PRODUCT OF CONFORMAL RADII OF OPEN SETS HAVING A FIXED AREA

Abstract. In this talk, our main focus will be on different versions of the following problem.

Problem: Given positive number $A > 0$ and a set of points a_k, $k = 1, \ldots, n$, on $\mathbb{C}$, find an open set D^* with area A that maximizes the product of conformal/inner radii with respect to the points a_k taken over all open sets D with area A; i.e. find D^* such that

$$\prod_{k=1}^{n} R(D^*, a_k) = \max_{D\colon \operatorname{area}(D)=A} \prod_{k=1}^{n} R(D, a_k).$$

This problem is a more general version of the following classical problem solved by George Pólya, who proved the following result: Among all the simply connected domains $D \ni 0$ having a fixed area, the disk centered at 0 has the maximum conformal radius with respect 0.

The problem stated above remains open even for two points. In this talk we mention few special cases when this problem is solved and discuss some geometric properties of extremal open sets in the general case.