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Preservice Teachers' Perspectives on Their Opportunities to Learn about Algebra

Hyunyi Jung
Department of Mathematics,
Statistics and Computer Science
Marquette University

Sarah L. Strikwerda
North Carolina State University

Joyce (JuHye) Park
Calvin College

Jill Newton
Department of Curriculum and Instruction
Purdue University

Abstract

This study investigates secondary mathematics preservice teachers' (M-PSTs') opportunities to learn about algebra as reported by five teacher preparation programs. Ten focus group interview transcripts were analyzed using systematic grounded theory. When M-PSTs mentioned specific algebra tasks during the focus group interviews, the instructors' interview data and course materials were used to support the M-PSTs' assertions. M-PSTs presented diverse learning opportunities that aligned with current policy recommendations, but also reported their preferences to learn about algebra in particular ways. Listening to voices of future teachers has the potential to improve teacher preparation programs, ultimately impacting secondary students' opportunities to learn algebra.

Keywords. *Algebra; Opportunity to learn; Mathematics preservice teachers; Teacher preparation programs*

Introduction

Learning opportunities experienced in teacher preparation programs vary greatly in terms of the ways in which content and pedagogy are shared, field experiences are organized, and professional knowledge is conveyed (e.g., Gansle, Noell, & Burns, 2012). There is a need to examine how features of teacher preparation programs influence preservice teachers' perceptions of their learning and teaching (e.g., Clark, Byrnes, & Sudweeks, 2015). Investigating the opportunities in teacher preparation programs and preservice teachers' perceptions can provide critical information on progress and changes that teacher preparation programs make, along with identifying strengths and weaknesses and using that information for further improvement (e.g., Herman, Klein, & Abedi, 2000). One way to investigate such opportunities is by paying attention to what future teachers have to say about their opportunities to learn about content and pedagogy in their programs. In this study, we investigated future teachers' opportunities to learn algebra in secondary teacher preparation programs in the United States.

As we listened to secondary mathematics preservice teachers' (M-PSTs') voices, we focused on their opportunities related to learning algebra, given the importance of algebra as a "civil right" to which all students should have access (e.g., Moses, 1995; Moses & Cobb, 2001). We specifically investigated the aspects of algebra learning opportunities M-PSTs reported and worked from the assumption that the experiences they remembered may influence their future algebra teaching. Recognizing the importance of M-PSTs' opportunities to learn algebra, we aim to answer the following research question: "*Which opportunities to learn about algebra in secondary mathematics teacher preparation programs were noteworthy to preservice teachers?*"

Algebra Learning and Teaching

Algebra is widely used in science and engineering (van der Kooij & Goddijn, 2011) and is the basis of advanced mathematics and many professional disciplines (Veneciano & Heck, 2016). In 1998, the National Council of Teachers of Mathematics (NCTM) initiated a panel discussion to debate whether all students should learn algebra. All panel members supported algebra-for-all because of the perceived importance of learning algebraic language (Morgatto, 2008). In spite of the importance of algebra, many students do not have sufficient opportunities to investigate its structure, symbols, and applications (e.g., Banerjee & Subramaniam, 2012; Kieran, 1992; Loveless, 2008; MacGregor & Stacey, 1997). Concerns about students' achievement in algebra has led to recommendations that all students should learn algebra prior to secondary school in order to make connections between arithmetic and algebra (Usiskin, 1987).

Given the significance of learning algebra along with the daily struggle that many students face in algebra classes, teachers must be more effectively prepared for teaching algebra to all students. Borko et al. (1992) found that some teachers who knew higher-level mathematics, such as calculus and advanced modern algebra, struggled to answer their students' lower-level mathematics questions because they had not had opportunities to learn about the conceptual difficulties students might face in learning school algebra. In addition, policy documents have provided recommendations for content knowledge that teachers need in order to teach algebra effectively. For example, NCTM (2000) concluded that teachers need opportunities to develop a broad conception of the algebraic concepts students learn prior to each class as well as the concepts that students will likely learn in other courses. Similarly, the Conference Board of the Mathematical Sciences (CBMS) (2001, 2012) recommended that teacher preparation programs provide opportunities for M-PSTs to learn about algebra with multiple representations and technologies. Diverse opportunities to learn about algebra should be studied in terms of the perceptions that M-PSTs expressed about the opportunities.

