THE CONCEPTUAL KNOWLEDGE OF PRESERVICE SECONDARY MATHEMATICS TEACHERS: HOW WELL DO THEY KNOW THE SUBJECT MATTER THEY WILL TEACH?

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Abstract

The success of the current reform movement in mathematics education depends on teachers' conceptual knowledge of the school mathematics subject matter. The prospective secondary mathematics teacher typically completes a major or minor in mathematics in order to gain certification. Through the process of interviews in order to explore and describe the knowledge of the school mathematics subject matter held by nine preservice secondary mathematics teachers at varying stages in the completion of their college-level content requirements, it was found that the subject matter knowledge of these prospective teachers was generally lacking in conceptual depth.

Introduction

> Central to the preparation for teaching mathematics is the development of a deep understanding of the mathematics of the school curriculum and how it fits within the discipline of mathematics. Too often, it is taken for granted that teachers' knowledge of the content of school mathematics is in place by the time they complete their own K-12 learning experiences. Teachers need opportunities to revisit school mathematics topics in ways that will allow them to develop deeper understandings . . . (NCTM, 1991, p. 134)

No one would question the centrality of subject matter knowledge for one who, at any level of education, teaches a particular subject. After all, how can one teach what one does not know? Yet, despite the admonition cited above from the NCTM's *Professional Standards for Teaching Mathematics*, the content requirements for prospective teachers of secondary mathematics at teacher education institutions have included primarily topics beyond the usual secondary subject matter, with little opportunity offered for these future teachers to revisit the secondary curriculum at a deeper conceptual level. This may be based on the presumption cited above about teachers' knowledge of school mathematics prior to beginning the content coursework of their certification programs. Or, it may be based on the belief that prospective secondary mathematics teachers will sufficiently deepen their understandings of school mathematics subject matter through the study of advanced mathematics during the content coursework of their certification programs. In either case, the tacit assumption appears to be that, by the time preservice secondary mathematics teachers have completed their college mathematics coursework, they will have the understandings of school mathematics subject matter requisite for teaching that subject matter. In order to examine the validity of this assumption, nine prospective secondary mathematics teachers, at varying stages in the completion of their university mathematics content requirements, were interviewed in a sequence of four in-depth interviews, with the goal of exploring and describing their conceptual knowledge of secondary mathematics subject matter. These explorations and descriptions were guided by the fundamental question of this study: Should the required content coursework for prospective secondary mathematics teachers include specific opportunities to revisit and reconstruct (or perhaps construct for the first time) the content of the school mathematics subject matter?

Participants and Procedures

The participants for this study came from the pool of all secondary mathematics education undergraduate students, classified as juniors or seniors, at a major university in Texas during the spring semester of 1996. Eighteen such students were identified as the population of potential participants and each of these students was sent a letter inviting them to participate in the study. Nine of the eighteen students, eight females and one male, agreed to participate. Each participant, including the one male, was given a female pseudonym in order to protect the anonymity of all participants. Background information was gathered at an initial meeting with each participant; this information is summarized in Table 1.

TABLE 1
Background Information for Interview Participants

	Number of Math Courses[A]:				
Name	Completed	Completed or in Progress	Still to be Completed	Math GPA	Names of Math Courses Completed / in Progress
Carla	4	7	5	3.50	Calculus I & II, Linear Algebra, Statistical Methods / Calculus III, Modern Geometry, Introduction to Analysis
Rita	6[B]	7	2	2.52	Calculus I, II, & III, Linear Algebra, Combinatorics & Algebra, Statistical Methods / Modern Geometry
Shea	6	8	5	3.75	Calculus I, II, & III, Linear Algebra, Elementary Statistics, Statistical Methods / Introduction to Analysis, Modern Geometry
Kerri	7	9	4	3.50	Calculus I, II, & III, Linear Algebra, Combinatorics & Algebra, Differential Equations, Numerical Methods / Introduction to Analysis, Modern Geometry
Kami	8	10	2	3.50	Calculus I, II, & III, Linear Algebra, Statistical Methods, Numerical Methods, Numerical Analysis, Math Modeling / Differential Equations, Modern Geometry
Leah	3	5	4	3.00	Calculus I, II, & III / Linear Algebra, Statistical Methods
Amy	7	8	1	2.40	Calculus I, II, & III, Linear Algebra, Combinatorics & Algebra, Statistical Methods, Differential Equations / Modern Geometry

