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First Generation Common Core Era Teachers' Understanding of Fractions

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Abstract

Implementation of the Common Core Mathematical Standards and Practices (CCSS) has emphasized the importance of instruction that develops both procedural knowledge and conceptual understanding. Novice teachers have experienced the implementation of the CCSS during their PreK-12 experience, as well as within their teacher preparation program. It may be that the renewed emphasis on productive mathematical dispositions and conceptual understanding might be realized in the content and pedagogical knowledge of these novice teachers. This study intends to contribute to the research on teacher knowledge by focusing on the knowledge of fractions held by first generation Common Core Era novice teachers.

Keywords: Mathematical Knowledge for Teaching, Fraction, Teacher Preparation

Introduction

The learning environment provided by a classroom teacher significantly influences the depth, rigor, and quality of a student's educational experience (Sanders, Wright, and Horn, 1997). The instructional decisions made by teachers impacts student learning opportunities in ways such as task selection and implementation, mathematical discourse, and formative assessment. To what extent a teacher is prepared to provide an enriching learning environment is dependent on multiple variables, including their own experiences as a student (Lortie, 1975), quality of their teacher preparation program (Swars, Smith, Smith, & Hart, 2009), and their own content knowledge (Hill, Rowan, and Ball, 2005). Large scale efforts to reform the content and teaching of mathematics education can have a direct impact on each of the above variables. The impact of these changes is more acutely felt by pre-service and novice teachers, whose experience with these reforms are integrated into their role as a student, pre-service and in-service teacher. Such is the case for current novice teachers in the United States whose PreK-12, university, and current professional role have been influenced by the adoption of the Common Core State Standards (National Governors Association, 2010).

In this article, I explore the content and pedagogical knowledge held by the first generation of CCSS students to enter the teaching profession. The lens chosen for this investigation was the understanding and instruction of fractions. I examined the ways in which novice teachers approach fraction concepts as doers of mathematics and how these approaches relate to the ways in which they provide instruction on these topics. I then further examine to see what may influence the novice teachers' understanding and pedagogical choices. The decision to focus on fraction concepts was due to the historical challenge the topic has presented for teachers both in their own understanding and in their instruction (Clarke & Roche, 2009; Strother et al., 2016). The product of this work will contribute to the literature by a) developing a better understanding of the relationship between novice teachers' knowledge of fractions and the application of this knowledge in their instruction, b) describing ways in which novice teachers' knowledge aligns with principles of the CCSS, and c) suggest potential CCSS related learning experiences that may impact novice teacher instruction.

Research Background

Effect of PreK-12 Student Experience. Past research suggests that teachers' intentions to root mathematics instruction in conceptual understanding may be supported or impeded by their own content knowledge and pedagogical beliefs (Borko, Eisenhart, Brown, Jones, and Agard et.al, 1992; Fuller, 1996). The basis of the knowledge and beliefs held by teachers is partially formed prior to any formal teacher training. As described by Lortie (1975), the apprenticeship of observation that pre-service teachers experience as students can affect what and how they instruct as practicing teachers (Chicoine, 2004). It is reasonable to assume that not all of the early observations are of a positive nature and do not necessarily build effective understanding of, and beliefs about, mathematics. Early negative experiences with mathematics may lead to long-term math anxiety and detrimental consequences on mathematical abilities (Hembree, 1990). This can have dire consequences for those who pursue a career in teaching, and for the students of these teachers.

The connections between how a teacher understands mathematics is interrelated to how they perceive themselves as doers of mathematics and of mathematics as a discipline. Ball (1990) found particularly troubling patterns in this area when examining pre-service teachers' knowledge for teaching. In this study, Ball found that "only half of the elementary teacher candidates said they enjoyed and were good at mathematics; over a third of them felt they were not good at math and said they tried to avoid it" (p. 461). As Hilton (1980) suggests, the responses of these teachers is not likely caused by an innate inability, but by "bad teaching, bad text and bad educational instruments (e.g. the standardized test) as among the principal causes of math incompetence and math avoidance" (p. 176).