Opportunities to Learn (OTL)

The OTL construct has been interpreted in a variety of ways by researchers. In the First International Mathematics Study, the construct was described as "whether or not…students have had the opportunity to study a particular topic or learn how to solve a particular type of problem" (e.g., Husen, 1967, pp. 162-163, cited in Floden, 2002). In the 1980s, international research also began focusing on additional OTL indicators such as how content was presented and who was presenting it (McDonnell, 1995). Similarly, Brewer and Stacz (1996) presented overlapping categories for OTLs, which included content and instructional strategies. Content included the extent to which learners had experienced specific topics and how instructional strategies related to the types of teaching experiences that would help them learn the topics.

The notion of OTL continued to evolve when it was included in legislation such as *Goals 2000: Educate America Act* (Guiton & Oakaes, 1995). These changes involved more attention to the psychology of learning. Additionally, OTLs were divided into intended opportunities and achieved opportunities. Gee (2008) emphasized that exposure to the same content does not mean

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that the learners have the same OTL; and accordingly, social and emotional aspects of learners and learning should be considered when researchers assess OTLs.

Based on these different approaches to OTL, we began with M-PSTs' perspectives on their opportunities to learn about algebra. We provide additional algebraic task examples, along with the instructors' intentions and descriptions of teaching the tasks in order to comprehensively describe M-PSTs' opportunities to learn particular topics (e.g., Husen, 1968) and how the topics were presented (e.g., Brewer & Stacz, 1996; McDonnell, 1995).

Methods

This study, focused on M-PSTs' opportunities to learn algebra, took place as part of a larger study, *Preparing to Teach Algebra* project, which used both a national survey of 128 secondary mathematics teacher education programs and an in-depth case study of five programs. We have named these five programs Great Lakes University (GLU), Midwestern Research University (MRU), Midwestern Urban University (MUU), Southeastern Research University (SRU), and Western Urban University (WUU). The selected programs and universities varied across several dimensions, including the number of students at the university ranging from approximately 9,000 to 32,000, the admittance criteria ranging from a minimum overall GPA 2.5 to 2.75, and the number of program graduates ranging from 12 to 39 per year. Three programs (i.e., GLU, MUU, WUU) were housed in Masters-granting universities and two in Doctorate-granting universities (i.e., MRU, SRU). At each university, we conducted two focus group interviews each with 3-4 M-PSTs who were near the end of their program (i.e., 10 focus group interviews total). These M-PSTs were recruited by site coordinators who was knowledgeable about their respective secondary mathematics education program with an effort to maximize gender, race, and ethnic diversity; it was not a requirement that the M-PSTs took courses from the set of instructors interviewed in the study.

We also conducted individual interviews with 48 instructors of required courses in these five programs. The number of required courses for M-PSTs to learn algebra and learn to teach algebra varied across the five programs, and each course was selected by the site coordinator in the program. We interviewed 10-13 instructors in four of the five programs; the fifth program was a post-baccalaureate program and we interviewed three instructors. The selected courses included mathematics courses (e.g., Linear Algebra, Abstract Algebra), mathematics-for-teachers courses specifically designed to connect college mathematics with school mathematics (e.g., Algebra in the Curriculum), mathematical education courses (e.g., Secondary Mathematics Methods), and education courses on algebra equity issues (e.g., Teaching in a Diverse Society). Moreover, we collected course materials (e.g., syllabus, tasks) from each instructor.

Data Collection. We used a focus group protocol in order to facilitate discussion among M-PSTs and examine their individual and shared understandings related to opportunities offered in their program (e.g., Flores & Alonso, 1995; Gibbs, 1997). In each focus group interview, the M-PSTs were given a list of required courses in their specific program. We asked them to indicate any experience that provided opportunities to learn algebra and/or to learn how to teach algebra and to make notes. After a few minutes, the group discussed these opportunities. Whenever possible, the facilitator allowed the discussion to continue without interruption; if the discussion strayed away from the topic of opportunities to learn algebra, the facilitator would ask questions to refocus the conversation. Following these discussions, the facilitator would ask a series of set questions to further discussion.

Our instructor interview protocol included similar questions to the focus group protocol. Each instructor and focus group interview took place for 60-90 minutes, and included two pages of facilitator instructions, four pages of 40 interview questions, and three-page handouts describing various aspects of algebra learning and teaching. All interviews were transcribed, reviewed by the research team for validity, and utilized as the primary data source of this analysis. All course materials were also organized into categorized folders.

