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ELEMENTARY SCHOOL TEACHER CANDIDATES' PERCEPTIONS OF GOOD PROBLEMS

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Abstract

This study describes a classroom action research activity regarding a group of elementary school teacher candidates' perceptions of good mathematics problems. A questionnaire containing 20 problems was given, and the candidates were asked to rate the quality of each problem on a 5-point scale. The results revealed that the majority of the teacher candidates considered typical routine problems good and showed strong resistance to some non-routine problems that have atypical characteristics. Although the need of new perspective towards the nature of problem solving was identified throughout the reflection process, the teacher candidates expected difficulties in utilizing atypical problems in their future classrooms due to the lack of systemic support or individual teacher's confidence. Implications of the results and the need for new directions are discussed.

Introduction

It is imperative to provide quality problems in mathematics class. In the past three decades, the professional community emphasized the importance of problem solving (NCTM 1980, 1989, 2000). In particular, the recent NCTM's (2000) standards identified problem solving as one of five process standards, stating, "problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program" (p.52).

This focus on problem solving is reiterated in teacher education programs for teacher candidates and teachers. However, it is too broad of an area to define the nature of the "problem," and it is too complicated to decide which types of problems are appropriate for which concepts and in what contexts. Even, we are not quite sure whether we are on the same page when referring to the "good problem."

This article reports on a classroom action research activity involving a group of elementary school teacher candidates, asking their perceptions of good mathematics problems. The purpose of this study is to examine how the teacher candidates define "good problems"; how they justify their choices of good problems; how their justifications can be interpreted; and what are the implications for the teacher education program.

Meanings and Criteria of Good Problems

Our life consists of a series of problems. There is always "a situation that causes difficulties" and "a question that must be answered" (Longman Dictionary, 2001). Mathematics educators' definitions of a problem have fully reflected these everyday meanings and refined them in their own terms. These terms include highlighting its difficulty, the unavailability of immediate solutions, and demand significant effort on the part of the problem solver (Buchanan, 1987; Blum & Niss, 1991; Charles & Lester, 1982; Sheffield & Cruikshank, 2005; Van de Walle, 2001).

These definitions identify a problem as a different entity from an exercise, drilling, or practice. Also, they imply that the distinction between a problem or drilling or exercise can be determined by the individual solvers' previous knowledge and experiences. For instance, it is a challenging *problem* to find the product of 25 and 32 for the student who has just learned the concept of multiplication. However, it is not a problem anymore for students who have already mastered the multiplication algorithm.

Based on this definition of a problem, leading educators have suggested some guidelines for selecting appropriate problems for the problem solving instruction (e.g., Cathcart, Pothier, Vance, & Bezuk, 2000; Hyde & Hyde, 1991; NCTM, 2000; Van de Walle, 2001). They have some common elements as follows:

- It integrates multiple topics and involves significant mathematics.
- It begins where the students are.
- The problematic or engaging aspects of the problem must be due to the mathematics that the students are to learn.
- It requires justifications and explanations for answers and methods.
- It provokes students' interest in pursuing it.
- It is solvable in many ways.
- It sometimes contains missing, extraneous, or contradictory information.
- It makes connections between different mathematical fields.

Purpose of the Study

The importance of problem solving has been continuously addressed for several decades, and it is widely shared within our professional community as an important goal. The purpose of this study is to investigate how elementary school teacher candidates internalize this important goal in the preparation of their professional life.

In Thompson's (1989) study on teachers' conceptions of a mathematical problem, she initially reported that "5 out of 16 teachers conceived a problem task as essentially the description of a situation involving stated quantities, followed by a question about some relationship among the quantities whose answer called for the application of one or more arithmetic operations" (p.235). Also, these teachers gave 'story' or 'word' problems as examples of a problem task. Thompson concluded the study, saying, "I hope that ten years from now we will be more knowledgeable about effective ways and techniques of teaching problem solving and have a better sense of how to go about preparing teachers in their use. I also hope that by then we will have had an opportunity to examine curricular materials for teaching problem solving and how they are used by teachers in their classrooms" (p. 243).

Definitely, more resources and studies are available now than when Thompson conducted her research. Does it necessarily mean that teachers' perceptions about problem solving have changed accordingly? This study investigates a group of elementary school teacher candidates' perceptions of good mathematics problems. Utilizing clinical interviews, it examines the teacher candidates' definitions of "good problems," their selection criteria, and justifications behind them. The results will show how much the message of "mathematics through problem solving" has been delivered to the pre-service teaching profession.

