GSRD Participants

Zhuanzhuan Ma

Research area: Statistics.

Title : Sparse Bayesian Variable Selection in High Dimensional Regression with Correlated Priors.

Abstract : With the rapid development of computational technology, Bayesian methods have received much attention in the model selection for high-dimensional data, which are frequently collected in various research and industrial areas. This presentation consists of two distinct but related research projects. In the first project, we construct a Bayesian hierarchical modeling framework for the problems of variable selection in logistic regression of high-dimensional settings, in which the model dimension exceeds the sample size. We assigned a modified global-local shrinkage prior including horseshoe and normal-gamma priors on the regression coefficients, and proposed an efficient sampling algorithm based on the Gibbs sampler to draw samples from the full conditional posterior distributions to make the posterior inference. We proved that under some regularity conditions, the proposed Bayesian method enjoys the standard model selection consistency in large k and small n settings. Simulation studies and two real-data applications are conducted to compare the performance of the proposed Bayesian method and existing ones in the literature. Given the potential ability of the proposed Bayesian method to be extended to other situations, in the second project, we actively invested how to generalize our computational framework for parameter estimation and variable selection in Bayesian binary quantile regression. Generalizing this method for high dimensional multi-response problems (e.g., quantitative and qualitative responses) will be an area of future investigation.

Rahnuma Islam

Research area: Applied Mathematics.

Title: Einstein's Brownian motion model in Chemotaxis system and its traveling band form.

Abstract: We study the movement of the living organism in a band form towards the presence of chemical substrates based on a system of partial differential evolution equations. We incorporate Einstein's method of Brownian motion to deduce the chemotactic model exhibiting traveling band. It is the first time that the Einstein method has been used to motivate equations describing mutual interaction of the chemotactic system. In addition to considering Chemotactic response and the random motion of organisms arising from the chemotactic response, we also consider the formation of the crowd by organism via interactions within or between the community. This crowding effect can also be seen as an organism traveling or migrating in a herd/group in search of food. We have shown that under specific hypnotize on the expected value of free jump of living organism from Einstein approach results in the model similar to Keller- Segel model. On the other hand, under no specific adjustment, the traveling band has been yielded and explained accordingly.

Isanka Garli Hevage

Research area: Applied Mathematics.

Title: Stability analysis of nonlinear Einstein model of Brownian motion.

Abstract: We consider the degenerate Einstein's Brownian motion model when the time interval τ of free jumps (particle-jumps before the collisions), reciprocals to the number of particles per unit volume $u(x,t) \geq 0$, at the point of observation x at time t. The parameter $\tau \in (0, C]$, which controls the characteristics of the fluid, "almost decreases", with respect to u, and converges to ∞ as $u \to 0$. This degeneration leads to the localization of the particle-distribution in the media. We study the stability for the governing partial differential inequality problem. In this talk we will report our new result on asymptotic stability with respect to L^p norm for the solution of the corresponding IBVP.

Jonathon Loftin

Research area: Applied Mathematics.

Title: Exact CutFEM Polynomial Integration.

Abstract: The implementation of discontinuous functions occurs in many of today's state-of-the-art partial differential equation solvers. However, in finite element methods, this poses an inherent difficulty: there are no quadrature rules readily available when integrating functions whose discontinuity falls in the element's interior. Many approaches to this issue have been developed in recent years. Among them is the equivalent polynomial technique. This method replaces the discontinuous function with a polynomial, allowing integration to occur over the entire domain rather than integrating over complex subdomains. Although eliminating the issues involved with discontinuous function integration, the equivalent polynomial tactic introduces its problems. In particular, either adaptivity is required to capture the discontinuity or error is introduced when some regularization of the discontinuous function is implemented. In the current work, we eliminate both of these issues. The results of this work provide exact algebraic expressions for subdomain and interface polynomial integration, where the interface represents the boundary of the cut domain. We also provide algorithms for implementing these expressions for standard finite element shapes in one, two, and three dimensions, along with a hypercube of arbitrary dimensions.

Nadeesha Jayaweera

Research area: Statistics.

Title: Confidence Envelope for Parametric Model Selection Criterion.