While up to this point I have focused on the possible negative consequences of early experiences (prior to formal teacher training) with mathematics, I propose the potential for these experiences to support a positive view of mathematics, and belief in one's own perceptions as a doer of mathematics. The National Council of Teacher Mathematics (2000) suggests that the "students' understanding of mathematical ideas can be built throughout school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge" (p. 21). It is reasonable to suggest that this is a necessary step in positioning future teachers (and all students) to have a positive disposition towards mathematics. However, as Sengupta-Irving and Enyedy (2014) argue "promoting mathematical proficiency without developing students' interest (or better yet, passion) marks a failure in the design of the learning opportunity" (p. 2). The authors suggest one possible approach to develop students' interests is to situate the learning opportunities within the current CCSS through the application of student-driven instruction, as opposed to highly guided instruction. Within this study, the possible implications of the CCSS are considered, and will be addressed later in this paper. Next, I will address the role that teacher preparation programs play in the development of teachers' conceptions of mathematics and themselves as doers of mathematics.

Effect of Teacher Preparation Program. By its nature, effective teaching is a task that may appear easy, but is actually complex and difficult (Lortie 1975). Preparing pre-service teachers is an equally daunting task that requires supporting the growth of future teachers' knowledge of a subject area, applying the best practice to teach this content, and manage the learning of a group that may not be intrinsically interested in learning what is being taught. As Labaree (2000) aptly describes, teacher education programs are tasked, "to provide ordinary college students with the imponderable so that they can teach the irrepressible in a manner that pleases the irreconcilable and all without knowing clearly either the purposes or the consequences of their actions" (p.231).

Within the field of mathematics education, researchers have worked to identify knowledge and skills necessary to effectively prepare and support mathematics teachers by identifying overarching necessary knowledge and skills. Shulman (1986) contributed to this area by identifying the need to define teacher knowledge, suggesting a specialized knowledge for teaching that integrated subject matter and pedagogical knowledge that he identified as pedagogical content knowledge. Ball, Thames, and Phelps (2009) extended Shulman's work and identified a framework of mathematical knowledge for teaching defined as "the mathematical knowledge that teachers need to carry out their work as teachers of mathematics" (p. 4).

The work of preparing teachers has further been challenged by the adoption of the CCSS (National Governors Association 2010). The CCSS have realigned the mathematics topics that are taught and the grades in which they are to be introduced. In addition, the CCSS also calls on teachers to assure that students develop certain mathematical expertise throughout their PreK-12 experience. This expertise is defined in the CCSS as the Mathematical Practices.

Common Core State Standards. The CCSS is a product of the work of state school chiefs and governors to establish consistent learning goals in mathematics and language arts for grades kindergarten to twelfth grade (NGA 2010). Within the mathematics content, the CCSS "provide clarity and specificity rather than broad statements…not only stressing conceptual understanding of key ideas, but also by continually returning to organizing principles" (NGA 2010, para. 3). Built within the CCSS for mathematics are Mathematical Practices, types of expertise students are expected to develop. The Mathematical Practices extend beyond knowledge of content to promoting productive approaches to learning mathematics, such as developing perseverance in problem solving, reasoning abstractly, and critiquing the reasoning of others. The development of these productive approaches are designed to be applicable beyond the K-12 experience.

We are now seeing the first generation of CCSS students entering the teaching profession, and utilizing their K-12 experience to inform their own instruction. This provides a rich opportunity to reflect on the potential long-term influence of CCSS. For teacher educators, examining this population can aide in the development of a better understanding of the knowledge for teaching held by these novice teachers. The product of this work will be informative to teacher preparation programs and in-service professional development.

Methods

Research Question. The objective for this study was to contribute to the literature on novice teacher knowledge of mathematics for teaching, and how this knowledge may be impacted by prior experiences. A novice teacher was defined as a practicing teacher with five or fewer years of experience. The choice to narrow the focus of mathematical knowledge for teaching to fraction operations, was due to the this being an area of difficulty for students and teachers (Clarke & Roche 2009; Strother et al., 2016). From this perspective, the following research questions were developed.

1) What approaches do novice teachers utilize when they are solving word problems requiring fractional computation and how do these approaches compare to their instructional approaches on these same topics?

2) In what ways do novice teachers explain what influences their decision-making process when teaching fraction concepts?

Participants. Participating teachers for the study were recruited via email. Thirty-three novice-practicing teachers participated anonymously. Of the 33 teachers; nine taught in the grade span PreK-4th, five taught in the grade span 5th to 6th, and 22 taught in the grade span from 7th to 12th. In terms of certifications, 19 were certified in PreK-6th, 12 were certified in 4th-8th, and 17 were certified in 7th-12th. Several of the teachers held more than one certification and taught in more than one grade span.