Methods

Participants

The interview took place at a teacher education institution located in a southern state. A total of twenty two elementary school teacher candidates who enrolled in Methods of Teaching Elementary Mathematics participated in the interview process. All of them completed four mathematics classes in the mathematics department as prerequisites for this class and as a requirement of being a highly qualified teacher. Also, they completed four educational foundations courses before this class as a requirement for admission to the professional education courses. Most of them were taking other methods classes at the same time and were planning to take the internship course within the following two semesters.

Interview

The first five sessions of the course covered NCTM principles and standards, learning theories, and the issues in the instructional planning. Although problem solving was not the main issue in these sessions, some students revealed their knowledge, concerns, and past experiences regarding problem solving. In particular, while sharing their autobiographies in the second session, many students expressed their negative past experiences in problem solving and asserted the need for new directions in elementary mathematics instruction.

The sixth session started with a survey questionnaire. The survey asked the teacher candidates to rate how an example represents the category it belongs in on a 5-point scale. For example, when the category is fruit, is an apple a good representation of the category? How about a tomato? If they feel the example is very good, it would rate a ‘5.’ If they feel the example is poor, or the example does not belong to this category, it would rate a ‘1.’ A ‘3’ means that the example fits moderately well. Before getting into the main interview, the teacher candidates practiced rating by using the categories of their choices such as ‘vehicle’, ‘good singers’, ‘sports’, etc. Some categories needed scientific knowledge to classify, and some of them asked for the interviewee’s personal judgment.

The interview questionnaire consists of 20 examples and the category was “Good Mathematics Problems” (see Appendix A). Some of them are very typical calculations or word problems and the other problems have somewhat unusual characteristics. Appendix B shows the types of problems and their characteristics:

First, the teacher candidates rated and solved the problems individually. Then, they were paired up and interviewed each other. Each person justified their ratings to the interviewer. The answers for the problems were not discussed in the interview session as to whether they were correct or not. The interviewer’s job focused on identifying the interviewee’s criteria of good mathematics problems. After individual interviews, the interviewers wrote a summary of the interviewee’s criteria of good mathematics problems.

Post- conference

In the following class session, the interview results were shared and discussed. Also, the class examined the previous research conducted by leading educators in problem solving and compared their recommendations with the class interview results. At the end, the instructor asked the following questions for further reflections: 1) Do you want to change any of the ratings you made last session? Why or why not? 2) Do you think you will be able to utilize some non-routine problems in your mathematics classroom? How? What will be the biggest obstacle or concern for you?

Results

As shown in Appendix C, the teacher candidates perceived that traditional routine problems are better than non-routine ones. Each interviewer's report provided more detailed reasons for the ratings. Although it was not a structured interview, many phrases and opinions overlapped. The following statements are the major responses for each sub category.

Routine Problems

The majority of the justifications from the participants who rated these problems as '4' or '5' emphasized the clear and typical structure of the problem. Some comments included: "It is straight to the point." "It is specific." "It told exactly what was needed to know." "It is very specific." "It is not confusing." "It has one answer." "It is simple to the point." "It is a good multiplication problem should be." "It has all the components." "It is good because it has a formula to go by. [This response was for the problem #9.]"

The responses from the participants who rated these problems as '1' or '2' revealed two extreme judgments for the difficulty level of the problem. Some said that the problem was too easy, and the others said the same problem was too difficult. Also, some justifications came from individual teacher candidate's distaste for a specific concept or problem type. The justifications in this category included: "It did not really challenge students." "It is too simple." "It is too easy." "It is too complicated for elementary level." "I don't think it is appropriate for elementary age group." "It is too complicated for K – 4 students." "I don't like fractions." "I just don't like this problem."

Non-routine Problems

The majority of the teacher candidates rated these problems as '1' or '2', perceiving that these are not good problems to utilize in the elementary school classroom. Appendix D summarizes the common themes in their justifications and some specific comments.

Summary of Interviews

Each teacher candidate interviewed the partner and summarized the results of the partner's criteria of good mathematics problems in a few sentences. The major themes were covered in their justifications, including straightforward and simple format, one-question-one-answer problem, appropriate difficulty level, real-life connections, math-oriented problems, clear directions, challenging problems, useful information only (no extra, missing, contradictory information), motivational aspect, and so on.

