Stochastic Modeling of Seasonal Influenza: Parameter Estimation and Application to Epidemiology

Abstract

Seasonal variation in the environment impacts many populations through the growth and maturation of plants, the behavior of animal populations, and the outbreaks of human infectious diseases. Periodic mean-reverting stochastic differential equations (SDEs),

$$dX(t) = r_p(\beta(t) - X(t))\, dt + d\beta(t) + \sigma_p X^p(t)\, dW(t), \tag{1}$$

with $r_p, \sigma_p > 0$ for $p \in [1/2, 1]$ and periodic mean $\beta(t)$, are mathematical models that realistically capture seasonal variability. We estimate parameters and develop a methodology to construct confidence regions for the model parameters, specifically accounting for the effects of seasonality through the fit of seasonal influenza data. Parameters in the SDEs in Eq. (1) are fit to the data in two steps. First, the proposed periodic function $\beta(t)$ is fit to the mean of the data with a least squares method. Second, the parameters r_p and σ_p of the SDEs are fit to the data with a maximum likelihood method. In addition, we demonstrate a method to fit the seasonal influenza data to an epidemic model with a periodic mean-reverting transmission rate and waning immunity. Periodic mean-reverting SDEs can be used to model many other biological systems to investigate seasonal or cyclical patterns.