Data Collection Instrument. The data collection instrument for this study was an anonymous survey administered using Qualtrics. The survey consisted of 14 questions, including multiple-choice responses, open-ended response, and mathematical tasks. Background data collected included education, certification, and curriculum resources available in their school district. Participants were asked to complete three mathematical tasks, showing all work. The three mathematical tasks are listed in below.

1) Inez read 96 of the 315 pages in her social studies book. She also read 112 of the 432 pages of her science book. Which book is she further along in?
2) The local animal shelter has 3/4 of a bag of food. Each day they go through 1/16 of a bag in order to feed the puppies. How many days will it take them to finish the bag?
3) Dominic began the day with a container of water. During the day, he drank 5/6 gallons of water and was left with 1/3 of a gallon. How much water did Dominic start the day with?

Following the mathematical tasks, the teachers were asked clarifying questions related to their choice of approach to solving, both as a doer and teacher of the mathematics. The three questions as given to the teachers are below.

1) For the three problems you uploaded, did you solve any in a way that is different from how you teach your students? If so, why?
2) What influences the way in which you choose your method(s) to teach fraction concepts?
3) How does your own K-12 experience with learning fractions compare to the way in which your current students are learning these concepts?

Data Analysis. Data analysis for this project was completed in three steps. The first step addressed organizing the categorical data and assigning an anonymous identifier to each teacher. This was accomplished using the resources within the Qualtrics software, and was then imported into a spreadsheet allowing for the additional codes from the other data points within the instrument.

The second step of the data focused on the responses to the fraction tasks. Analysis for these was based on the work of Newton (2008) who classified data involving fraction computation by using a flexibility score which included "finding alternate methods to find solutions to routine problems" (p. 1090) and Whitacre and Nickerson (2016) whose coding approach included "identifying valid strategies that led to correct responses" (p. 66). Building off of this work, the three mathematical tasks were assigned a code in three categories. The first category focused on the whether the answer to the task was the correct answer. If the answer was incorrect, it was then analyzed to see if the error was due to a procedural or conceptual issue. The second category focused on the approach to completing the task. The third code assigned the approach as traditional (T) or non-traditional (NT). Table 1 contains categories for potential approaches for each task.

Table 1
Categories of Possible Responses for Fraction Tasks

Task One	Task Two	Task Three
Common Denominator-T	Invert and Multiply-T	Common Denominator-T
Conversion to Decimal-T	Repeated Subtraction w/model-NT	Equation-T
Greater Number, Bigger Size-NT	Repeated subtraction no model-NT	Model-NT
Conversion to Percent-T	Proportion-T	
Approximation-T	Equation-T	
Benchmark-NT	Repeated Addition-NT	
Cross Multiplication-T	Common Denominator Division-NT	

The third step of the data analysis focused on open-ended responses completed by the teachers following the mathematical tasks. To code this data, we applied inductive grounded theory (Strauss and Corbin, 1994). As data was collected, the data was analyzed for patterns and key words. Using the notes from the coding of the initial collection of data, open codes were developed. As the remaining data was collected, open codes were revised with the aim of developing what Birks and Mills (2015) refer to as a storyline. Table 2 contains the categories for the three open-ended questions.

Table 2
Categories of Responses to Open Ended Questions

Question One Comparing approach to instruction	**Question Two** Influences of approach to instruction	**Question Three** Comparison to K-12
Different-teacher chose faster/simpler approach	Student knowledge	Similar-Positive
Different -teacher use of technology	University coursework	Similar-Negative
Different -student uses of technology	Standardizes testing	Different-Positive
Different -student representations	Curriculum and State Standards	Different-Negative
Same-technology	Personal experience as a student	
Same-refers to specific aspect of approach		
Same-use of representations		

To support the reliability of the data analysis, multiple researchers independently examined codes and data. Initial categories and preliminary codes were developed by several undergraduate assistants. The Principal Investigator refined the initial categories and established a coding schema. A researcher, not associated with this project, analyzed a sample of the data and codes to assure alignment. Any differences were discussed and codes were refined prior to completion of the data analysis.

Results

Response to Task One. In task one, teachers were asked to solve the following question: "Inez read 96 of the 315 pages in her social studies book. She also read 112 of the 432 pages of her science book. Which book is she further along in?" All teachers correctly solved this task. The breakdown of frequency for each solution strategy can be seen in Table 3.

