *Elementary teachers' pedagogical content knowledge of mathematics.* Chicago, Illinois: Mid-Western Educational Research Association Conference (ERIC Documentation Reproduction Service No. ED 401270).

Gibson, S., & Dembo, M. (1984). Teacher efficacy. A construct validation. *Journal of Educational Psychology*, *76*(4), 569-582.

Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal*, *28*(5), 689-704.

Graeber, A. (1999). Forms of knowing mathematics: What preservice teachers should learn. *Educational Studies in Mathematics, 38*(3), 189-208.

Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning, 8*(1), 37-63.

Hill, H., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.

Jacobs, V. R., & Phillips, R. A. (2004). Mathematical thinking: Helping prospective and practicing teachers focus. *Teaching Children Mathematics, 11*(4), 194-201.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Knight, E. Q. (1993). *Teacher preparation in elementary math: Students' response to an emphasis on understanding*. Unpublished doctoral dissertation. University of Illinois at Chicago. Chicago, Illinois.

Kolstad, R.K. & Hughes, S. (1994). Teacher attitudes toward mathematics. *Journal of Instructional Psychology. 21*(1), 44-48.

Leapard, B. B. (2000). *Affective, metacognitive, and conceptual effects of an emerging scholars program on elementary teacher preparation: An application of the Treisman workshop model*. Unpublished doctoral dissertation. University of Toledo. Toledo, Ohio.

Leonard, J. & Joergensen, P. (2002, January). *Empowering all elementary preservice teachers to teach children mathematics*. Paper presented at the 6th Annual Conference of the Holmes Partnership, San Antonio, TX. (ERIC Documentation Reproduction Service No. ED 401270).

Mead, J. V. (1992). Looking at old photographs: Investigating the teacher tales that novice teachers bring with them (Report No. NCRTL-RR-92-4). East Lansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction Service No. ED346082)

Lubinski, C. A., & Otto, A. D. (2004). Preparing K-8 preservice teachers in a content course for standards-based mathematics pedagogy. *School Science and Mathematics, 104*(7), 336-350.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates.

MacNab, D. S., & Payne, F. (2003). Beliefs, attitudes and practices in mathematics teaching: Perceptions of Scottish primary school student teachers. *Journal of Education for Teaching, 29*(1), 55-68.

Mapolelo, D. C. (2003). Case studies of changes of beliefs of two in-service primary school teachers. *South African Journal of Education*, *23*(1), 71-78.

McGinnis, J.R., Kramer, S., Shama, G., Graeber, A.O., Parker, C.A., & Watanabe, T. (2002). Undergraduates' attitudes and beliefs about subject matter and pedagogy measured periodically in a reform-based mathematics and science teacher preparation program. *Journal of Research in Science Teaching, 39*, 713-737.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, Va: National Council of Teachers of Mathematics.

Pâolya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Garden City, N.Y.: Doubleday.

Quinn, J., R. (1997). Effects of mathematics methods courses on the mathematical attitudes and content knowledge of preservice teachers. *The Journal of Educational Research, 91*(2), 108-113.

Randhawa, B. S., Beamer, J. E., & Lundberg, I. (1993). Role of maths self-efficacy in the structural model of maths achievement. *Journal of Educational Psychology, 85*(1), 41-48.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula, T. J. Buttery & E. Guyton (Eds.), *Handbook of research on teacher education* (2nd ed., pp. 102-119). New York: Simon & Schuster Macmillan.

Rowan, B., Chiang, F.-S., & Miller, R. J. (1997). Using research on employees' performance to study the effects of teachers on students' achievement. *Sociology of Education*, *70*(4), 256-284.

Sprinthall, R.C. (2000). *Basic Statistical Analysis* (6th Edition). Boston: Allyn & Bacon.

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K., & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Teacher Education, 4*(3), 205-225.

Stoddart, T., Connell, M., Stofflett, R., & Peck, D. (1993). Reconstructing elementary teacher candidates' understanding of mathematics and science content. *Teaching and Teacher Education*. *9*(3), 229-241.

Swars, S. L. (2004). *Mathematics teaching efficacy beliefs of elementary preservice teachers and their relationship to mathematics anxiety.* Unpublished doctoral dissertation, University of Alabama.