Data Analysis. As part of the larger study, we have previously reported on M-PSTs' opportunities to learn about: (a) algebraic modeling (e.g., Jung, Mintos, & Newton, 2015); (b) equity issues (e.g., Mintos, Hoffman, Kersey, Newton, & Smith, 2018); and (c) algebraic connections (e.g., Newton, Jung, Stehr, & Senk, 2015). For this paper, we focused on the algebra OTLs that M-PSTs remembered from their program. We did not intend to evaluate individual programs in this study, rather present the overall impressions of the M-PSTs.

Using systematic grounded theory (e.g., Strauss & Corbin, 1998), two researchers first coded the focus group interview transcripts on the paragraph level. Then, they coded each excerpt indicating whether the M-PST described their opportunities as positive or negative. Each excerpt was coded as "positive" when phrases such as these were used: "huge for me," "biggest one," "save us," "improved," "becomes clear," "helps," and "got me so strong." For example we coded this excerpt as "positive":

> *We used GeoGebra and we would use the functions and kind of manipulate them on there, which helps because then if I teach it, now I know how to use GeoGebra and will use it. (GLU)*

This was coded as "positive" because the M-PST stated that learning about GeoGebra was helpful in preparing him or her to teach algebra using the technology in the future.

Explicit negative evaluation of the opportunities, coded as "negative," included phrases such as "not exactly going to bring that to my students," "not really learning," "don't support," "poorly explained," "not realistic," and, "feel ill prepared." For example we coded this excerpt as "negative":

> *Theoretically, the [Algebra, Modeling, and Calculus] in the Curriculum classes are supposed to be between college level and school. But I don't think they do the greatest job. They talk a lot more about the school like Algebra, specifically the Algebra I, talked more about the school Algebra but it didn't really bring in any of the topics from our college level Algebra. (MRU)*

This was coded as "negative" because the M-PST stated that her program did not "do the greatest job."

After independently reading the coded excerpts (149 paragraphs in total), we discussed subcategories that would capture the common ideas of the excerpts; these subcategories were modified until we achieved consensus (Strauss & Corbin, 1998). We present the finalized positive aspect subcategories in Table 1 and describe the negative aspect subcategories in our findings section. More positive aspects of algebra learning opportunities were reported (125 times) than negative aspects (24 times). Two of our team members individually coded the

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excerpts and had 90-98% agreement across the five teacher preparation programs. A third team member reviewed the coded data and discussed the data with the other two members until a consensus was reached. We then counted the number of times each aspect appeared in the interviews from the five programs.

Once the transcripts were coded, we searched for algebra tasks and instructor interviews describing the tasks, in order to include details related to the positive aspects of opportunities that several M-PSTs mentioned. All the algebra tasks were reviewed in order to select ones that clearly show positive algebra opportunities M-PSTs described, along with instructor's purposes and instructional strategies used to implement the tasks. We used the tasks, and participants' reports to describe how the algebra topics were presented, implemented, and perceived (e.g., Brewer & Stacz, 1996; Gee, 2008; Husen, 1968; McDonnell, 1995).

Findings

We aimed to answer our research question (i.e. "Which opportunities to learn about algebra in secondary mathematics teacher preparation programs were noteworthy to preservice teachers?") using the opportunities reported by M-PSTs as a proxy for "noteworthy." Accordingly, we describe the evaluation of opportunities reported by M-PSTs in five secondary mathematics teacher preparation programs and sample tasks that reveal the positive aspects of those learning opportunities.

Positive Aspects of Opportunities Reported in Teacher Education Programs. When analyzing the positive aspects of teacher education programs, several aspects were repeated frequently. These included (a) use representations to make concepts visual; (b) develop a coherent view of algebra and its place in mathematics; (c) use tools or technologies to learn or teach the content; (d) apply the concepts in real-world contexts; (e) engage in discovering the concepts; (f) understand students' approaches to the content; (g) collaborate to learn the content; and (h) teach algebra lessons in secondary classrooms or to other M-PSTs. We present sample statements coded for these aspects of opportunities in Table 1.