Table 3
Percentage Per Each Solution Strategy for Task One

Solution Strategy	Frequency Applied
Common Denominator	12.5%
Conversion to Decimal	56.25%
Greater Number, Bigger Size	3.125%
Conversion to Percent	12.5%
Approximation	3.125%
Benchmark	6.25%
Cross Multiplication	6.25%

Conversion to decimal, followed by conversion to percent and common denominator were the most common approaches applied by the teachers. In determining the percentage read for each book, teachers also calculated the decimal equivalent. The ways in which decimal equivalents were determined did vary within the data. As seen in Figure 1, one teacher used a partial quotients approach (sometimes referred to as the intermediate algorithm for division), where the majority used the standard division algorithm. The teacher that used the partial quotients approach teaches in the PreK-4th grade span using the Math in Focus resources, while the teacher using the traditional algorithm teaches in the 7th-12th grade span using Glencoe Math resources.

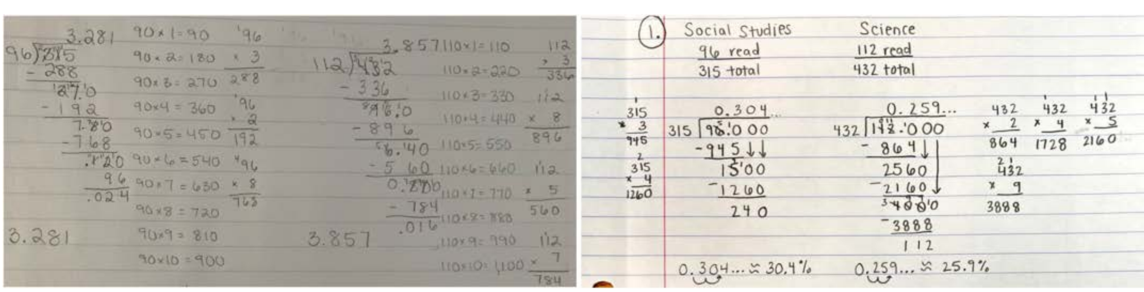

Figure 1
Two Approaches to Finding Decimal Representation

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Two solution approaches that were coded as non-traditional were the greater number bigger size (GNBS) and benchmark strategies. GNBS is a strategy to compare two fractions where one of the fractions has both more pieces (numerator) and the size of these pieces are larger (denominator). For example, 4/7 is greater than 3/8, because 4/7 has more pieces (4 > 3) and larger pieces (1/7 > 1/8). Benchmark strategy is used to compare fractions, by comparing each fraction to another fraction often between the two given fractions. For example, 4/9 is smaller than 9/16 because 4/9 is less than a 1/2 and 9/16 is greater than a 1/2. Figure 2 contains examples of teachers using each of the strategies to solve task one. The teacher using the GNBS strategy teaches in the 7th-8th grade span using a McGraw-Hill resources, and the teacher using the benchmark strategy teaches in the PreK-4 grade span using Math Expressions resources.

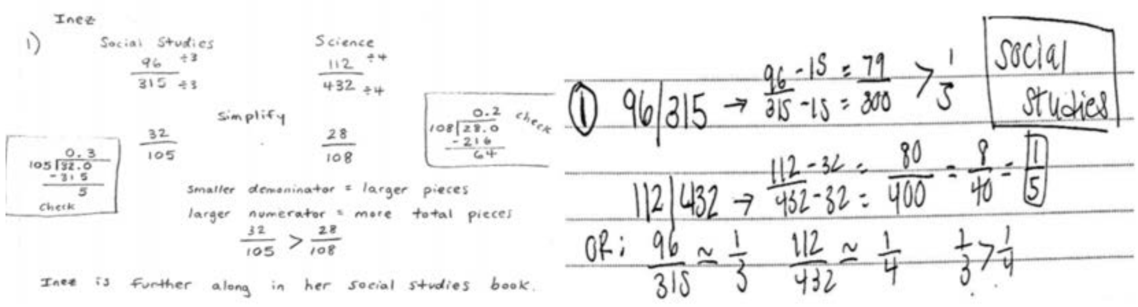

Figure 2.
Non-traditional Approaches

Response to Task Two. In task two, teachers were asked to solve the following question: "The local animal shelter has 3/4 of a bag of food. Each day they go through 1/16 of a bag in order to feed the puppies. How many days will it take them to finish the bag?" All teachers correctly solved this task. The breakdown of frequency for each solution strategy can found in Table 4.