Post-conference

The class discussion focused on the meaning of the "problem," which should be differentiated from drilling or practice, and instructional strategies in a problem solving class. Also, the value of using non-routine problems was discussed. For example, an activity called "trap hunting" utilizes nonsensical problems to enhance children's problem analyzing and posing skills. In this activity, the children were asked to identify trap problems, which contained contradictory or missing information, and to revise them into sensible problems (Lee, 2005). In addition, some research results were examined, which revealed the procedural-oriented problem solving with the absence of the sense-making process (e.g., Schoenfeld, 1991).

After the class discussion, the teacher candidates answered two questions: 1) Do you want to change any of the ratings you made last session? Why or why not? 2) Do you think you will be able to utilize some non-routine problems in your mathematics classroom? How? What will be the biggest obstacle or concern for you? They anonymously wrote the answers.

For the first question, 21 teacher candidates answered that they would change the ratings on some of the problems. The major explanations included that mathematics problems are different from the practice or drilling and the problems should be challenging and encourage students to think. However, one teacher candidate would not change any ratings, saying, "Now I learned what experts say about problem solving and can state what good problem solving instruction should be. However, my feelings towards the problems I rated are still the same."

For the second question, 20 teacher candidates responded that they expected some kind of difficulties. Ten of them were concerned about the resistance from other people in the learning community, including veteran teachers, administrators, and parents. Especially, some of them pointed out that it would be very hard to utilize a variety of problems in mathematics class when typical types of test problems exist. Seven teacher candidates were concerned about their own teaching strategies. They were not quite sure about how they could reach all children in problem solving instruction and how to develop appropriate teaching strategies which they were not used to. Three teacher candidates were concerned about specific concepts such as fractions and measurement. They thought that it would be hard to provide lots of examples and situations for children to better understand since the concept itself is too complicated. Two teacher candidates expected a promising future, saying, "I think I could carry on new strategies to help the children understand better than I did when I was their age", "I believe that there wouldn't be any difficulties on the condition that children are taught well."

Discussion

A teacher candidate is in the transition phase between the student's status and the teacher's. Thus, it is very important for them to not only do mathematics but to also internalize the meaning underlying the concepts and instructional processes. In this sense, the results of this study lead to some implications for teacher education in general, and mathematics teacher education in particular.

First, it is notable to see that the same old perceptions still exist. Researchers reported numerous students' and teachers' misconceptions on mathematics problems, such as "all problems must have one correct answer."; "There is only one correct way to solve a problem."; "An answer to a mathematical question is usually a number."; or "Every context (problem statement) is associated with a unique procedure for "getting" answers." (e.g., Baroody, 1987; Thompson, 1989). The teacher candidates' ratings for routine problems and their justifications are not much different from what was heard a couple of decades ago. It implies that the discussion on problem solving instruction has not quite reached the classrooms.

Second, there is a gap between 'knowing' and 'doing' in the teacher candidates' expectations. After in-depth discussions on problem solving instruction and the review of the research, 21 participants out of 22 replied that they would change the ratings because their views changed towards the nature of mathematics problems. However, 20 participants out of 22 expected that the future of this new knowledge is not promising. Whether the obstacles lie in the lack of systemic support or individual teachers' inability, the teacher candidates somehow foresaw that their teaching profession would be filled with various "I-Know-But" situations.

Third, the teacher candidates' justifications for good problems led the need for strengthening professional self-esteem. Many participants explained that good problems are simple, specific, and easy. These problems do not need serious thinking processes. One participant's comment during the interview process was rather shocking, stating, "For K-3 students, they need problems that really do not require much thinking. For 4 – 6 grade students, they can have more difficult problem but need to include all needed information." Elementary mathematics is not a collection of simple, easy, and basic examples that anybody can teach. It is important to let the elementary school teacher candidate know that they are teaching significant mathematics and realize their job is more than delivering simple mathematics rules.

Concluding Remark

Research on mathematics teacher education clearly shows the need of new perspective on the problem solving instruction. In response to this need, the activity described in this paper intended to initiate teacher candidates into the art of reflective practice, which in turn will help to transform mathematics classroom from one that is drill driven to one that promotes critical thinking.

The challenge for all teachers is to renew and refine their knowledge and practice throughout the on-going professional development and to actively participate in the reflective decision-making process in their instruction. In the end, teacher reflection can help to promote teacher thinking in mathematics instruction, which in turn may help to improve student attitudes toward this important subject.