Table 1

Aspects of Opportunities to Learn Algebra Reported as Positive by M-PSTs

Category	Sample statement
Use representations to make concepts visual	Again, I would say in my Discrete Math class with set theory, seeing the relation "A union B," or something, being able to draw a Venn diagram to represent A as this part of circle, B as this part of the circle and therefore A union B is the part of the circle, so that helps. (MRU)
Develop coherent view of algebra and its place in mathematics	S2: I was thinking a lot about the connections between college and school algebra and so like Abstract Algebra and Advanced Calculus are both focused a lot on proving commutative laws stuff like that and like integers and real numbers. And those are things that like, in grade school we learn

	the definitions and we can repeat them, but I guess, I probably didn't understand them as well." IN: "And do you think it's purposeful if the professor is trying to help you I mean, is there an effort to try to help you connect what you're doing now to what you did then?" S2: "Um, I want to say no, but for me personally it helped me understand just everything that I've learned and things started making a lot more sense and I think that's really important for a teacher to know. Um, because you can repeat a definition out to a student but if you really understand the properties of it, if they get stuck you can explain it better. (MUU)
Use tools or technologies to learn or teach the content	We used different, we used GeoGebra and we would use the functions and kind of manipulate them on there, which helps because then if I teach it, now I know how to use GeoGebra and use it. (GLU)
Apply concepts in real-world contexts	One of the things I thought too, I forget if it was Mathematics for Secondary Teachers or Middle School Mathematics Methods, one of the thing from Dan Meyer was the problem where we all know about the problem where a cylinder fills up with water this fast, how long does it take to fill up? And we looked at this one blog off of Dan Meyer, which said this is what you want to show students. Show students a picture -- or show students a video of a can filling up with water. Don't give them any information, just the video, and then ask them how long does it take for them to fill it up? And I think that as a general idea kind of helped frame the idea when we want to teach modeling to students, we don't necessarily have to provide them with all of the exact necessary information that we typically would see in a story problem. (GLU)
Engage in discovering the concepts	And it was partly due to the professor. He was just great about whatever. We could have been learning about algebra or geometry and he was just... He really looked in depth. We discovered everything. (SRU)
Understand students' approaches to the content	One of the projects we had within that class is to go observe an ESL student at a school and based on that we wrote a paper and she asked us to reflect what we would do for that specific student as math teachers. So we looked at their English level, their background, their family, where are they coming from, and I believe we did have a discussion on that, so it helped us. Once you have that specific student, then I think that's when you start working. Okay, what would I specifically do? (WUU)

Collaborate to learn the content	You know I've only taken a handful of these math classes, but in Discrete Math, it is very group orientated and I do really appreciate that. I benefit a lot from the group mentality. (MRU)
Teach algebra lessons in secondary classrooms or to peer M-PSTs	I mean the field experiences were obviously the most...far and away the most useful things for me. You just got to see people actually teaching algebra and then you got a chance to try it yourself, so that was definitely the most helpful. (MRU)

We also summarized the total number of positive aspects of opportunities reported by M-PSTs in the five programs. (See Table 2.)

Table 2

Positive Aspects and Extent of Opportunities Reported by the M-PSTs

Reported Positive Aspects of Opportunities to Learn Algebra	*n*	(%)
a) Use representations to make concepts visual	26	(22.2%)
b) Develop a coherent view of algebra and its place in mathematics	24	(20.5%)
c) Use tools or technologies to learn or teach the content	20	(17.1%)
d) Apply concepts in real-world contexts	13	(11.1%)
e) Engage in discovering the concepts	10	(8.5%)
f) Understand students' approaches to the content	10	(8.5%)
g) Collaborate to learn the content	9	(7.7%)
h) Teach algebra lessons in secondary classrooms or to other M-PSTs	5	(4.3%)
Total	117	(100%)

Note. n represents the number of interview segments coded

Overall, using representations to make algebraic concepts visual was mentioned most often, followed by developing a coherent view of algebra and its place in mathematics, and using tools or technologies to learn or teach algebra; these opportunities were mentioned as positive by M-PSTs in nearly every program. Other unique positive opportunities were not listed in Table 2 because they were only reported by M-PSTs in one program (8 excerpts). For instance, M-PSTs at GLU emphasized the use of multiple strategies to solve a problem involving algebraic concepts. Among several positive learning opportunities mentioned by M-PSTs, there were algebraic tasks that both the M-PSTs and instructors described during the interview. In the following section, we describe sample tasks that the instructors provided, followed by corresponding comments from both the instructors and M-PSTs.