Table 4
Percentage Per Each Solution Strategy for Task Two

Solution Strategy	Frequency Applied
Invert and Multiply	33.3%
Repeated Subtraction-w/model	10%
Repeated Subtraction-no model	6.6%
Proportion	26.6%
Equation	6.25%
Repeated Addition	6.25%
Common Denominator	6.25%

Use of the invert and multiply solution strategy (top left example in Figure 3) was most likely to be applied by teachers across all grade spans. Applying a proportion as a solution strategy (bottom two examples in Figure 3) was most often observed in teachers on the 5th-6th grade level. Setting up an equation to solve the task (top right of Figure 3) was almost exclusively applied by teachers with secondary certifications (7th-12th grade).

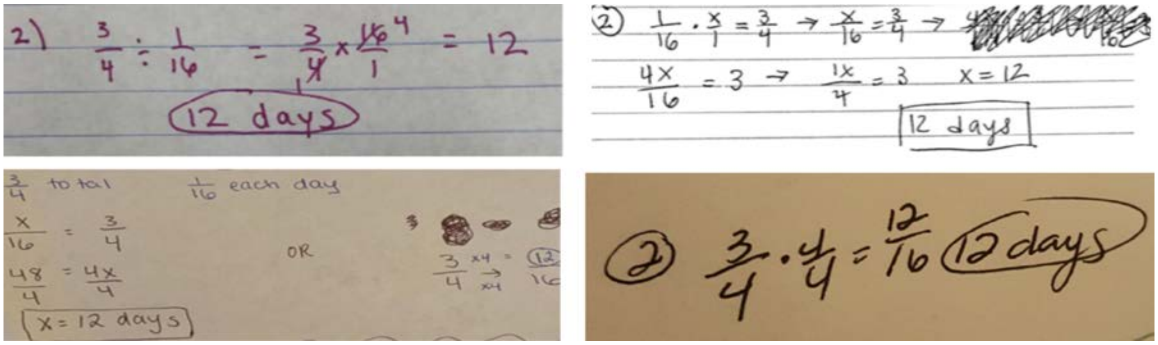

Figure 3
Traditional Approaches to Solving Task Two

The two approaches to solving task two that were coded as non-traditional were use of a pictorial model and use of a common denominator. The top two examples in Figure 4 illustrate teachers using a pictorial model of repeated subtraction to illustrate that there are 12 copies of 1/16 in 3/4. The teacher whose work appears on the top left teaches on the PreK-4th grade level using Math in Focus resources, and the teacher whose work appears on the top right teachers on the 7th-8th grade level using College Preparatory Mathematics resources. The teacher in the bottom left of Figure 4 applied a repeated addition approach; counting the number of copies of 1/16 it would take to make 3/4. This teacher works on the PreK-6 level using the Math in Focus curriculum. The teacher whose work is at the bottom right of Figure 4, uses a common denominator approach to division. In this approach, two fractions who have common denominators have equal size pieces, so when dividing the focus is only on the number of pieces (numerator). This teacher works on the PreK-4th grade level and uses the Math Expressions resources.

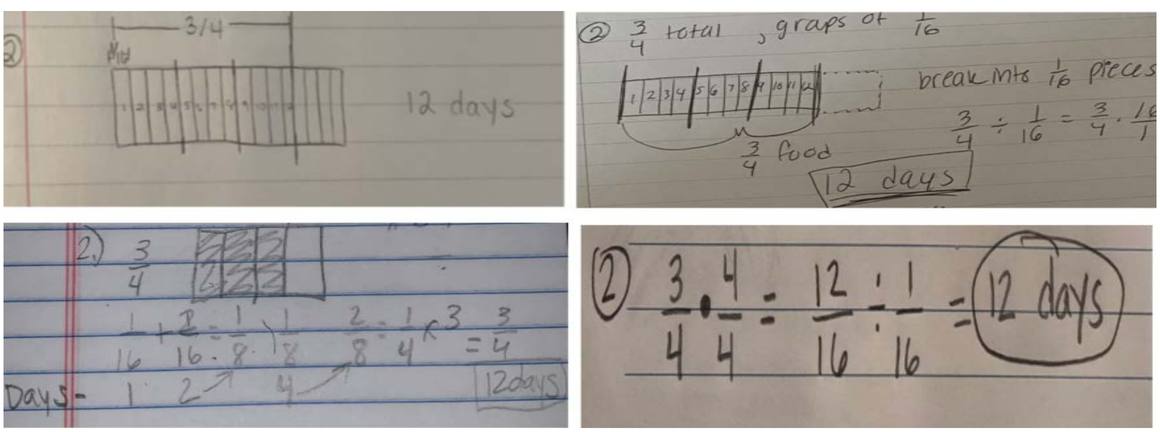

Figure 4
Non-traditional Approaches to Solving Task Two

Response to Task Three. In task three, teachers were asked to solve the following question: "Dominic began the day with a container of water. During the day, he drank 5/6 gallons of water and was left with 1/3 of a gallon. How much water did Dominic start the day with?" All teachers correctly solved this task. The breakdown of frequency was common denominator 57.5%, setting up an equation 33.3%, and representation with a model 6%.

