

Adaptive Techniques to Robust Upper Limit Estimation of the Total Signal Rate in an Inhomogeneous Poisson Process with Unknown Background

Abstract

Detecting weak signals in the presence of uncertain or poorly characterized backgrounds is a central challenge across many scientific fields, including particle physics, astronomy, epidemiology, and environmental science. Classical likelihood-based approaches, such as the Likelihood Ratio Test, rely on strong parametric assumptions about the background and may provide unreliable results when those assumptions are violated.

This study develops robust, nonparametric procedures for setting conservative upper limits on the total signal rate in an inhomogeneous Poisson process where the signal density is known but both the background density and background rate are unknown. The framework is designed to be adaptable across one- and higher-dimensional settings.

Two complementary strategies are proposed. The first is a spacing-based approach in one dimension, which leverages order statistics and non-decreasing intensity estimation, and naturally extends to higher dimensions through Voronoi-based methods. The second is a binning-based regression approach, where the domain is partitioned into Voronoi bins, observed event counts are regressed against local signal mass, and bootstrap techniques are applied to construct upper limits.

To benchmark performance, these methods are compared against an oracle likelihood ratio test, in which the background distribution is assumed to be known up to its parameters. Through extensive simulation studies under both background-only and signal-present scenarios, and across diverse background structures, we assess the behavior of median upper limits and median ratios under different signal-to-background regimes.

Results show that the spacing-based method provides limits competitive with the oracle benchmark, even under challenging conditions, while the regression approach offers computational efficiency but is more sensitive to background structure. The proposed procedures are broadly applicable and provide robust, conservative, and practically useful upper limits in settings where background characteristics are highly uncertain.