Algebraic tasks recommended by M-PSTs and instructors. An instructor of mathematics-for-secondary teachers course at GLU described a task that required M-PSTs to use algebra tiles to develop conceptual understanding related to multiplying and factoring two polynomials. The task included a sentence: “algebra is one of the points where many students are turned off by math.” Students discussed the following question: “Why do you think that might be?” M-PSTs were then asked to collaboratively use algebra tiles, sketch rectangles that represent several polynomial combinations (e.g., x-1 by x+2, x-2 by 2x-1), and find the area of the rectangles. They were also asked to arrange the algebra tiles to represent the polynomials (e.g., $x^2 + 4x + 4$), which helps them visualize factoring the polynomials.

The instructor described algebra tiles as a tool to represent algebraic symbols and relationships. He said

> *It emphasizes the multiplicative structure of algebra...So we spend a lot of time thinking about ‘what does it mean to be factorable?” and we connect it with specific skills like polynomial division or synthetic division and try to tie it to the graphical representation.*

The instructor’s emphasis on developing M-PSTs’ conceptual understanding of factoring polynomials was confirmed by the M-PSTs in his course. An M-PST at GLU stated,

> *Well, like completing the square and just - like using the algebra tiles to multiply two polynomials or even factoring two polynomials. I think that was huge because in my mind the algebra always just clicked but if somebody asked me how I did it, I'd just be like, "It just works!" So..."Don't you see my work?" But the algebra tiles I think they helped create a visual that explains what's happening algebraically.*

This alignment of the task provided by the instructor and the comments from both the instructor and M-PSTs address the following positive aspects of opportunities listed in Table 2: (a) Use representations to make concepts visual; (b) Develop a coherent view of algebra and its place in mathematics; (c) Use tools or technologies to learn or teach the content; and (g) Collaborate to learn the content. M-PSTs saw the value of using representations (e.g., rectangle sketches) to visualize symbols, developed a coherent view of algebra by connecting polynomials with area models, used tools (e.g., algebra tiles) to learn about the concepts, and collaboratively represented and discussed the algebraic relationships.

Another noteworthy task was provided by a Modeling in the Curriculum course instructor at MRU. In the task, groups of M-PSTs burned a candle and recorded its height as it burned. They recorded two variables (i.e., the height of the candle and the time) and sketched their predictions of what the results might look like. They also defined the dependent and independent variables, and described the relationship between the variables. Then they found an equation that seemed to be a good fit for the data and graphed the equation with the data.

The instructor described his instructional strategies to implement this task. He commented

,

> *I have ten different sized jars, and they light a candle and they cover it with the jar and they time how long it takes to go out. So we have, our data is, volume is the independent variable, and time is the dependent variable, which is kind of a weird thing in itself, because time is almost always [the independent variable], but it's the volume of the jars that's determining the time. We have lots of those discussions too about dependent, independent, how important it is for kids to think about that. And the reason I specifically*

> *use that experiment is it's not clean data. It's linear, but there's so many other things going on, and we talk about simplifying assumptions, "What are some underlying variables that are making it not linear? And they conjecture what those might be. But, ok, but we are gonna say we want it to be linear so our data is like scattered. And so I present it to them as, first of all you can't use any magic buttons, I call them on the calculator, but I want you to find an equation, and we are gonna see who can come up with the best equation. So they find equation we kind of compare them…and we put all their equations onto a spreadsheet or sometimes we do it in the calculator with the list, and then we figure, ok from their equation, what would we expect the times to be? So we have the actual times and their expected times.*

M-PSTs saw the value of this task and reported several positive experiences. When one M-PST said that discussing functions using the data they produced helped them learn about algebra, the other M-PSTs in the focus group agreed that they preferred a realistic approach to encounter algebraic concepts in situations. The other MRU focus group also appreciated the OTLs in this course; they mentioned the efficient use of Excel spreadsheets to represent the collected data.

This candle task and the corresponding comments from both the instructor and M-PSTs also address several positive aspects of opportunities included in Table 2: (a) Use representations to make concepts visual; (c) Use tools or technologies to learn or teach the content; (d) Apply concepts in real-world contexts; (e) Engage in discovering the concepts; and (g) Collaborate to learn the content. M-PSTs appreciated their opportunities to represent algebraic concepts (e.g., functions and equations) with graphs. They used technology (e.g., Excel spreadsheets) to represent their own data in the real-world context (burning candles). As they collaboratively produced the data, they engaged with discovering the algebraic equations from the representations of the data.