The common denominator approach (example on left side of Figure 5) was the most common

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approach, and was represented in teachers in all grade spans. Middle and high school teachers were more likely to represent the situation using an equation (right side of Figure 5), than PreK-4th grade teachers. The example on the left in Figure 5 is from a teacher on the 7th-12th grade level using a McGraw Hill resources, and the example on the right is from a teacher on the 5th-6th grade level using Pearson resources.

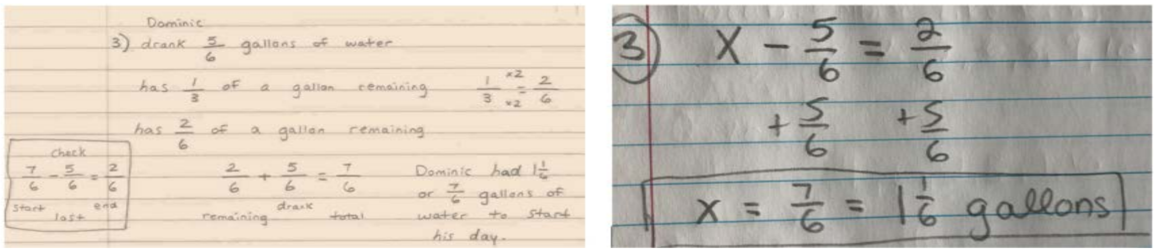

Figure 5
Example of Traditional Solution Strategies for Task 3

Two teachers utilized a pictorial approach as part of their solution strategy. The response on the left in Figure 6 utilizes a bar model to represent part-part-whole relationship by creating equal sized pieces. This teacher works on the PreK-4th grade level and uses Math in Focus resources. The example on the right shows a diagram being used to help make sense of the tasks, but not directly being used to arrive at a solution. This teacher works on the 5th-6th grade level and uses Everyday Mathematics resources.

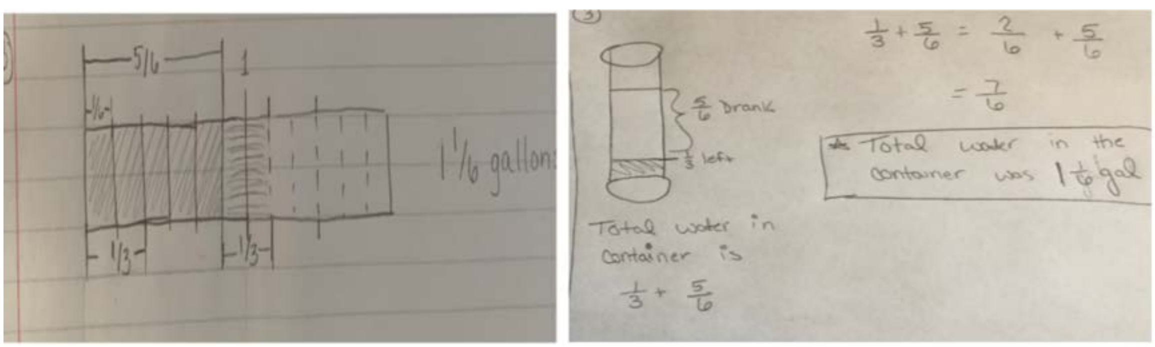

Figure 6
Pictorial Solution Representations for Task Three

Task three provided the only incorrect solution across all of the data. Figure 7 illustrates the teacher misinterpreting the 5/6 as the whole within the problem. This teacher works on the PreK-4th grade level and uses Math Expressions resources.

