Discounted Cash Flow Analysis I

Definition

For give cash flows $\{R_t : t = 0, \ldots, n\}$ (positive or negative), the net present value (NPV) is the sum of all present values of R_ts, i.e.

$$NPV = P(i) = \sum_{t=0} v^t R_t, \qquad v = \frac{1}{1+i}.$$

The interest rate i to make the net value zero at any given time, in particular, to make

$$NPV = P(i) = \sum_{t=0} v^t R_t = 0,$$

is called the yield rate, or internal rate of return (IRR).

Discounted Cash Flow Analysis II

The yield rate is the leveled interest rate such that current values of investments and returns are equal at any given time.

A financial calculator can be used to compute *NPV* or *IRR*.

Discounted Cash Flow Analysis III

Example

[Exam FM Sample Problem 147] Company X received the approval to start no more than two projects in the current calendar year. Three different projects were recommended, each of them requires an investment of 800 to be made at the beginning of the year.
The cash flows for each of the three projects are as follows:

End of year	Project A	Project B	Project C
1	500	500	500
2	500	300	250
3	-175	-175	-175
4	100	150	200
5	0	200	200

The company uses an annual effective interest rate of 10% to discount its cash flows. Determine which combination of projects the company should select.

Discounted Cash Flow Analysis IV

Example (Exam FM Sample Problem 97)

Five deposits of 100 are made into a fund at two-year intervals with the first deposit at the beginning of the first year. The fund earns interest at an annual effective rate of 4% during the first six years and at an annual effective rate of 5% thereafter. Calculate the annual effective yield rate eared over the investment period ending at the end of the tenth year.

Discounted Cash Flow Analysis V

Example (Exam FM Sample Problem 173)

An insurer enters into a four-year contract today. The contract requires the insured to deposit 500 into a fund that earns an annual effective rate of 5.0%, and from which all claims will be paid. The insurer expects that 100 in claims will be paid at the end of each year, for the next four years. At the end of the fourth year, after all claims are paid, the insurer is required to return 75% of the remaining fund balance to the insured. To issue this policy, the insurer incurs 100 in expenses today. It also collects a fee of 125 at the end of two years. Calculate the insurer's yield rate.

Dollar-weighted and Time-weighted Yield Rates I

The above yield rate is also called the dollar-weighter yield rate.

Definition

The dollar-weighted yield rate is the leveled annual interest rate which will generate the same return.

To find the dollar-weighted yield rate i_d, we assume $u = 1 + i_d, v = 1/u$, choose a comparison point, let the values of investments and returns equal at the comparison point, and solve for i_d.

For example, if the beginning invest among is A, C_t are invested at the time t, and the ending balance (return) after n years is B, then the dollar-weighted yield rate i_d satisfies

$$B = Au^n + \sum_t C_t u^{n-t} = A(1 + i_d)^n + \sum_t C_t (1 + i_d)^{n-t}.$$

Dollar-weighted and Time-weighted Yield Rates II

The dollar-weighted yield rate can be calculated by a IRR solver if the cash flows are periodic, and can also be solved by a TVM solver if also the intermediate cash flows are leveled. If it is not the case, an iteration method on a computer needs to be used to solve it exactly.

We can derive an approximated formula by using the simple interest approximation if the $\{C_t\}$ are invested in a year. From

$$B = A(1 + i_d) + \sum_t C_t(1 + i_d)^{1-t}.$$

Dollar-weighted and Time-weighted Yield Rates III

and replacing the ending balance B by $B = A + \sum_t C_t + I$, where I is the interest, and $(1+i_d)^{1-t}$ by $(1+i_d)^{1-t} \approx 1 + (1-t)i_d$, we have

$$\begin{aligned} I &= Ai_d + \sum_t C_t((1+i_d)^{1-t} - 1) \\ &\approx Ai_d + \sum_t C_t((1+(1-t)i_d) - 1) \\ &= Ai_d + \sum_t C_t(1-t)i_d \end{aligned}$$

Simple interest approximation of the dollar-weighted yield rate

$$i_d \approx \frac{I}{A + \sum_t C_t(1-t)}.$$

Dollar-weighted and Time-weighted Yield Rates IV

Definition

The time-weighted yield rate is the leveled interest rate such that the accumulation functions are the same.

Suppose for the kth period, the beginning balance before deposit is B_{k-1}, C_{k-1} is the new investment at beginning, and the ending balance is B_k. Then the accumulation function for the period k is

$$\frac{B_k}{B_{k-1} + C_{k-1}}$$

Dollar-weighted and Time-weighted Yield Rates V

Therefore

$$(1+i_t)^n = \left(\frac{B_1}{B_0+C_0}\right)\left(\frac{B_2}{B_1+C_1}\right)\cdots\left(\frac{B_m}{B_{m-1}+C_{m-1}}\right)$$

and

the annual time-weighted yield rate i_t is given by

$$i_t = \sqrt[n]{\left(\frac{B_1}{B_0+C_0}\right)\left(\frac{B_2}{B_1+C_1}\right)\cdots\left(\frac{B_m}{B_{m-1}+C_{m-1}}\right)} - 1.$$

Dollar-weighted and Time-weighted Yield Rates VI

Example (Exercise 7.25)

Deposits of \$1000 are made into an investment fund at time 0 and time 1. The fund balance is \$1200 at time 1 before deposit and \$2200 at time 2.

a) Compute the annual effective yield rate computed by a dollar-weighted calculation.

b) Compute the annual effective yield rate which is equivalent to that produced by a time-weighted calculation.

Dollar-weighted and Time-weighted Yield Rates VII

Example (Exercise 7.26)

You invest \$2000 at time t=0 and an additional \$1000 at time $t = 1/2$. At time $t = 1$ you have \$3200 in your account. Find the amount that would have to be in your account at time $t = 1/2$ before deposit, if the time-weighted rate of return over the year is exactly 0.02 higher than the dollar-weighted rate of return. Assume simple interest in calculating the dollar-weighted return.

Dollar-weighted and Time-weighted Yield Rates VIII

Example (Exercise 7.27)

You invest \$2000 at time $t = 0$ and an additional \$1000 at time $t = 1/2$. At time $t = 1/2$ you have \$2120 in your account and at time $t = 1$ you have \$3213.60 in your account.

a) Find the dollar-weighted rate of return on this investment. Do not use the simple interest approximation for fractional periods.

b) Find the time-weighted rate of return on this investment.

Dollar-weighted and Time-weighted Yield Rates IX

Example (Exercise 7.28)

An investor deposits 50 in an investment account on January 1. The following summarizes the activity in the account during the year:

Date	Value Immediately Before Deposit	Deposit
March 15	40	20
June 1	80	80
October 1	175	75

On June 30 the value of the account is \$157.50. On December 31 the value of the account is X. Using the time-weighted method, the equivalent annual yield during the first 6 months is equal to the the time-weighted yield during the entire 1-year period. Calculate X.

Dollar-weighted and Time-weighted Yield Rates X

Example (Exercise 7.29)

You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y. Calculate Y.