Erin	7[C]	11	1	2.40	Calculus I, II, & III, Linear Algebra, Combinatorics & Algebra, Statistical Methods, Introduction to Analysis / Abstract Algebra, Modern Geometry, Operations Research, Matrix Theory
Kay	9[D]	11	1	3.63	Calculus I, II, & III, Linear Algebra, Combinatorics & Algebra, Statistical Methods, Numerical Methods, Introduction to Analysis, Advanced Calculus I / Differential Equations, Modern Geometry
Averages	6.3	8.4	2.8	3.13	

[A]Those participants showing only 9 mathematics courses required for their respective degree programs were preparing for a second teaching field in addition to mathematics. Rita's second teaching field was Human Performance, Leah's was French, and Amy's was Spanish. Four participants showed a requirement of 12 mathematics courses. Shea, who indicated a requirement of 13 courses, took Elementary Statistics which was not a degree requirement. Kerri, who also indicated a requirement of 13 courses, was a mathematics major who intended to obtain secondary teaching certification at a later date.

[B]Three of Rita's completed courses (Calculus I, II, and III) were taken at another university.

[C]Three of Erin's completed courses (Calculus I, II, and III) were taken at a community college.

[D]One of Kay's 9 completed courses was actually AP Calculus for which she received college credit for Calculus I.

As Table 1 shows, although the nine participants were at varying stages in the completion of the mathematical coursework required of their respective degree programs, almost all were beyond the halfway mark; most were well beyond. For the group as a whole, the 6.3 mathematics courses completed represents 56% of all mathematics course requirements, while the 8.4 mathematics courses completed or in progress represents 75% of all mathematics course requirements. (The number of courses completed or in progress is important to note because the interviews were conducted during the latter half of the semester when those courses in progress were more than half completed.) An average of only 2.8 mathematics courses remained to be completed after the semester during which the interviews took place. The participants' grade point averages for mathematics courses completed ranged from a minimum of 2.40 to a maximum of 3.75, with an average GPA of 3.13. As of the end of the semester during which the interviews took place, none of the participants had done any student teaching nor had they taken a "methods" course specific to the teaching of secondary mathematics.

Four interviews, each about one hour long, were conducted with each participant over a six-week period during the latter part of the spring semester of 1996. Access to paper and pencil was provided for the participants throughout the interviews, and they were encouraged to write at any time in support of their verbal explanations. Each of the 36 interviews was audiotaped and subsequently transcribed to make it easier to analyze the results. At an average of about 7 transcribed pages per interview, there were about 250 total pages of transcribed interviews to analyze. During the analysis, the transcribed verbal responses were juxtaposed with the written responses in order to get a complete picture of both what was said and what was written.

The mathematical topics explored during the interviews and outlined in the list below were chosen because they are typically topics first encountered at the precollege level. The phrase "mathematical ideas" will be used to refer to the properties, definitions, theorems, formulas, and algorithms comprised in this list.

1. Exponents: the meaning of zero, negative, rational, and irrational exponents; exponential properties.
2. Division and fractions: division by 0, division by a fraction, multiplication by a fraction.

3. Operations with integers: subtraction of a negative integer, product of integers.
4. Slope and lines: slope of a horizontal or vertical line, slope-intercept equation of a line, slopes of parallel and perpendicular lines.
5. Other topics from algebra: multiplication of two binomials (FOIL), the quadratic formula, the distance formula, transformations of the graph of an equation.
6. Trigonometry: the Pythagorean Identity.
7. Formulas from geometry: area of a triangle, circumference and area of a circle.