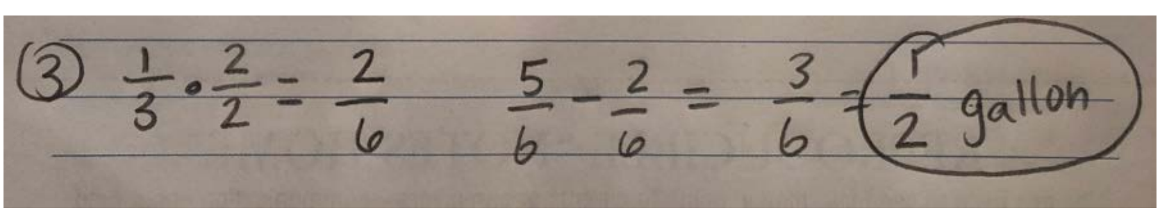

Figure 7
Incorrect Solution for Task Three

Reflection on Instruction to Content Question. Teachers were evenly divided when comparing how their solution strategy for the three tasks aligned or did not align with how they would instruct their students on the same topic. For teachers whose approach did align with their instruction, a theme emerged that focused on strategies that support not only their students' learning, but their own understanding as well. Examples of two responses that represent this theme are below. Note that the examples span the PreK-12 grade span.

> I solved the same way I would teach my students. I find that it is easier to understand the problem when I solve it the way I teach them and not only do my students have an easier time with the problems, but I do as well. (PreK-4, Math in Focus)

> I solved them in a way that I would teach my students, a picture is worth a thousand words and for most students that is the most important idea to instill in them when they need to solve a word problem. You must start with a picture if it is at all possible. (Grade 7-12, Pearson)

The second theme that emerged was teachers voicing concern over the complexity of the task, particularly in task one. Teachers suggested that the complexity drove the way in which they would solve the task and the ways in which they would instruct their students. In the quote below, a teacher explains that due to the size of the denominators the common denominator strategy used in their instruction would be difficult to utilize, so instead the teacher used cross multiplication. It is worth noting here that the Common Core Standards does not recommend the use of this strategy for students as they develop their conceptual understanding of fraction concepts.

> I found the answer to #1 differently than how I would teach my students because the denominators were so large…I also find that cross-multiplying is much faster and more efficient than finding a common denominator and changing the fraction to compare. (PreK-4, Math in Focus)

The use of technology was a prevalent theme to address the perceived complexity of task one. Several teachers voiced concern over the abilities of their students to navigate the computation. This response was evident across grade levels including high school, as can be seen in the excerpt below.

> Yes, I would allow my students access to a calculator. Mental math ability has decreased significantly the more technology is introduced into the classroom. I would also let students change the fractions to decimals to solve since their previous 11 years of education has allowed for that and, in general, they have very weak basic fractional skills. (7-12, College Preparatory Mathematics)

Reflection on Influence. When asked what influences the methods the most common responses were student knowledge, curriculum, and state standards (see Table 5).

References to student knowledge were broadly presented in two ways. The first was the intent to support conceptual understanding of students, building on pictorial and concrete representations. An example of this type of response is below.

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> Fractions are a complex idea and while I am in a place in the curriculum where students are expected to apply their knowledge of fractions and their operations, I still think it is important that they understand WHY things work the way they do. I do not accept "tricks" such as "cross multiply" or "butterfly method" or KCF, because they don't show students what math is happening, and for students to fluently use these operations, they need to be able understand why they work because otherwise they are at risk of over-generalizing procedures. (7-12, Desmos/Illustrative Mathematics)

Table 5

What influences instructional methods for teaching fractions

Influence	**Frequency**
Student Knowledge	43.3%
University Coursework	10%
Curriculum, State Standards, Standardized Testing	30%
Personal experience as a student	16.7%

A second way in which teachers referred to student knowledge focused on the perceived deficits in knowledge and the need to remediate in way that may focus less on developing conceptual understanding. An example of this can be seen in the quote below.

> Background knowledge. There are A LOT of gaps in understanding of fractions, so I spend the majority of our unit just reviewing what fractions are rather than how to divide them (like we should be). Keep in mind, our math classes our grouped homogeneously and I teach the lowest sixth grade math class. Manipulatives that are available also restrict teaching practices. (5-6, EngageNY)

Personal experience also appeared to play a role in teachers' instructional decision process, specifically experiences as a student in the grade that they are currently teaching. These experiences appear to reinforce the need to develop conceptual understanding, and influence the methods to achieve this goal. Examples of this pattern can be seen in the quotes below.

> I remember elementary teachers using manipulatives to teach fractions which helped me to deepen my understanding of the concept. (Math Expressions PreK-4)

> The students in my classroom influence it. I try to have a variety of manipulative for students to use. Sometimes my own understanding can influence my teaching too. Using a double bar graph is a method I tend to shy away from to solve fraction problems because they confused me as a student. (5-6, Envision Math)

The influence of school-based curricula and state based standardized testing clearly influenced teachers. Several teachers voiced concern that straying from their provided curricula, and the included instructional methods, they would be putting their students at a deficit. As one teacher stated "The way the manual tells is how I teach. If I stray from that version, the tests and corresponding workbook pages will be confused" (PreK-4, Everyday Mathematics).

