Yield Curves and Spot Rates I

Some terminologies about interest rates :

LIBOR: London Interbank Offered Rate.

Federal Discount Rate: The discount rate is the interest rate that Federal Reserve Banks charge when they make collateralized loans-usually overnight-to depository institutions (banks, savings and loans, and credit unions).

Prime Rate: The term "prime rate" (also known as the prime lending rate or prime interest rate) refers to the interest rate that large commercial banks charge on loans and products held by their customers with the highest credit rating. Typically, the customers with high creditworthiness are large corporations that are

Yield Curves and Spot Rates II

borrowing from commercial banks in order to finance their operations with debt.

Federal Funds Target Rate: The target rate is specified by the members of the Federal Open Market Committee (FOMC). The effective and target rates do not always coincide. The Federal Reserve may engage in open market operations (buying or selling the US Treasury securities) to eliminate the discrepancy between the two rates.

Yield Curves and Spot Rates III

Federal Funds Rate: Also known as the Federal Funds Effective rate. It is the interest rate that depository institutions (such as banks and credit unions) charge other depository institutions for overnight lending of capital from their reserve balances on an uncollateralized basis.

Base Point: A base point is 0.01%, i.e. 100 bps = 1%.

Credit Spread: An extra rate charged to compensate default, usually expressed in continuous per annum form. For example, if the target continuous rate is 5% and it is estimated 10% loans will default, then the credit spread ϵ satisfies

$$e^{0.05t} = 0.9e^{(0.05+\epsilon)t}$$

Yield Curves and Spot Rates IV

and $0.05 + \epsilon$ is the interest rate charged.

Real Interest Rate: An interest rate with inflation protection.

Nominal Interest Rate: An interest rate without inflation protection.

Yield Curves and Spot Rates V

In practice, the interest rates for different CD terms are different. For example, a bank may offer 2% for 1-year certificate deposit and 2.5% for 5-year deposit.

Definition

A yield curve is a relationship between rates of interest and the term of the investment. The interest rates on the yield curve are called the spot rates.

Yield Curves and Spot Rates VI

Following are examples of Daily Treasury Par yield curves on the web site of the U.S. Department Of The Treasury: ▸ Link

Select type of Interest Rate Data

Daily Treasury Par Real Yield Curve Rates

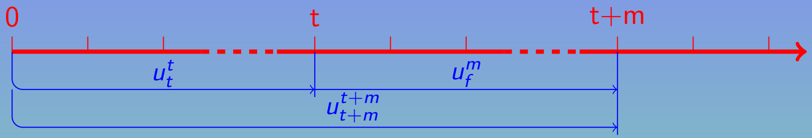
Select Time Period

Current Month

Apply

Date	5 YR	7 YR	10 YR	20 YR	30 YR
09/01/2022	0.87	0.84	0.81	0.95	1.07
09/02/2022	0.75	0.74	0.73	0.89	1.01
09/06/2022	0.87	0.86	0.85	0.99	1.11
09/07/2022	0.86	0.84	0.82	0.95	1.07
09/08/2022	0.93	0.91	0.88	1.01	1.11
09/09/2022	0.94	0.92	0.91	1.03	1.14

Forward Rates I



where u_t, u_{t+m} are the accumulation factors for the spot rates at t and $t+m$, respectively, and u_f is the accumulation factor for the forward rate in $[t.t+m]$.

Definition

Let the spot rate at times t and $t+m$ be s_t and s_{t+m}, respectively. Then the m-year annual forward rate f, deferred t years, is defined by the equation

$$(1+s_{t+m})^{t+m} = (1+s_t)^t(1+f)^m.$$

Forward Rates II

m-year Annual Forward Rate, Deferred t Years

The m-year annual forward rate f, deferred t years, is given by

$$f = \left(\frac{(1 + s_{t+m})^{t+m}}{(1 + s_t)^t} \right)^{\frac{1}{m}} - 1.$$

In particular, the one year forward rate, deferred t years, is given by

$$f = \frac{(1 + s_{t+1})^{t+1}}{(1 + s_t)^t} - 1.$$

Forward Rates III

Example (Exam FM Sample Question 92)

You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%
6	10.00%

Calculate the one-year forward rate, deferred four years, implied by this term structure.

Forward Rates IV

Example (Exam FM Sample Question 119)

The one-year forward rates, deferred t years, are estimated to be:

Year (t)	0	1	2	3	4
Forward Rate	4%	6%	8%	10%	12%

Calculate the spot rate for a zero-coupon bond maturing three years from now.

Interest Rate Swaps I

There are two kinds of loans: a fixed interest rate loan and a variable interest rate loan. An interest rate swap is an agreement between two parties in which each party makes periodic interest payments to the other party based on a specified principal amount. One party pays interest on a variable rate while the other party pays interest on a fixed rate.

Interest Rate Swaps II

Terminologies:

Counterparties: The two parties in a interest swap agreement.

Payer: The counterparty who pays the swap (fixed) rate.

Receiver: The counterparty who receives the swap (fixed) rate.

Notional amount: The principle amount used to calculate the interest.

Settlement dates: The dates the counterparties exchange the payments.

Settlement period: The time between the settlement dates.

Swap tenor: The specified period of the swap, is also called swap term.

Amortizing swap: Notional amounts are decreasing.

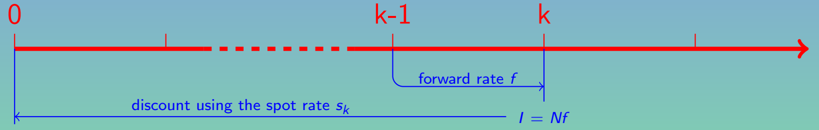
Interest Rate Swaps III

Acreting swap: Notional amounts are increasing.