Abstract : In choosing a candidate model in likelihood-based modeling via an information criterion, the practitioner is often faced with the difficult task of deciding just how far up the ranked list to look. Motivated by this pragmatic necessity, we construct an uncertainty band for a generalized (model selection) information criterion (GIC), defined as a criterion for which the limit in probability is identical to that of the normalized log-likelihood. This includes common special cases such as AIC BIC. The method starts from the asymptotic

normality of the GIC for the joint distribution of the candidate models in an independent and identically distributed (IID) data framework, and proceeds by deriving the (asymptotically) exact distribution of the minimum. This is a non-standard result from the theory of order statistics, since although the original data are IID, the sample of GIC values are in fact dependent. The calculation of an upper quantile for its distribution then involves the computation of multivariate Gaussian integrals, which is amenable to efficient implementation via the R package "mvtnorm". The performance of the methodology is tested on simulated data by checking the coverage probability of nominal upper quantiles, and compared to the (parametric) bootstrap. Both methods give coverages close to nominal for large samples, but the bootstrap is two orders of magnitude slower. The methodology is subsequently extended to two other commonly used model structures: regression and time series. In the regression case, we derive the corresponding asymptotically exact distribution of the minimum GIC invoking Lindeberg-Feller type conditions for triangular arrays, and are thus able to similarly calculate upper quantiles for its distribution via multivariate Gaussian integration. The bootstrap once again provides a default competing procedure, and we find that similar comparison performance metrics hold as for the IID case. The time series case is complicated by a far more intricate asymptotic regime for the joint distribution of the model GIC statistics. Under a Gaussian likelihood, the default in most packages, one needs to derive the limiting distribution of a normalized quadratic form for a realization from a stationary series. Under conditions on the process satisfied by ARMA models, a multivariate normal limit is once again achieved and proved for implementation. The bootstrap can however be employed for its computation, whence we are once again in the multivariate Gaussian integration paradigm for upper quantile evaluation. Comparisons of this bootstrap-aided semi-exact method with the full blown bootstrap once again reveals a similar performance, but faster computation speeds.

Shadi Heenatigala

Research area: Applied Mathematics.

Title: A charge-conserving low-rank tensor method for the Vlasov–Maxwell system.

Abstract: Collisionless Plasma dynamics is described by the celebrated Vlasov-Maxwell (VM) system. The Vlasov equation is defined in 6D phase space, and hence VM simulations are extremely expensive due to the curse of dimensionality. In this work, we exploit the low-rank structure of the Vlasov solution to address the challenge. In particular, hierarchical Tucker decomposition is adopted to express the low-rank solution in high dimensions. The approach takes advantage of the fact that the differential operators in the Vlasov equation are tensor friendly, based on which we propose a novel way to dynamically and adaptively build up low-rank solution basis by adding new basis functions from the discretization of the PDE and removing basis from an SVD-type truncation procedure. To enforce the local charge conservation for the low-rank method, we further adopt a novel conservative truncation technique. An extensive set of benchmark tests are performed to demonstrate the efficiency and efficacy of the proposed charge-conserving low-rank VM solver.

Karthik Kumar Vasudeva

Research area: Applied Mathematics.

Title: On a finite element model for two-dimensional thermoelastic problem.

Abstract: The study of elastic materials under thermal loading has been an intense area of research from past several decades. The issues of expansion and contraction of materials when exposed to external mechanical forces and extreme temperatures are of notably important in designing aircrafts and in the development of novel materials. Traditionally, the response of an elastic material is modeled within the framework of linearized theory of elasticity which is a first-order relationship between Cauchy stress and stretch tensors. However, the same relationship predicts singular strain when applied to model cracks and fractures. This clearly a violation of the basic assumption with which the theory is predicated upon. In this talk, a main focus is on studying the numerical solution to a nonlinear thermoelastic

model which is a coupled linear-quasilinear partial differential equation (PDEs) system. Due to the nonlinearity in the model, obtaining a closed form solution is a daunting task within the purview of the new framework developed in this research. Therefore, we seek a stable numerical solution to the partial differential equation system by using conforming bi-linear finite element method. The numerical results depict a marked contrast to the classical linearized description of the material body. Finally, intention is to provide a theoretical basis to develop physically meaningful models to study the evolution of complex network of cracks induced by thermal shocks.

Md Sakhawat Hossain

Research area: Statistics.

Title: Parameter identification for a model for multi-functional materials with hysteresis and thermodynamic compatibility

Abstract: Multifunctional materials such as magnetostrictive, electrostrictive, and piezolectric materials have tremendous potential for engineering applications as they are able to convert mechanical to electro-magnetic energy and vice-versa. In recent years, they have found a niche in the area of energy harvesting. One of the features of this class of materials is that they show significant hysteresis which needs to be modeled correctly in order to maximize the energy harvesting potential. A method of modeling multifunctional materials that exhibit the phenomenon hysteresis and is compatible with the laws of thermodynamics was developed recently. The model is based on the Preisach hysteresis operator and its storage function. The difficulty is that the parameters in the model appear in a non-linear fashion, and there are several constraints that must be satisfied by the parameters for thermodynamic compatibility. In this article, we present a novel methodology that uses the rate independent memory evolution properties of the Preisach operator to split the parameter estimation problem into two linear least squares problems with constraints. Then we used alternative direction methods of multipliers(ADMM) algorithm and accelerated proximal gradient method to estimate the Preisach weights. Numerical results are presented for Galfenol.