<u>2026 SIAM Graduate Student Research Day (GSRD)</u>

Speaker Name: Karthik Vasudeva

Title: Mixed Finite Elements for Optimal Control Problems Constrained by the Biharmonic Equation

Time: March 24 (12.30PM – 12.50PM)

Abstract: Optimal control problems constrained by the biharmonic equation arise in applications such as thin plate modeling, vibration control, and structural design. The fourth-order nature of the governing partial differential equation poses significant analytical and numerical challenges, including the requirement of C^1-conforming discretizations and, in boundary control settings, the presence of fractional Sobolev trace spaces.

Mixed finite element formulations provide a natural framework for addressing these difficulties. In particular, the Hermann–Miyoshi approach introduces a stress variable identified with the Hessian of the state, reformulating the biharmonic equation as a system of second-order equations. This reduction permits the use of standard finite element spaces and leads to stable saddle-point formulations under suitable inf–sup conditions, whereas the Ciarlet–Raviart approach provides an alternative mixed formulation without explicitly introducing the Hessian.

In this talk, I will discuss how this mixed framework allows for the unified analysis and discretization of distributed optimal control problems governed by the biharmonic equation. The resulting optimality systems couple state, adjoint, and, when using Hermann–Miyoshi, stress variables, and they admit consistent and efficient finite element approximations with provable stability and convergence properties.

I will also present how the framework extends to boundary optimal control problems. A lifting-based reformulation transforms boundary controls into equivalent volumetric variables, yielding an optimal control problem posed entirely in integer-order Sobolev spaces at both the continuous and discrete levels. This approach avoids the explicit use of fractional Sobolev norms and leads to stable and convergent numerical schemes for both the state and auxiliary variables. Numerical experiments illustrate the effectiveness of the proposed methods and demonstrate their robustness for optimal control problems constrained by fourth-order partial differential equations.

Speaker Name: Kaveendri Rathnayake

Title: The Dynamics of Intraguild Predation under Stoichiometric Conditions

Time: March 24 (12.50PM – 1.10PM)

Abstract : Intraguild predation (IGP) is a common and taxonomically widespread interaction in natural food webs, in which species that exploit the same prey, also prey upon one another. This blend of competition and predation is a contributing factor to niche shifts, cascading interactions and alternative stable states in food webs [2]. Ecological stoichiometry provides a reliable framework for modeling these complex dynamics. Contradictory conclusions about the effects of IGP in previous studies motivated us to investigate how the optimal strength of IGP influences coexistence and community structure.

This study develops a simple stoichiometric model for a three-species system consisting of one producer and two herbivores to address the question of how variation of the intraguild predation strength shapes food web dynamics across two trophic levels. In the absence of intraguild predation, the system reduces to a two-dimensional Lotka–Volterra type LKE (Loladze–Kuang–Elser) competition model [1]. Introducing IGP links, defined as feeding interactions within the same trophic level, eliminates limit cycles, which is a striking observation. As the strength of the IGP link increases, the limit cycles disappear, and species coexistence is only possible when the interaction strength (the feeding preference of the consumer species on the resource species) lies between 0.5 and 0.7. Moreover, coexistence occurs at higher carrying capacities within this range. Another key finding is that the bifurcation diagrams show cycles at low IGP strength; these cycles disappear at intermediate strengths and collapse at 0.7. When the direction of IGP link is reversed, the dynamics are also reversed. However, unlike the previous case, the cycles do not collapse as the strength increases.

Overall, our findings indicate that both the strength and direction of intraguild predation directly correlate with system stability, coexistence patterns, and bifurcation structure, highlighting the importance of stoichiometric constraints in shaping food web dynamics across trophic levels.

Keywords: Intraguild predation strength, stichometry, coexistence, bifurcation, phosphorus

References

[1] Irakli Loladze et al. "Competition and stoichiometry: coexistence of two predators on one prey". In: Theoretical Population Biology 65.1 (2004), pp. 1–15.

[2] Gary A Polis and Robert D Holt. "Intraguild predation: the dynamics of complex trophic interactions". In: Trends in ecology & evolution 7.5 (1992), pp. 151–154.

Speaker Name: Kalana Kushan Munasinghe

Title: Robust Upper Limit Estimation for Signal Rates in an Inhomogeneous Poisson Process with Unknown Background

Time: March 24 (1.10PM – 1.30PM)

Abstract: Detecting weak signals in the presence of unknown or poorly characterized background noise is a fundamental challenge in fields such as particle physics, astronomy, epidemiology, and environmental science. Classical likelihood-based procedures require strong parametric assumptions about the background distribution, which may be unrealistic or unverifiable in practice.

This talk presents a robust, nonparametric framework for constructing conservative upper limits on the total signal rate in an inhomogeneous Poisson process when the signal density is known but both the background density and background rate are completely unknown.

In one dimension, we introduce a spacing-based method that exploits the geometry of order statistics. By trimming a fraction of the domain determined by a tuning parameter β, we construct a non-decreasing test statistic based on cumulative spacings and invert its empirical distribution to obtain upper limits.

This geometric idea extends naturally to higher dimensions through Voronoi-based intensity estimation. In two and higher dimensions, we estimate local event intensity using Voronoi tessellations, construct signal-mass trimming regions, and define an analogous test statistic based on integrated intensity over retained regions.