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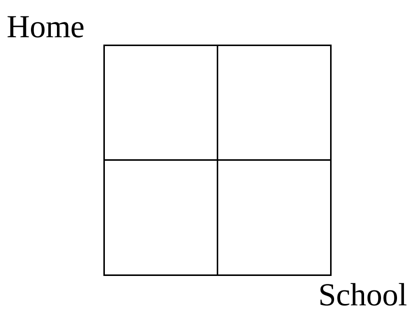
Appendix A: Questionnaire

Category: GOOD MATHEMATICS PROBLEMS

Please indicate your rating to the left of each instance below and provide your answer for the problem.

Rating	Example	Answer
( )	#1. Subtract: 95 – 38	
( )	#2. Jerry bought a cake for \$10. It's 1/5 of his monthly allowance. Tom ate 2/5 of the cake and Sue ate another 2/5 of the cake. How much of the cake did Tom and Sue eat?	
( )	#3. Johnny had '***a***' notebooks. Mary has 2 more notebooks than Johnny does. How many notebooks does Mary have?	
( )	#4. There are 8 pitchers for sale at the market. Mrs. Brown buys 2 pitchers. How many pitchers left?	
( )	#5. Which of the following numbers does not belong with the others? Explain. 4, 36, 25, 11	
( )	#6. For a salad, Mary used 3 ½ pounds of apples, 1/3 pounds of nuts, and ¾ pounds of raisins. How many pounds of the salad did Mary make in all?	
( )	#7. Find two fractions that are between ¼ and ½.	
( )	#8. You are about to ask your friend to guess the secret number you wrote in the paper, which is 25. One clue is given for your friend here. Make up two more clues: "The number is a less than 30."	
( )	#9. Fill the blank: 30 is _____ percent of 120.	
( )	#10. Make up a problem about students in the classroom using the following formula: 30 – x = 28	
( )	#11. Sandy earned \$350 and spent 3/5 of it. How much money did she have left?	

Rating	Example	Answer
( )	#12. A man can build a garage in 12 days. If 12 people who have the equal ability as the man work together, how long will it take to build the garage? If 288 people work together how long will it take to build the garage?	
( )	#13. Calculate: 3/5 + 2/5 + 1/5	
( )	#14. Decode the following message: “NZGS RH UFM!”	
( )	#15. Which unit would you use to find the length of a pencil? (a) cups (b) inches (c) pounds (d) yards	
( )	#16. There were 8 flower pots in the garden. Limmy put more flower pots in the garden and now there are 5 flower pots in the garden. How many flower pots did Limmy put in the garden?	
( )	#17. You are about to ask your friends to think of a number between 2 and 10. You want to ask them to conduct the following operations: “Multiply the number by 9. Add the digits in the product. Then, take away 5 from the sum. What is your answer?” How many different answers do you expect? Explain.	
( )	#18. The following map shows all the roads from my home to school. How many different paths can you find without backtracking?  	
( )	#19. I bought five T-shirts for $10 for each. How much money did I spend?	
( )	#20. How much did your household pay for the electricity last year? What is the monthly average cost?	

Appendix B: Types of Problems in the Questionnaire

Sub Category		Characteristics	Example
Routine Problem		• Problems that can be solved by calculating the given numeric information using one or more operations. • Problems that follow predictable pattern and can be solved without actually reading the whole problem situation.	#1, #4, #6, #9, #13, #15, #19
Non –Routine Problems	Many Possible Answers	• Problems that produce more than one answer.	#7, #8
	Puzzles	• Game-like problems. • Problems that may not require any mathematical knowledge.	#14, #18
	Multi Step Word Problem	• Problems that require more than one step operation. • It is required to read the context carefully to make a plan of solution. • Operation with the given numeric data only can result partial results.	#11
	Contain Extra Information	• Problems that have unnecessary information in producing the answer.	#2
	Algebraic Expression	• Problems that contain letter data, instead of numeric data.	#3
	Non-realistic Situation	• Problems that can be calculated but the problem situation cannot be happen in real life.	#12
	Nonsensical Information	• Problems that have contradictory information. • Problems that have missing information.	#16
	Posing Problem	• Problems that ask to pose the question.	#10
	Justification	• Problems that require the explanation of why the particular decision was made.	#5, #17
	Application	• Problems that require gathering necessary data and making a decision base on the data.	#20

