PRODUCT OF CONFORMAL RADII OF OPEN SETS HAVING A FIXED AREA

ABSTRACT. In this talk, our main focus will be the following conjecture suggested by Dr. Solynin.

Conjecture 1. Consider the family of convex domains \mathcal{F} having the following properties.

(1) For every $D \in \mathcal{F}$, Area(D) = A.

(2)
$$-1, 1 \in D$$
 for every $D \in \mathcal{F}$.

Let $D^* \in \mathcal{F}$ be the convex domain that maximizes the following product,

 $R(D^*, 1, -1) = \max_{D \in \mathcal{F}} R(D, 1)R(D, -1)$

Prove or disprove that D^* is an ellipse whose focii is ± 1 .

If this conjecture is true, that would be another interesting result of the focus of an ellipse. Moreover, the conformal representation of an interior of an ellipse contains Jacobi Elliptic functions. Hence, some inequalities related to special functions can be obtained by using the properties of conformal radius of an ellipse with respect to a given point.