To evaluate performance, we compare these background-agnostic procedures to an oracle Garwood benchmark in which the background distribution is assumed to be fully known. Simulation studies across multiple signal-to-background regimes are summarized using contour plots of mean upper limits and relative difference surfaces.

Results show that the proposed spacing–Voronoi framework produces conservative upper limits that remain competitive with the oracle benchmark, even when no background information is used. The approach provides a geometrically intuitive and dimension-adaptive solution for robust signal inference under severe background uncertainty.

Speaker Name: Shashipraba Rajakaruna

Title: Topological functional analysis of longitudinal brain MRI data

Time: March 24 (1.30PM – 1.50PM)

Abstract: Neurodegenerative diseases such as Alzheimer's disease evolve gradually, with structural brain changes accumulating over years. Conventional longitudinal MRI analyses based on voxelwise comparisons or region-level summaries may miss multiscale structural patterns that emerge across spatial scales. In this work, we develop a longitudinal topological framework to characterize disease related brain dynamics using functional summaries derived from persistent homology. From serial T1-weighted MRI scans, we compute Betti curves across multiple homology dimensions to quantify the evolution of connected components, loops, and higher-order cavities across filtration thresholds. These curves provide interpretable multiscale signatures of brain structure for each subject and visit. After aligning curves on a common grid, we apply longitudinal functional principal component analysis to decompose variability into subject specific random intercept and slope effects, visit level deviations, and residual noise, producing low-dimensional modes that capture dominant patterns of topological change over time. We then use the resulting principal component scores to examine associations with clinical and demographic factors and to evaluate group differences in longitudinal trajectories. As ongoing work, we assess whether the population mean Betti curve profile is time invariant and examine how this assumption affects the extracted modes of variation and group-level inference. Overall, the proposed approach provides a principled way to study disease evolution through the longitudinal dynamics of multiscale topological summaries derived from structural MRI, quantifying both progression and heterogeneity across individuals.

Keywords: Persistent homology; Betti curves; Longitudinal functional data analysis; Functional principal component analysis; Alzheimer’s disease

Speaker Name: Sikha Saha

Title: Modeling and Simulation of Measles Transmission Dynamics in the United States: An Age-Structured Mathematical Modeling Approach.

Time: March 24 (1.50PM – 2.10PM)

Abstract: The United States has a long-standing issue with measles, even with high levels of vaccination. This is primarily due to immunity gaps, population heterogeneity, and the risk of severe outcomes requiring hospitalization. In this study, we develop and analyze a deterministic mathematical model to investigate measles transmission dynamics in a heterogeneous population. We divide the population into children, low-risk adults, and high-risk adults, and further classify individuals in each group into susceptible, vaccinated with imperfect protection, exposed, infectious, hospitalized, and recovered compartments. To capture both the burden of disease and the transmission, we include some of the most important epidemiological and demographic processes, such as recruitment, childhood to adulthood ageing, transition between adult risk classes, and mortality due to disease. We define the basic analytical properties of the model, such as positivity and boundedness of solutions, and calculate the basic reproduction number to describe the potential of transmission. We examine both local and global stability of the disease-free

equilibrium and establish the conditions under which measles can be eliminated, which includes a clear herd immunity level. We supplement the analytical results with the numerical simulation and the global sensitivity analysis to study the impact of the important parameters, determine the effect of the vaccination coverage, and the impact of the intervention strategies. Our results give a quantitative understanding of the role of age and risk heterogeneity in determining the processes of measuring the transmission and control of measles, and offer an informative framework to guide the processes of vaccination planning and decision-making in health on the part of the public in a heterogeneous population.

Speaker Name: Amit Kumar Saha

Title: Analytical Modeling of Microplastic Transport in Rivers: Incorporating Sinking, Removal, and Multi-Phase Dynamics

Time: March 24 (2.10PM – 2.30PM)

Abstract: Microplastics (MPs) are transported through rivers acting as major conduits to oceans, yet standard transport models often fail to capture polymer-specific dynamics like settling and removal. This study proposes two novel analytical frameworks to address this: a Modified Advection-Dispersion Equation (ADE) incorporating first-order sinking and removal, and a Multi-Phase Model accounting for hydrodynamic-particle coupling. We derived exact closed-form solutions for a finite pulse input and validated the baseline model against established results. Our results demonstrate that the conventional ADE significantly overestimates peak MPs concentrations, while the Modified ADE reveals a "stretching" effect that extends the duration of ecosystem exposure. Our analysis indicates that sinking is the primary driver of mass loss to sediments, with higher sinking rates reducing aqueous concentrations by approximately 50% compared to non-settling scenarios. However, removal employs negligible influence during the initial pulse phase but shows cumulative impact over long transport distances. The study highlights the critical need to incorporate sediment accumulation terms into risk assessments, as ignoring sinking leads to underestimating benthic pollution and overestimating marine flux. Additionally, the multi-phase formulation provides a theoretical basis for modeling dense plastic spills where particles alter flow momentum.

Speaker Name: Syed Ehsan Ar Rafi

Title: Estimation of unknown sample size when only the empirical cdf is known: MLE approach

Time: March 24 (2.30PM – 2.50PM)

Abstract: In many modern data environments, complete datasets are inaccessible, and analysts must rely solely on aggregated information such as empirical proportions or grouped counts. When the underlying population distribution is unknown and the total sample size is itself unobservable,

statistical inference becomes challenging. Motivated by this problem, we propose a parametric maximum likelihood–based estimator for an unknown sample size using only estimated category probabilities derived from an assumed parametric model.