Deferred swap: The 1st settlement period starts later than the time 0.

Interest Rate Swaps IV

The principle to compute the swap rate is that the total present value of the future interest payments are equal, where a future interest payment is estimated using the forward rate, and the spot rates are used to calculate the discounts.



If $\{t = m, \ldots, t = n\}$ are the settlement dates, N_k is the notional amount at time k, $f_{[k-1,k]}$ is the forward rate for the settlement period $[k-1, k]$, and s_k is spot rate for the time k with the corresponding annual accumulation factor u_k. Then

$$f_{[k-1,k]} = \frac{u_k^k}{u_{k-1}^{k-1}} - 1 = \frac{v_{k-1}^{k-1}}{v_k^k} - 1$$

Interest Rate Swaps V

and the present value of interest at the time k is

$$N_k f_{[k-1,k]} v_k^k = N_k \left(\frac{v_{k-1}^{k-1}}{v_k^k} - 1 \right) v_k^k = N_k \left(v_{k-1}^{k-1} - v_k^k \right)$$

So the present value of the variable interests is

$$\sum_{k=m}^{n} N_k \left(v_{k-1}^{k-1} - v_k^k \right)$$

and the swap rate R satisfies

$$\sum_{k=m}^{n} N_k R v_k^k = \sum_{k=m}^{n} N_k \left(v_{k-1}^{k-1} - v_k^k \right).$$

Interest Rate Swaps VI

Swap Rate

The swap rate R is given by

$$R = \frac{\sum_{k=m}^{n} N_k (v_{k-1}^{k-1} - v_k^k)}{\sum_{k=m}^{n} N_k v_k^k}$$

where $v_k = \frac{1}{1+s_k}$ is the annual discount factor corresponding to the spot rate s_k. If the notional amounts are leveled, i.e., $N_k = N$ for all k, then

$$R = \frac{v_{m-1}^{m-1} - v_n^n}{\sum_{k=m}^{n} v_k^k}$$

If in additional, the first settlement period starts now, i.e., $m = 1$, then

$$R = \frac{1 - v_n^n}{\sum_{k=1}^{n} v_k^k}$$

Interest Rate Swaps VII

Example (Exam FM Sample Question 155)

You are given the following spot rates:

Years	1	2	3	4	5
Spot Rate	4%	4.5%	5.25%	6.25%	7.5%

You enter into a 5-year interest rate swap (with a notional amount of $100,000$) to pay a fixed rate and to receive a floating rate based on future 1-year LIBOR rates. If the swap has annual payments, what is the fixed rate you should pay?

Interest Rate Swaps VIII

Example (Exam FM Sample Question 156)

Company ABC has an existing debt of 2,000,000 on which it makes annual payments at an annual effective rate of LIBOR plus 0.5%. ABC decides to enter into a swap with a notional amount of 2,000,000, on which it makes annual payments at a fixed annual effective rate of 3% in exchange for receiving annual payments at the annual effective LIBOR rate. The annual effective LIBOR rates over the first and second years of the swap contract are 2.5% and 4.0%, respectively. ABC does not make or receive any other payments. Calculate the net interest payment that ABC makes in the second year.

Interest Rate Swaps IX

Example (Exam FM Sample Question 197)

Katarina has borrowed 300,000 from Trout Bank. Katarina will repay 100,000 of principal at the end of each of the first three years.
Katarina will pay Trout Bank a variable interest rate equal to the one year spot interest rate at the beginning of each year.
Katarina would like to have a fixed interest rate so she enters into an interest rate swap with Lily. Under the interest rate swap, Katarina will pay a fixed rate to Lily, and Lily will pay a variable rate to Katarina. The variable rate will be the same rate that Katarina is paying to Trout Bank.
The other terms of the swap will mirror the loan that Katarina has.
You are given the following spot interest rates:

Years	1	2	3	4	5
Spot Rate	4.3%	4.6%	5.1%	5.4%	5.6%

Calculate the swap interest rate for Katarina's swap.

Interest Rate Swaps X

Example (Exam FM Sample Question 198)

You are given the following spot interest rates:

Years	1	2	3	4	5
Spot Rate	4.3%	4.6%	5.1%	5.4%	5.6%

Tommy purchases a deferred interest rate swap with a term of five years. Under the swap, there is no swapping of interest rates during the first two years. During the last three years, the settlement period will be one year. Under this swap, Tommy will be the payer. The variable interest rate will be based on the one year spot rate at the start of each settlement period. The notional amount of this swap is 500,000.

Calculate the swap rate for this swap.

Interest Rate Swaps XI

Example (Exam FM Sample Question 199)

Miaoqi and Nui entered into a four year interest rate swap on May 5, 2015. The notional amount of the swap was a level 250,000 for all four years. The swap has annual settlement periods with the first period starting on May 5, 2015.

Under the swap, Miaoqi agreed to pay a variable rate based on the one year spot rate at the beginning of each settlement period. Nui will pay Miaoqi the fixed rate of 4% on each settlement date.

On May 5, 2017, the spot interest rate curve was as follows:

Time	1	2	3	4	5
Spot Rate	3.8%	4.1%	4.3%	4.5%	4.7%

Miaoqi decides that she wants to sell the swap on May 5, 2017. Calculate the market value of the swap on May 5, 2017, from Miaoqi's position in the swap.