The conceptual knowledge of the interview participants for the mathematical ideas described in this list was the focal point of the explorations conducted during the interviews. Conceptual knowledge of a mathematical idea has been described as an understanding of why that idea is deemed warranted and how the idea is connected or related to other mathematical ideas (Hiebert, 1986; Shulman, 1986; Skemp, 1987). Having a sound conceptual knowledge base for mathematical ideas typically taught at secondary (or even presecondary) levels is arguably necessary, at least desirable, for any future teacher of secondary mathematics. However, in light of calls for reform in school mathematics teaching (NCTM, 1989, 1991, 1995; NRC, 1989), where classroom instruction is problem-centered and inquiry-based and where conceptual understanding (rather than just procedural knowledge of rules and algorithms) is the goal of instruction, possession of a sound conceptual understanding of the school mathematics subject matter becomes even more crucial for future teachers of that subject matter.

In order to access the conceptual knowledge of the participants regarding a particular mathematical idea, two other aspects of their knowledge surrounding that idea were explored first. Initially, they were asked to demonstrate their procedural knowledge of the idea at hand (e.g., providing the number to which 2^0 is equivalent, finding the product of two given binomials, supplying the statement and purpose for the quadratic formula). For any occasion when a participant failed to show procedural proficiency for an idea, the deficiency was ignored if it did not prevent further discussion of a conceptual nature. Participants were then asked whether a particular mathematical idea could even be explained in a conceptual manner (e.g., whether there is a warranted consistency behind the choice of 1 as equivalent to 2^0, whether there is a justification for the FOIL process, whether there is a derivation for the quadratic formula). A participant's denial of the existence of a conceptual explanation for a particular mathematical idea (e.g., a participant's claim that "This is arbitrary--something you just have to memorize.") certainly precluded any further exploration of the participant's conceptual knowledge for that idea. Finally, for those participants unencumbered by procedural difficulties or conceptual denials, conceptual knowledge regarding a particular mathematical idea was revealed and explored by asking them to explain why the idea is warranted or how it is justified (e.g., why 2^0 is equivalent to 1, why the FOIL process works, where the quadratic formula comes from).

For any occasion when a participant acknowledged and considered the conceptual basis of one of the mathematical ideas explored during the interviews, the participant's response was categorized in one of three ways. When a participant was unable to offer a conceptual explanation (i.e., basically responded "I don't know why that is so.") or when a participant's comments seemed more appropriately characterized as superficial rather than conceptual, then such a response was considered to be a "no explanation" response. For any occasion when a participant made comments that seemed to be directed toward providing a legitimate conceptual explanation for a mathematical idea, yet the would-be explanation fell short in some way(s) of correctly, completely, and validly explaining why the given mathematical idea is true, then such an attempted explanation was categorized as "flawed." Finally, for any occasion when a participant provided an explanation that appeared to be correct, complete, and valid for a given mathematical idea, then such an explanation was categorized as "conceptually sound."

Results

Table 2 shows a summary of participant responses for the mathematical ideas explored during the interviews. The following legend explains the acronyms used in Table 2 to describe the categorization of responses:

SPP	represents	"Showed Procedural Proficiency"
CCB	represents	"Considered Conceptual Basis"
ONE	represents	"Offered No Explanation"
OFE	represents	"Offered Flawed Explanation"
OSE	represents	"Offered Sound Explanation"

Out of a potential for 279 occasions for the 9 participants to demonstrate their conceptual knowledge for the 31 different mathematical ideas shown in Table 2, participants acknowledged and considered the conceptual basis of one of these mathematical ideas on 231 occasions. (On 2 occasions, participants did not have the opportunity to consider an idea conceptually due to unusual circumstances; on 46 occasions, participants did not attempt a conceptual explanation for an idea due to procedural difficulties or failure to acknowledge a conceptual basis for the idea.) A careful analysis of the interview transcriptions shows that for those 231 occasions when participants acknowledged and considered the conceptual basis of one of the mathematical ideas explored during the interviews, they basically had "no explanation" for the mathematical idea being considered 85 times (37%), they offered an explanation that was "flawed" in one or more ways another 85 times (37%), and they were successful in offering an explanation categorized as "conceptually sound" only 61 times (26%). When considering all potential opportunities to offer a conceptual explanation for a mathematical idea, some of which were not enacted due to procedural difficulties or failure to acknowledge a conceptual basis for the idea, the participants' "success rate" drops to 22% (61/277).