Comparison to PreK-12 Experience. When asked to compare their own experience with fraction concepts on the PreK-12 level to the way in which their current students are experiencing the same content, 80% suggested that there was a difference, while 20% suggested a similarity. Within both of these categories, the teachers discussed the alignment or non-alignment in both positive and negative ways. For example, the teacher below refers to her current teaching in more positive manner than her experience as a student, in terms of currently focusing on conceptual understanding, not just procedural knowledge.

> I learned everything almost entirely procedurally, whereas now we are teaching both to develop the Conceptual Understanding and Procedural Fluency. This has resulted in greater thinking "ability" for my students; however, I do notice that their ability to work with fractions and their comfort level is not the same as me and my peers. I think it is better to have students understand what fractions are and how they work, however having taught higher levels of math it is also key that students have a solid procedural knowledge as well. (7-12, Desmos/Illustrative Mathematics)

Not all teachers viewed their current instructional experience as an improvement to their experiences as a student. For example, the teacher below voices concern over current students being exposed to multiple solution strategies.

> There are WAY more strategies out there than when I was a kid. I think that in some ways, that can be a burden to educators and students. We are encouraged to try new ways constantly. Sometimes, the traditional methods work! (PreK-4, Math in Focus)

Within the data, there were also examples of teachers who viewed positively the alignment between their own PreK-12 experience and their current teaching of fractions. The following teacher reflects back to her elementary teacher’s concrete approach to engaging with fractions. “I remember elementary teachers using manipulatives to teach fractions which helped me to deepen my understanding of the concept. I also try to use manipulatives as much as possible when teaching my students math.” (PreK-4, Math Expressions).

Discussion

The impetus for this study was an interest in examining the ways in which novice teachers understand fraction concepts, how this knowledge aligns with the CCSS, and what may influence their pedagogical decision process. I was specifically interested in novice teachers, as they have experienced the mathematical education reforms present in the CCSS, both as a student and as a teacher. The results of this study suggests partial evidence that novice teachers’ approaches to fractional computation align with the principles of the CCSS.

Within the CCSS, there is renewed emphasis on modeling mathematics and applying varied solution strategies, (National Governors Association, 2010). Analysis of the mathematics tasks showed evidence that some novice teachers in this study were able to apply, unprompted, non-traditional approaches including partial quotients and the use of common denominator to solve division of fractions. When discussing their approach to instructing fraction concepts, teachers in the study highlighted the need for their students to be aware of varied solution strategies. As one teacher reported, “I try to teach multiple ways to work with fractions.” A similar pattern occurred in using pictorial models to solve the math tasks. Without prompting, the data showed (limited) evidence of the use of pictorial models, particularly when representing the repeated subtraction model of division. Teachers commented of the importance of their own students modeling mathematics using bar models and manipulatives. As one teacher suggests “You must start with a picture if at all possible.” Teachers in the study also often referenced the need for their students to not only develop procedural fluency, but also develop a deep conceptual understanding, a significant element within the implementation of CCSS.

Establishing a clear cause and effect between any one variable and a teacher’s approach to doing and teaching mathematical concepts is a complex goal. The results of this study can not definitively connect teacher experiences with the CCSS and their knowledge of fractions. The results of the study do suggest that novice teacher may be positively influenced by variables related to CCSS, including personal experience as a student (PreK-University) and curriculum/textbook resources. Further developing a better understanding of how these novice teachers integrate second-generation CCSS aligned curricula into their own mathematical understanding would be useful in supporting pre-service and practicing teacher professional development.

The data in this study also suggest that some teachers may not view the CCSS emphasis on models and multiple solution strategies as an improvement upon their own PreK-12th grade experience. Within this study, several teachers utilizing CCSS aligned resources suggested, “While I do value those visuals, it confuses some of my students more than helps them” and “I teach math the old ways instead of new ways. I am permitted to do so because I teach functional students.” Research focusing on understanding why teachers who themselves as students were exposed to the CCSS approach to mathematics, are hesitant to implement these approaches in their instruction may also help to better inform the professional development of pre-service and practicing teachers to support the implementation of CCSS with fidelity.

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