These two task examples and their implementations reported by both instructors and M-PSTs provide evidence of the positive aspects of learning opportunities described by M-PSTs. M-PSTs also expressed their concerns about the lack of opportunities or less helpful opportunities to learn about algebra. We describe the negative aspects of opportunities described by M-PSTs in the following section.

Negative Aspects of Opportunities Reported in Teacher Education Programs. Overall, the M-PSTs at the five teacher education programs mentioned negative aspects of opportunities 24 times across all interviews. Some M-PSTs reported that their program missed learning opportunities that they felt were important such as learning to teach proofs, functions, algebraic connections, and technology. One M-PST expressed the common concern of not learning how to teach proofs as follows:

> *We do learn what goes in to a proof. I can sit down for an hour and write up a two-page proof on some random algebra theorem and it would be correct, but we never learned how can we teach students how to write just a simple geometric proof and I find that to be a little difficult in the classes I'm teaching.*

M-PSTs also reported that they would like to understand more about functions. An M-PST at GLU said,

> *We sort of use functions but don't really learn a whole lot about them. I mean we only learn about certain types of functions: bijective, subjective, and all that stuff. But it didn't really seem like a lot of stuff that I'm thinking, like my knowledge of functions seems like it just came from prior knowledge from when I came to college.*

In terms of making algebraic connections, M-PSTs mentioned that they had opportunities to learn about school algebra, but less opportunities connecting school level algebra with college level algebra. M-PSTs were also expressed that the technology they learned about was difficult to put "into action in an actual classroom." Finally, M-PSTs expressed concern for what they perceived as the overuse of lectures in courses, stating that they struggled to learn in this way. An M-PST from MUU expressed, "I struggled a lot in calculus and I wish group work would have been more the case rather than lecture, putting up problems on the board, and then, you know, that's the end of class." These concerns related to the lack of opportunities to learn about certain topics (e.g., proof, functions) and too many lectures were reported by multiple M-PSTs across universities.

Discussion and Implications

The purpose of this study was to investigate opportunities to learn about algebra that were noteworthy to M-PSTs at five purposefully selected teacher preparation programs. We reported the aspects of opportunities they reported as positive or negative and described examples of positive aspects of learning opportunities. Overall, positive aspects of opportunities were mentioned much more often by M-PSTs than negative aspects. A limitation of these results is that we did not explicitly ask M-PSTs to report positive or negative aspects of opportunities. We focused more on the *specific aspects* of positive or negative opportunities provided by M-PSTs rather than the extent to which M-PSTs reported positive or negative aspects of opportunities. When we examined the aspects of opportunities that M-PSTs reported as positive, for example, we found that all such aspects were valuable for teacher educators to consider (i.e., use representations to make the concepts visual, develop coherent view of algebra and its place in mathematics, use tools or technologies to learn or teach the content, apply concepts in real-world contexts, engage in discovering the concepts, understand students' approaches to the content, collaborate to learn the concepts, and teach algebra lessons in secondary classroom or to peer M-PSTs). These positive aspects of opportunities have also been recommended by professional organizations, such as CBMS (2012) and the National Council for Accreditation of Teacher Education (2011). Some aspects were mentioned by M-PSTs at multiple universities, but not all universities (e.g., develop coherent view of algebra and its place in mathematics, apply the concepts in real-world contexts). Instructors may use these aspects reported by M-PSTs to reflect on their courses and programs as they consider the nature of their own program.

Additionally, some M-PSTs mentioned that their teacher education programs lacked certain opportunities including the following: to learn functions thoroughly, to learn about how to teach proofs effectively, or to learn algebraic concepts through exploration. Despite the research studies and policy documents supporting the necessity of these types of learning opportunities (e.g., CBMS, 2012; NCTM, 2011), several M-PSTs reported feeling unprepared for teaching algebra in these areas. The argument made by Borko et al. (1992) and Turnuklu and Yesildere (2007) that M-PSTs lack opportunities related to making connections between mathematics and pedagogy still seems to apply to teacher preparation programs. Future research is needed to examine the extent and nature of M-PSTs' opportunities to draw connections between content

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and pedagogy. Although our five case study programs varied across several dimensions (e.g., location, size), we acknowledge that a variety of secondary education programs exist in the U.S. Given that our report did not distinguish middle school and high school M-PSTs, future research may focus on the comparison between OTLs reported by middle and high school M-PSTs. Listening to and valuing the voices of future teachers provides possible directions for future research and potential opportunities for improving mathematics teacher preparation.

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