TABLE 2
Summary of Responses

Mathematical Idea	Number of Participants Who: SPP	CCB	ONE	OFE	OSE
$2^0 = 1$	9	8	4	1	3
$2^{-3} = \frac{1}{2^3}$	9	5	3	1	1
$2^{1/2} = \sqrt{2}$	7	5	2	3	0
$2^{3/2} = \sqrt{2^3}$ or $(\sqrt{2})^3$	9	6	1	1	4
2^π or $2^{\sqrt{2}}$ is real	7	7	0	7	0
0^0 is indeterminate	2	2	1	0	1
$b^m \cdot b^n = b^{m+n}$	9	9	2	3	4
$b^m \div b^n = b^{m-n}$	8	8	2	4	2
$(b^m)^n = b^{m \cdot n}$	9	9	2	3	4

2 ÷ 0 is undefined	9	8	3	3	2
0 ÷ 0 is indeterminate	5	4	0	3	1
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	9	9	3	4	2
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	9	9	6	2	1
$2 - (-3) = 6$	9	8	2	5	1
$2 \cdot -3 = -6$	9	7	4	1	2
$-2 \cdot -3 = 6$	9	7	5	2	0
Slope: Horizontal Line	9	8	0	0	8
Slope: Vertical Line	9	8	0	0	8
y = mx + b: "b"	9	8	3	3	2
y = mx + b: "m"	9	9	3	6	0
Slope: Parallel Lines	9	9	0	9	0
Slope: Perpen. Lines	6	8	0	8	0
FOIL	9	8	3	5	0
Quadratic Formula	6	9	9	0	0
Distance Formula	8	8	2	2	4
Vert. Shift: $y = x^2 + 4$	9	9	0	3	6
Hor. Shift: $y = (x+4)^2$	2	2	0	1	1
$\sin^2 x + \cos^2 x = 1$	9	9	5	2	2
$A = \frac{1}{2}bh$	9	9	4	3	2
$A = \pi r^2$	9	8	8	0	0
$C = 2\pi r$	9	8	8	0	0
Total	249	231	85 (37%)	85 (37%)	61 (26%,22%)
Total Opportunities	279	277	231	231	231 or 277

Certainly, a summary of how responses were categorized as is given in Table 2 serves only to provide a rough glimpse of the wealth of information about these nine preservice teachers' conceptual knowledge that was revealed in the extensive dialogues contained in the interview transcriptions. For one to get a more complete sense of the nature of the interview participants' understandings, or lack thereof, an examination of a more extensive anthology of responses would be in order (see Bryan, 1997). However, an elaboration of the responses for several of the mathematical ideas explored, accompanied by actual quotes from the participants, can serve to illuminate some of the more general findings of this study.

The Quadratic Formula. Six of the nine participants were successful in writing the quadratic formula; the other three needed some help in correctly completing this task. The participants were unanimous in their expression of appropriate knowledge of the purpose for the quadratic formula and in their agreement that the quadratic formula is based on some logical proof or derivation. However, not one of them had any clue as to how this formula is derived. The participants were about equally split among those who claimed they had never seen such a derivation, those who believed they had seen a derivation and didn't remember any details, and those who just weren't sure either way. Perhaps Rita best typifies the participants' beliefs and knowledge regarding the quadratic formula with the following statement: "I'm sure that somewhere a couple of hundred years ago somebody had a proof for it, but I have not ever seen it."

