Homework 2

Due 10/27 Monday

1. Suppose f and g are two measurable functions on $(\Omega, \mathcal{F})$. Prove that $f+g$ is also measurable.

2. State Fatou's lemma. Give an example that Fatou's lemma fails if $f_n \geq 0$ is not satisfied.

3. State the dominant convergence theorem. Give an example that the theorem fails if g is not integrable.

4. In the probability measure space $(\Omega, \mathcal{F}, P)$ (i.e. $P(\Omega = 1)$). Prove that if $X_n \to X$ in probability and $|X_n|$ is uniformly bounded, then $\mathbb{E}X_n \to \mathbb{E}X$.

5. Suppose X_n is a uniformly bounded sequence of real-valued random variables. If $X_n \Longrightarrow X$, then $\mathbb{E}X_n \to \mathbb{E}X$.

6. Present an example that $\{f_n\}$ is uniformly integrable, while $\{f_n\}$ can not be bounded by an integrable function.

7. Prove that if $X_n \to^P X$ and $Y_n \Longrightarrow Y$, then $X_n + Y_n \Longrightarrow X + Y$.

8. Provide an example that $X_n \Longrightarrow X$ and $Y_n \Longrightarrow Y$, while $X_n + Y_n \Longrightarrow X + Y$ fails.