White, M. A. F. (1986). *Preservice teachers' achievement in the essential elements of elementary school mathematics: The development of an evaluation instrument.* Unpublished doctoral dissertation. University of Houston. Houston, Texas.

Zazkis, R., & Campbell, S. (1996). Divisibility and multiplicative structure of natural numbers: Preservice teachers' understanding. *Journal for Research in Mathematics Education. 27*(5), 540-563.

Appendix A

Mathematical Content Knowledge for Elementary Teachers

1. In the sentence $18 \div 6 = 3$, 18 represents a number of cookies. Which of the following statements is true?
    a. Neither 6 nor 3 can represent a number of children.
    b. If 6 represents a number of cookies, then 3 represents a number of cookies.
    c. If 6 represents a number of children, then 3 represents a number of cookies.
    d. 6 must represent a number of cookies.

2. The number $10 \div \frac{1}{2}$ represents the solution to the problem. Which of the following statements can represent the problem?
    a. How many boys came to the club meeting if half of the ten children present were boys?
    b. With 10 sticks of gum, how many children can have gum if each gets ½ of a stick?
    c. Give each of 2 children half of a box of 10 apples. How many apples will each get?
    d. Divide 10 crayons equally between two boxes. How many crayons will be in each box?

3. To estimate 43 x 28 by rounding to the nearest 10, think
    a. 40 x 20.
    b. 40 x 30.
    c. 45 x 25.
    d. 50 x 30.

4. 5% of $170 indicates the amount you would have if you
    a. divide $170 into 100 equal parts and take 5 of the parts.
    b. divide $170 into 5 equal parts and take 1 of the parts.
    c. divide $170 into 10 equal parts and take 2 of the parts.
    d. take 5 times $170 and move the decimal point 2 places to the right.

5. A piece of tape 1.6 meters in length is to be cut in equal lengths measuring 20 centimeters each. How many pieces of tape can be produced?
    a. 8
    b. 0.8
    c. 32
    d. 12.5

6. A strip of paper 3 yards, 5 inches long is taped on a wall for a mural. Another piece 1 yard, 2 feet, 7 inches long is taped end-to-end with the first piece, giving a total length of:
   a. 5 yards.
   b. 4 yards, 3 feet, 2 inches.
   c. 5 yards, 2 feet.
   d. 5 yards, 2 inches.

7. The average (mean) of 4 whole numbers is 16. Two of the numbers are 32 and 2. The other two numbers are
   a. both greater than 2.
   b. both less than 32.
   c. both 16.
   d. equal.

8. Which of the following conceptual models would be the least feasible for the concept of integers?
   a. a number line, i.e. a thermometer
   b. the charges of protons and electrons
   c. number of letters in the alphabet
   d. winning and losing money (i.e. a poker game)

9. Solve:

$$\begin{array}{r} 12\frac{13}{24} \\ -\quad 3\frac{10}{18} \\ \hline \end{array}$$

10. One of your students, Greg, explains that to solve the problem, 294×12, he thinks $3600 - 72 = 3528$. Greg says that this works because of the distributive law. Where does the 72 come from by Greg's reasoning?

    a. $300 - 294 = 6$ and $6 \times 12 = 72$.
    b. $2 \times 9 \times 4 = 72$.
    c. Since the first factor has 3 digits and the second has 2 digits, then $3 \times 2 \times 12 = 72$.
    d. Adding the digits in the ones place we get 2 + 4 = 6. Adding the digits in the other places we get 2 + 9 + 1 = 12. Now $12 \times 6 = 72$.

11. What number would go in the circle below to make the statement true?

$$\frac{\text{O}}{4} = \frac{2}{5}$$

12. Mary has socks in two drawers of her dresser. In the top drawer, one-third of the socks are white. In the bottom drawer, two-fifths of the socks are white. She has the same number of socks in both drawers. What portion (fraction) of Mary's socks are white?

13. Arrange the following from largest to smallest:
    .990 .099 .0991 .9909

14. Check that $4 + 5 + 6 = 3 \times 5$, $7 + 8 + 9 = 3 \times 8$, and $39 + 40 + 41 = 3 \times 40$. Write down a sentence that explains the pattern. Express your sentence using symbolic (algebraic) notation.

