Finalexam Practice Problems

April 26, 2025

1. Let (X, Y) admit the following joint density

$$f(x,y) = ke^{-x-y}, \ x \ge y \ge 0$$

(1) Find k.

(2) Find $f_{X|Y}(x)$.

(3) Find $\mathbb{E}[X|Y]$.

2. Let X_1, \dots, X_n be independent random variables with the same distribution

$$f(x) = e^{-x}, \ x \ge 0$$

Let $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ be the order statistics. Find the joint density of $(X_{(1)}, X_{(n)})$.

3. Let (X, Y) admit the following joint density

$$f(x,y) = 2e^{-x-y}, \ x \ge y \ge 0$$

Find the joint distribution of (XY, Y^2) .

- 4. Define the Binomial, Geometric, Poisson, Negative Binomial and Hypergeometric distributions.
- 5. Let (X, Y) admit the following joint density

$$f(x,y) = 2e^{-x-y}, \ x \ge y \ge 0.$$

- (1) Find $P(X \le 2Y)$.
- (2) Find Ee^Y .

6. Let X admits the following density function

$$f(x) = e^{-x}, \ x \ge 0.$$

Find the density of $U = X^2$.

7. A box contains 5 red balls, 3 blue balls and 1 black balls. Take two balls out randomly.

- (1) Find the sample space.
- (2) Find the probability that there exists at least a red ball.
- (3) Find the probability the two balls have two different colors.
- (4) Given one of the two balls is red, find the probability that the two balls have the same color.