The Significance of "m" and "b" in y = mx + b. When asked to identify the graphical significance of the constants $\frac{1}{2}$ and -3 in the equation $y = \frac{1}{2}x - 3$, all nine participants quickly and accurately identified $\frac{1}{2}$ as the slope and -3 as the y-intercept. When asked to provide conceptual background that explains why in each case, only two of the nine participants recognized that it must be the case in an equation of the form $y = mx + b$ that when the value of x is 0, the value of y, therefore the y-intercept, is always b; none of the nine participants could provide any explanation as to why the coefficient of x should be the slope. Kerri, like many of the participants with reference to this mathematical idea and numerous others as well, was unable to transcend the fact that "the whole thing was taught [to her] as a convention," something to be memorized.

Multiplication of Two Binomials or "FOIL". When asked to demonstrate the skill of multiplying two binomial factors, $2x + 3$ and $x - 4$, all nine participants did so quickly and accurately, each using the acronym "FOIL" in reference to the process used. However, none of the nine participants could offer a complete and correct justification for the "FOIL" process he or she had just used. In fact, five of the participants failed to acknowledge the role of the distributive property in any way. The following excerpt from Amy's interview is typical of this group:

Amy: Uh, why does it work (pause). Oh, goodness (laughs). I don't know how to . . .

Tom: How to justify it?

Amy: Yeah. I mean, I know that you're given each term which you multiply by the other terms, but I can't really . . .

Tom: Can you think of any properties or rules in mathematics that . . . justify what you were just describing?

Amy: Oh, something like commutative or something like that . . . Something like that . . . Nothing [else] is popping up.

Three participants all basically showed a similar two-step proof in order to demonstrate why the FOIL process works: $(2x + 3)(x - 4) = 2x(x - 4) + 3(x - 4) = 2x(x) + 2x(-4) + 3(x) + 3(-4)$. Each of the three specifically named the distributive property as justification for the second step of their proof; however, they all failed to acknowledge this same property as the source of justification for the first step. When challenged regarding this matter, Carla suggested "the associative property of addition or some type of property of addition." Kay responded, "I don't know. The associative property? I don't know." After a long pause, Rita simply mused, "I never really thought about this before."

Trigonometry: The Pythagorean Identity. All nine participants were successful in supplying the right side of the identity, $\sin^2 x + \cos^2 x = 1$, given the left side; furthermore, they all interpreted appropriately the meaning of an expression like $\sin^2 x$ as the square of sin x. However, only two participants were successful in offering a conceptually sound explanation for the derivation of this identity. Carla, who failed to offer any conceptual basis for the Pythagorean Identity, is a good example of the struggles participants experienced, as well as the confusing responses they gave, in trying to talk about this identity. When asked to describe what the expression "sin x" represents, the following excerpts from the dialogue that ensued indicate that Carla's inability to explain why the Pythagorean Identity is true may have stemmed from her muddled thinking regarding trigonometric functions in general:

Carla: It represents a triangle, well an angle of . . . If you want to find what it represents, you look at a right triangle (makes a sketch) and sin x is going to represent one of these two [acute] angles (pause). I think it's hypotenuse over opposite or is it . . .

Tom: So, are you telling me . . . that expression represents an angle . . . or does it represent that ratio?

Carla: It represents the ratio . . . [but] I'm thinking sine, sine theta . . .

Tom: Why is that different?

Carla: Theta is an angle and x is just x, it's a variable.

Tom: So would that prompt you to do something other than what you started here (referring to the right triangle)?

Carla: (long pause, writes $\sin x = \frac{\text{hyp}}{\text{opp}}$) This is a ratio. . . . Sine is a function.

Tom: Whose value is equal to a ratio? Is that . . . accurate?

Carla: I think so.

Tom: Okay, but yet you're having trouble with the x?

Carla: Well, I mean x is an angle but when you have a graph of it, it's not really an angle, it's a decimal number.

