Homework-2

November 14, 2024

1. Let X_n be a Markov chain transition probability

$$p_{ij} = \mathbb{P}(X_{n+1} = j | X_n = i).$$

Given the distribution of X_1 is $\mu_i = \mathbb{P}(X_1 = i)$, find $\mathrm{Cov}(X_1, X_2)$.

2. Consider the Markov chain with state space $\{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

How many irreducible classes does the chain have? Find the stationary distribution on each class if it exists.

3. Consider the following random walk with state space $\{0, 1, 2, 3, 4\}$ and the transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ p & 0 & q & 0 & 0 \\ 0 & p & 0 & q & 0 \\ 0 & 0 & p & 0 & q \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Find the expected time of absorption to state 0 or 4 given the initial state is $X_0 = k$.

4. Let ξ_n be a sequence independent Bernoulli random variable with parameter α. Let X_n be the number of consecutive success runs including the present trial as the last. The transition of X_n is

$$\mathbb{P}(X_{n+1} = i + 1 | X_n = i) = \alpha \text{ and } \mathbb{P}(X_{n+1} = 0 | X_n = i) = 1 - \alpha.$$

Find the limiting distribution if it exists.

5. For the Markov Chain in problem 2, find $f_{00}^{(3)}$.

6. Present an example of Markov Chain which is not irreducible and has one unique stationary distribution.

7. Suppose X_n and Y_n are two independent Markov Chains with valued in $\{1, 2, 3, \cdots\}$. Prove or disprove $\{X_n + Y_n\}$ is a Markov Chain.

8. Let

$$P = \begin{pmatrix} 0.4 & 0.5 & 0.1 \\ 0.05 & 0.7 & 0.25 \\ 0.05 & 0.5 & 0.45 \end{pmatrix}$$

Find the limiting distribution π.