Appendix C: Rating Results

Sub Category		Example	Rating 1	Rating 2	Rating 3	Rating 4	Rating 5
Routine Problem		#1	1	1	3	1	16
		#4	2	0	3	5	12
		#6	2	5	6	6	3
		#9	2	1	5	7	7
		#13	0	1	3	5	13
		#15	0	1	3	8	10
		#19	2	1	1	7	11
Non –Routine Problems	Many Possible Answers	#7	7	4	5	5	1
		#8	0	2	6	13	1
	Puzzles	#14	20	1	0	0	1
		#18	4	4	11	3	0
	Two Step Word Problem	#11	2	3	6	6	5
	Contain Extra Information	#2	9	2	6	4	1
	Algebraic Expression	#3	8	7	2	3	2
	Non-realistic Situation	#12	8	9	2	2	1
	Nonsensical Information	#16	16	5	1	0	0
	Posing Problem	#10	1	6	6	7	2
	Justification	#5	9	6	2	5	0
		#17	6	5	7	4	0
	Application	#20	13	6	3	0	0

Appendix D: Justifications for Non-routine Problems

Problem Type	It is not a good problem. (1 or 2 points were given)	It is a good problem. (4 or 5 points were given)
Many Possible Answers (#7, #8)	**Difficulty** "It is too complicated for elementary level." **Purposefulness** "No point in it." "There is no purpose." "It is not really a math problem" **Multiple answers** "Not a good problem because it has more than one answer."	**Emphasis on thinking process** "It is thought provoking." "It enables the child to think and understand another way to receive the answer." "It makes students think." **Creativity** "It allows students to think and integrate own creativity to solve problem." "It enables the children to think outside the box."
Puzzles (#14, #18)	**Purposefulness** "Not a math problem." "Ridiculous." "Has nothing to do with math unless there were numbers to find the answer." "What is the point?" "Vague, not relevant." **Complexity/Clarity** "Too complicated." "Too confusing." "Tricky."	**Organizing thinking process** "I made me keep track of possible solutions."
Two-step Word Problem (#11)*	**Clarity** "It is somewhat confusing."	**Clarity** "The problem is simple." "Have all the components." "To the point." **Problem solving skill** It is a great multi-step problem. It made students think." **Real-life connection** "Money is easy to relate."
Contain Extra Information (#2)	**Clarity** "Not worded clearly." "Too confusing." "Make info more clear." "The question is not clear." "Too complicated." "Two problems in one." "Misleading." "First part is irrelevant."	**Problem solving skill** "It had unnecessary information to eliminate."
Algebraic Expression (#3)	**Interpretation of the letter data** "What is 'a'?" "I don't know what 'a' is. If I knew the value of 'a', It would have been simple." "If 'a' was a number, the problem would be solved." **Purposefulness** "Pointless. It cannot be solved." "Pointless because it does not have one final answer." "Not sure of which concepts working with." **Difficulty level** "Too simple." "Children may not understand a concept." "Confusing for elementary students."	**Algebra connection** "It is a good problem to introduce algebra concepts."

Problem Type	It is not a good problem. (1 or 2 points were given)	It is a good problem. (4 or 5 points were given)
Non-realistic Situation (#12)	**Difficulty level** "It is very confusing and too hard." "It is not for children." **Insufficient information** "How long do they work a day?" "My answer is 1 day and 1 hour, if they work 24 hours a day. But it is not realistic."	**Difficulty level** "It is a challenging question." "It helps to understand that problems can be worked in many ways."
Nonsensical Information (#16)	**Reasonableness** "It doesn't make sense." "Part of the problem is missing." "Not enough information." "It is a trick question." "It cannot be solved."	
Posing Problem (#10)	**Difficulty level** "I did not think that children could understand."	**Understanding of word problems** "It is a good way to understand word problems." "It gives students opportunity to create problem to solve." **Real-life connection** "Children are able to relate their problem in an everyday way." **Thinking process** "It makes children think and encourages them to create."
Justification (#17)	**Clarity** "It is not very specific." "It is too broad." "Too confusing." "Unclear." "I feel that something is missing." "I don't understand what the problem is asking for." **Difficulty level** "It is not appropriate for elementary grade levels unless they know pattern." **Multiple answers** "So many answers. It is stupid." "There are a lot of answers."	**Multiple answers** "It is a good problem with more than one answer." "Good question. It makes students think of all possible answers." **Difficulty level** "It was challenging." **Connections** It is clear and many concepts are incorporated." "I had to go back and think through all properties."
Application (#20)	**Lack of information** "Need to provide amount." "Information is missing." **Relevance** "It is irrelevant for children." "How will a child know this?" "Out students' range of knowledge." "A child doesn't know income." "A child doesn't know how much utilities are."	

- For this problem, only 9 teacher candidates out of 22 produced the correct answer ($ 140). Six participants answered '$210' by multiplying two given numbers. Three participants produced other incorrect answers (e.g., $290, $162, $110), and four participants did not provide the answer.