At this point, the conversation with Carla transformed into a discussion of radian measure of angles and conversions between degrees and radians. After she successfully described several equivalencies using radians and degrees, Carla indicated that the radian measure of an angle must be expressed as a multiple of π and that "1" is not valid as the measure of an angle in radians. When the other eight participants were quizzed about radians as a way to measure angles, three others agreed with Carla that "1" is not a valid measure in radians. Of the five remaining participants, only two were able to expand on the meaning of 1 radian, with Rita drawing a sketch of an angle in standard position with this measure and Kay computing $180^{o}/\pi$, or a little less than 60^{o}, as its equivalent. Only Rita talked about the relationship between radian measure of an angle and arc length, but even she failed to describe just how arc length is used in establishing this unit of measurement called a radian.

Although Shea mentioned a connection between the identity $\sin^2 x + \cos^2 x = 1$ and the Pythagorean Theorem almost immediately, she could never make this connection explicit. In fact, her thinking regarding trigonometric functions in general was no less muddled than Carla's:

Shea: [The identity] has something to do with right angles (sketches a right triangle, labeling legs a and b, hypotenuse c). That's a right triangle, and you have the Pythagorean Theorem with $b^2 + a^2 = c^2$ and a^2 is just, this would be the sine (referring to a^2), right?

Tom: What would be the sine?

Shea: The length of this line (side a) is sine of c over a . . . and this (referring to side b) is the cosine of c over b. (At some point, despite what she has said, Shea writes $\sin \frac{a}{c}$ and $\cos \frac{b}{c}$ beside these two sides, respectively.)

Tom: Are you saying cosine <u>of</u> b over c?

Shea: Yeah. Am I on the right track and I'm just getting confused? . . .

At this point, the conversation focused specifically on an attempt to decipher just exactly how Shea was thinking about what sin x, and for that matter x itself, represents. At various times during the conversation, Shea indicated that sin x represents "the hypotenuse over the opposite angle"; the ratio of the hypotenuse over a side "opposite the hypotenuse"; the ratio of the hypotenuse over a side "opposite a certain angle"; and finally the ratio of two sides in a right triangle, one being the hypotenuse and the other being the side opposite x, one of the acute angles in that triangle. Shea was never sure whether it was opposite over hypotenuse or hypotenuse over opposite; furthermore, she never offered any deeper insights into how this ratio and the Pythagorean Theorem combine to form a justification for the identity $\sin^2 x + \cos^2 x = 1$.

Formula for Area of a Triangle. All nine participants offered the formula $A = \frac{1}{2}bh$ as the appropriate formula for the computation of the area of a triangle. However, four of the nine participants were unable to offer any justification whatsoever for the basis of this formula. Out of these four, only Erin seemed to be on the verge of making a discovery in the discussion of her knowledge regarding this formula:

Erin: Well, you're multiplying the length of this (referring to the base of her sketch of what appears to be an equilateral triangle) by its height. But then you're taking away, oh, you're taking away half of it. Because when you do that you find the area of like a square or a rectangle and you want to take off the half that's no longer there. (At this point, Erin draws a square whose "base" is the base of her triangle.)

Tom: But why is it a half?

Erin: I don't know. . . . If you know it's a square, then that's why you know it's a half. Well, no . . . I don't know why it's a half.

Although each of the other participants was able to offer a justification for this formula in the special case of a right triangle, only two of the participants were able to extend their explanation to work for any given triangle.

Discussion

Admittedly, some of the mathematical ideas explored during the interviews, although rather elementary in nature, are deceptively difficult to consider from a conceptual perspective. Furthermore, even though the interviewer went to great lengths to put the participants at ease, they were still put somewhat "on the spot" during the interviews. Nevertheless, the mathematical ideas explored are all clear examples of secondary, sometimes even presecondary, mathematics content ideas (i.e., ideas for which any secondary mathematics teacher should have an "expert" knowledge base, enabling the construction of sound conceptual explanations). The fact that <u>any</u> of the nine participants (moreover some or all of them) had basically a conceptual void for some of the ideas explored during the interviews is an amazing, not to mention troubling, revelation, especially for a group of university students whose prior experiences with mathematics had not only sustained their advancement to a higher plane of mathematical education but also prompted aspirations of teaching the subject themselves (for most of the participants, within a year or two). Furthermore, many of the "conceptually sound" explanations came after false starts and incorrect thinking by participants and clarification requests and prodding questions by the interviewer. Ironically, it appears that the participants' successful completion of university mathematics content courses not only failed to bring substantial insights into the secondary mathematics subject matter, but was attainable without a deep understanding of that presumably prerequisite subject matter--yet certifies the competence of these future teachers to teach the very subject matter that still remains largely memorized rather than understood.