15. Claudia has $11,372 to invest in stocks. She decides to purchase stock in Acme Automobiles, which is selling at the rate of $36 a share. Claudia did the following to determine the number of shares she could buy.

$$\begin{array}{r} 315 \\ 36\overline{)11372} \\ \underline{108} \\ 57 \\ \underline{36} \\ 212 \\ \underline{180} \\ 32 \end{array}$$

What does the "57" indicate?
a. The "57" means that Claudia has 572 shares of stock so far, because you still have to "drop the 2."
b. The "57" means that so far in the problem, Claudia has 57 dollars left.
c. The "57" means that so far in the problem, Claudia has bought 57 shares.
d. The "57" means Claudia would have at least $570 left if she bought only 300 shares of stock.

16. Calculate $\dfrac{(-40)\times 5 + (7 - (-3))}{(-10)} =$

    Show your steps.

17. Solve: $3\frac{2}{5} \div 2\frac{3}{7} = ?$

18. Which of the following statements about a prime number, *x*, is false?
    a. The greatest common factor of *x* and any other number is either x or 1.
    b. The least common multiple of *x* and any other number, call it y, is either y or their product, xy.
    c. The prime factorization of *x* is *x* numbers long.
    d. The only factors (or divisors) of *x* are 1 and *x*.

19. Your class has recently been learning the procedure for multiplying 2-digit numbers. One of your students, Gloria, multiplies 43 and 49 as shown below.

At first, her work looked like this

$$\begin{array}{rrr} & \overset{2}{4} & 3 \\ \times & 4 & 9 \\ \hline 3 & 8 & 7 \end{array}$$

Followed by this

$$\begin{array}{rrr} & \overset{1}{4} & 3 \\ x & 4 & 9 \\ \hline 3 & 8 & 7 \\ 1 & 7 & 2 \\ \hline 5 & 5 & 9 \end{array}$$

A few of her classmates show Gloria the traditional procedure correctly and offer the following advice, which you overhear. Which response is the most conceptually correct?

a. 559 divided by 49 is clearly not 43. What do you get when you divide these numbers?
b. 4 tens times 3 ones is 120 and 4 tens times 4 more tens is 1600 and 1600 plus 120 is 1720.
c. You see that you are on the second line in your answer. You have to shift over one place to the left every time you go down a line in your process.
d. Since the 7 goes beneath the 9, the 2 needs to go beneath the 4 because it is in the same place value.

20. You notice one of your students doing subtraction problems in an unusual way. The following two examples demonstrate the method used.

$$\begin{array}{rrrrr} & 4 & 2 & 9 & 6 \\ - & 1 & 7 & 7 & 1 \\ \hline \end{array} \longrightarrow \begin{array}{rrrrr} & 4 & \overset{12}{\cancel{2}} & 9 & 6 \\ - & \overset{2}{\cancel{1}} & 7 & 7 & 1 \\ \hline & 2 & 5 & 2 & 5 \end{array}$$

$$\begin{array}{rrrrrr} & 4 & 3 & 5 & 9 & 1 \\ - & 1 & 7 & 2 & 6 & 7 \\ \hline \end{array} \longrightarrow \begin{array}{rrrrrr} & 4 & \overset{13}{\cancel{3}} & 5 & 9 & \overset{11}{\cancel{1}} \\ - & \overset{2}{\cancel{1}} & 7 & 2 & \overset{7}{\cancel{6}} & 7 \\ \hline & 2 & 6 & 3 & 2 & 4 \end{array}$$

When asked about the process, the student replies, "My uncle says that if the top number is too little to subtract, then just put a "1" in front of the top number to make it big. But, every time you do that, then you have to go down and to the left and make that number one bigger, because otherwise you'll mess up the answer." Which of the following reasons validates this procedure for subtraction?

a. c(a-b) = ca-cb
b. (a-b) = (a+c) – (b+c)
c. By the associative property for numbers, one may add ten instead of subtracting ten during computation.
d. This is just another way of our usual "borrowing", as in taking away a ten to get more ones, taking away a hundred to get more tens, and so on.