Returning to the fundamental question of this study, it seems clear that the nine future teachers who participated in these interviews would have benefited greatly from opportunities during their university coursework to deepen their conceptual understandings of the content of the school mathematics curriculum. Carla acknowledged this issue and compared the difficulties she experienced trying to construct explanations during the interviews with the difficulties she experienced in her university mathematics classes:

> Carla: There is a certain feeling of insecurity when it is hard to explain something that's so easy, and I'm supposed to be able to do that. I'm working on very hard things right now and I'm having enough trouble getting that, but . . . it's harder to do this than some of the hard stuff that we're supposed to be doing. And this is what I should have a better focus on, on this type of information, because that's what I'm going to be teaching. . . . I've spent a lot of time in the schools observing and everything, but I haven't gotten to think this deeply about stuff.

But the conceptual shortcomings of a small sample of nine students at one teacher preparation program do not constitute grounds for generalizing to any system of mathematics teacher preparation beyond the campus where this particular study was conducted. However, a number of recently completed studies of the knowledge of prospective secondary mathematics teachers have shown the very same results--that prospective secondary mathematics teachers, even with a substantial amount of university mathematics coursework completed, still may not have a level of conceptual understanding of their future subject matter that seems requisite for the teaching of that subject matter, especially in ways consistent with those advocated by the current reform movement in mathematics education. (e.g., see Ball, 1988, 1990; Bush, Lamb, & Alsina, 1990; Even, 1993; Wilson, 1994; Wood, 1993)

Of course, there is still another possible assumption lurking here and not yet addressed--that teachers of mathematics will sufficiently deepen their level of conceptual understandings of subject matter while "on the job" teaching mathematics to their future students. While the presumption that teachers will learn a great deal about their subject from the teaching of the subject seems quite warranted, it is not at all clear just how deeply nor how fast such understandings will take hold. Nor is it practical to expect novice teachers, once again in light of the current reform movement, to relearn (or perhaps learn deeply for the first time) significant portions of the content they teach, while at the same time planning instruction that is quite different and more demanding than what most have generally experienced as students themselves. Furthermore, there is an ethical issue to consider: "Who decides whose children get shortchanged while waiting for teachers to develop [such] understandings?" (McDiarmid & Wilson, 1990, p. 102)

According to the Research Advisory Committee of the NCTM, an important goal of research in mathematics education is "to open new ways of seeing what is currently taken as simple and obvious" (Research Advisory Committee of the NCTM, 1995, p. 302). That students, who are not only qualified to study higher mathematics at the university level but also well on their way to successfully completing the requirements of such advanced study, have an adequate, perhaps excellent, conceptual understanding of secondary mathematics subject matter might seem a simple assumption. That the best content preparation for a future secondary mathematics teacher should be the same as for any other university mathematics major, involving the study of mathematics beyond the secondary level as much and as far as possible, might seem an obvious determination. However, such notions about the knowledge of prospective secondary mathematics teachers and their optimal preparation, particularly in light of the current reform movement in mathematics education, are neither simple nor obvious (e.g., see Ball & Wilson, 1990; Brown & Borko, 1992; Cooney, 1994; MSEB, 1996). While research on prospective secondary mathematics teachers' knowledge and beliefs, such as that done for this particular study, "may not offer a blueprint for revision of mathematics teacher education programs, . . . it certainly alerts us to question our current practices" (Thompson, 1992, p. 141). The questioning of current practices, a fundamental first step toward optimizing the preparation of secondary mathematics teachers at any point in time, is especially critical in the present era of change and reform.

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